Carroll *et al.* Reply: Comment [1] claims that the laser threshold emerging from a new coherent-incoherent model (CIM) [2] is "unattainable" when the term $\sum_{n\neq l}\delta\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$ is added to the equation for the photon-assisted polarization $\delta\langle bc^\dagger v\rangle$. Moreover, it identifies the classical polarization $|P|^2$ with $\sum_{n,l}\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$, thus claiming that neglecting $\sum_{n\neq l}\delta\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$ violates the quantum-classical correspondence.

Here we show that (1) the threshold exists, persists, and is *attainable* even with the wrong assumptions of [1]; (2) correctly taking into account terms of the order of $\sum_{n\neq l}\delta\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$ and the sum's spatial nonlocality confirms that the CIM provides accurate values of the laser threshold.

In nanolasers, terms like $\sum_{n\neq l}\delta\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$ are normally neglected. They represent collective effects, like superradiance, usually not observable in the presence of strong polarization dephasing due to high carrier density screening [3,4]. The CIM [2] matches the parameters of standard GaAs-based quantum dots (QDs), with a very rapid decay [5] and negligible correlations of the intrinsic polarization.

Furthermore, $|P|^2$ does not correspond to $\sum_{n,l}\langle c_l^\dagger v_l v_n^\dagger c_n\rangle$. Imposing operator normal ordering gives $\sum_{n,l}\langle c_l^\dagger v_l v_n^\dagger c_n\rangle = \langle c_l^\dagger c_l\rangle - \sum_{n,l}\langle c_l^\dagger v_n^\dagger v_l c_n\rangle \neq |P|^2$, where $\langle c_l^\dagger c_l\rangle$ is the excited state population and $\sum_{n,l}\langle c_l^\dagger v_n^\dagger v_l c_n\rangle$ is the sum of the expectation values of the product of polarizations between QDs placed at different positions: a spatially nonlocal term. This decomposition proves the point. The polarization is local, does not depend on population, and is related to $|\langle v^\dagger c\rangle|^2$, included in the CIM [[2], Eq. (2)] but arbitrarily and inconsistently removed from Eq. (1) in [1].

The correct dynamical form for $\langle c_l^\dagger v_n^\dagger v_l c_n\rangle$ is

$$\begin{aligned}(d_t + 2\gamma + i\Delta\varepsilon)\langle c_l^\dagger v_n^\dagger v_l c_n\rangle\\ = g_{ls}^*[\langle b_s^\dagger v_n^\dagger c_n\rangle(1-2\langle c_l^\dagger c_l\rangle)\\ -2\langle v_n^\dagger c_n\rangle\langle b_s^\dagger c_l^\dagger c_l\rangle + 2\langle b_s^\dagger\rangle\langle c_l^\dagger c_l\rangle\langle v_n^\dagger c_n\rangle]\\ + g_{ns}[\langle b_s c_l^\dagger v_l\rangle(1-2\langle c_n^\dagger c_n\rangle) - 2\langle c_l^\dagger v_l\rangle\langle b_s c_n^\dagger c_n\rangle\\ + 2\langle b_s\rangle\langle c_n^\dagger c_n\rangle\langle c_l^\dagger v_l\rangle],\end{aligned} \quad (1)$$

where the coefficients g_{ns} depend on the cavity-mode field at the QDs' positions [6]. Spatial nonlocality introduces into Eq. (1) products of coupling coefficients g_{ls} and polarization operators $v_n^\dagger c_n$ from different QDs. Neglecting these phase differences [1] assumes that QDs and g_{ls}, which depend on the mode [6], are identical. These extremely strict conditions cannot be satisfied by all QDs for physically realistic boundary conditions.

Adding Eq. (1) of [1] to the CIM [2] displaces the threshold (black star in Fig. 1) from its original position [2] (blue star), rendering the postbifurcation dynamics unstable due to the arbitrary removal of terms of comparable size. Consistently computing (as in [2]) the variables at the appropriate order [cf. Eq. (1) above], but keeping the unphysical assumption of identical QD coefficients [1] stabilizes the dynamics, moving the threshold to a lower pump (red star). Relaxing this unphysical condition returns the threshold to approximately the CIM value (red diamond and cross). In summary, thresholds leading to coherent fields can always be observed. Contrary to claims in [1], the model of [2] is correct and widely applicable.

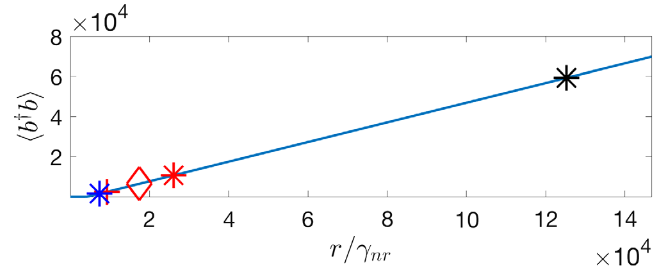

FIG. 1. Photon number versus pump for 40 QDs. The blue star is the laser threshold of the CIM [2], the black star of the CIM plus Eq. (1) of [1], the red star when variables ignored in [1] are included, the red diamond (red cross) assumes that only 90% (50%) of the QDs are identical. All parameter values are the same as in [2].

Note that neglecting $\delta\langle b^\dagger bc^\dagger c\rangle$ and $\delta\langle b^\dagger bv^\dagger v\rangle$ is standard procedure with cluster expansions [7] at the two-particle level [3]. Finally, there is a misinterpretation regarding the emission after the bifurcation in [1]: close to threshold only a fraction of the photon field is coherent and single frequency.

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