

Domain Textures in the Fractional Quantum Hall Effect

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Impacts of domain textures on low-lying neutral excitations in the bulk of fractional quantum Hall effect (FQHE) systems are probed by resonant inelastic light scattering. We demonstrate that large domains of quantum fluids support long-wavelength neutral collective excitations with well-defined wave vector (momentum) dispersion that could be interpreted by theories for uniform phases. Access to dispersive low-lying neutral collective modes in large domains of FQHE fluids such as long wavelength magnetorotons at filling factor $\nu = 1/3$ offer significant experimental access to strong electron correlation physics in the FQHE.

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Current geometrical theories of strongly correlated phases in fractional quantum Hall effect (FQHE) fluids of two-dimensional electron systems (2DES) identify low-lying neutral collective excitations (known as magnetorotons [1]) as chiral gravitons [2–7]. The bulk of FQHE fluids is nonuniform and formation of domain textures [8–11] have a significant impact on low-lying collective excitations [12–14]. The impact of bulk domain textures in strongly correlated phases is the subject of increasing attention, with prominent examples in FQHE fluids [15–20] and in high temperature superconductors [21–24]. To investigate the insulating bulk of quantum Hall phases, studies on thermal transport and electronic interference by edge modes are interpreted under bulk-edge correspondence [25–31]. Experimental methods that directly probe low-lying neutral collective excitations in the bulk of FQHE fluids are crucial tools in the quest to understand strong electron correlation physics.

In the second Landau level (SLL) at $\nu = 7/3$ a seemingly conventional FQHE is actually very distinct from its counterpart at $\nu = 1/3$ in the lowest Landau level (LLL) [32–38]. Even under a small in-plane magnetic field, the appearance of anisotropic longitudinal resistance indicates the emergence of domains of nematic phases [39,40] that coexist with domains of the bulk FQHE fluid [41]. Newly discovered collective modes under a small in-plane magnetic field are interpreted as long-wavelength plasmons of nematic domains in the filling factor range $2 < \nu < 3$ [42]. Studies of the plasmons under a small in-plane magnetic

field serve as a probe of the impact of domain textures on low-lying collective excitations.

Access to magnetoroton modes of FQHE fluids and to plasmons of nematic liquids is provided by resonant inelastic light scattering (RILS) methods [13,42]. Here, we report that RILS by nematic plasmons at $\nu = 7/3$ under a small in-plane magnetic field reveals a wide range of nematic domain sizes from around $1 \mu\text{m}$ to characteristic sizes larger than several microns, and establishes FQHE-nematic fluids at $\nu = 7/3$ as an ideal platform to study impacts of domain textures on low-lying neutral excitations. The domains with dimensions much larger than the inelastic scattering wavelength (about $1 \mu\text{m}$) support low-lying collective excitations with well-defined wave vector (momentum) dispersions. Very sharp dispersive long-wavelength magnetoroton modes have been reported in the LLL at $\nu = 1/3$ [13]. We surmise that the observed modes, with well-defined wave vectors, are excitations of the FQHE fluid in large domains. Results at $\nu = 1/3$ reported in previous works are examined to highlight possible experimental insights on long-wavelength magnetoroton excitations that are identified as chiral gravitons under strong electron correlation in recent theory for uniform FQHE phases [4–7].

Characteristic domain sizes are obtained by modeling the RILS intensity of nematic plasmons. While the matrix element depends on plasmon-electron interactions [12], we focus here on the photon frequency dependence and on breakdown of wave vector conservation due to formation of domain textures. We find that intensity is [43]

$$I \propto \left| \frac{1}{(\omega_I - \omega_1 - i\gamma_1)} \frac{1}{(\omega_o - \omega_2 - i\gamma_2)} \right|^2 \times \frac{1}{[\omega(q) - \omega(k)]^2 + (\Delta\omega/2)^2}, \quad (1)$$

ω_1 (ω_2) and γ_1 (γ_2) are the energies and broadenings of the incoming (outgoing) resonant channels of optical excitations of the GaAs quantum well that hosts 2DES. When both incoming (ω_I) and outgoing (ω_o) photon energies are in the vicinity of these optical excitations, a large intensity enhancement occurs through double resonant inelastic light scattering (DRILS). $\omega(q)$ is the plasmon energy at wave vector q [43]. Energy conservation in inelastic scattering requires $\omega(q) = \omega_I - \omega_o$. $\Delta\omega$ is the range of the plasmon energies due to breakdown of wave vector conservation.

The inelastic scattering wave vector \mathbf{k} is along the in-plane magnetic field [42]. Wave vector conservation ($\mathbf{k} = \mathbf{q}$) occurs for domain sizes $D \gg \lambda = 2\pi/k$, where λ is the inelastic scattering wavelength, $k = |\mathbf{k}|$ and D is a characteristic length of nematic domains along the direction of \mathbf{q} . Wave vector conservation is broken under a finite domain size, so mode wave vectors have an uncertainty $\Delta q = 2\pi/D$ [49]. We have

$$\Delta q = \left(\frac{dq}{d\omega} \right) \Big|_{q=k} \Delta\omega = 2k\Delta\omega/\omega(k) \quad (2)$$

for nematic plasmons with dispersion $\omega(q)$ [43]. Estimates of values of D can be obtained with Eq. (2) and determinations of $\Delta\omega$ from inelastic light scattering spectra.

At $\nu = 7/3$ there is a set of DRILS spectra for which the breakdown of wave vector conservation has very minor impact. Here, estimated nematic domain sizes are $D \approx 5.3 \mu\text{m} \gg \lambda = 1.2 \mu\text{m}$. In the same device, strong impact of the breakdown of wave vector conservation in another set of spectra is found for domains that have D comparable to λ . The optical resonances in large nematic domains involve quasiparticle states close to the Fermi level. This could be evidence showing that large nematic domains are responsible for anisotropic electrical conduction in the FQHE at $\nu = 7/3$ under tilted magnetic fields [41]. Domains of FQHE fluid with $D > \lambda$ and those with $D < \lambda$ should coexist in the nonuniform bulk of FQHE states. Albeit the nonuniformity, low-lying neutral collective excitations in large domains with $D \gg \lambda$ accessed by RILS and DRILS would manifest the fundamental correlation physics in the uniform bulk of FQHE phases in the LLL and the SLL.

The ultraclean 2DES is confined in a 30-nm-wide, symmetric, modulation-doped single GaAs/AlGaAs quantum well. Carrier mobility of the wafer measured by transport is $\mu = 23.9 \times 10^6 \text{ cm}^2/\text{Vs}$ at 300 mK. The sample is mounted on the cold finger of a $^3\text{He}/^4\text{He}$ -dilution refrigerator with windows for direct optical access and

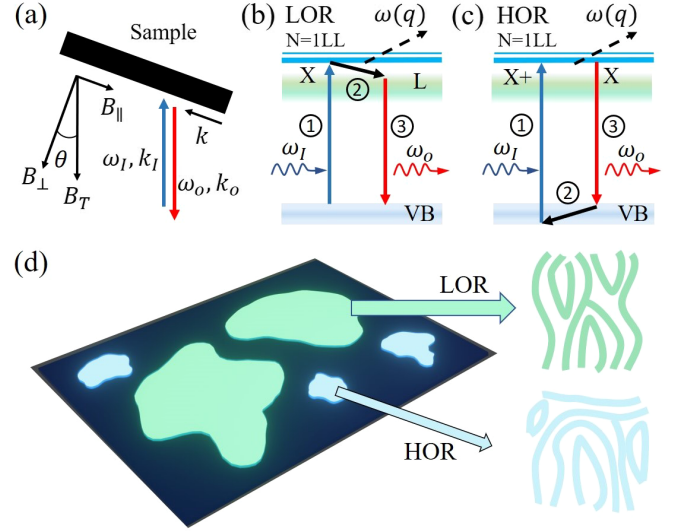


FIG. 1. (a) Schematic description of the experimental backscattering geometry at a tilt angle $\theta = 20^\circ$. Incoming and outgoing photons have energies ω_I , ω_o and wave vectors k_I , k_o , respectively. The total magnetic field B_T produces a perpendicular component B_\perp and a parallel component B_\parallel . The inelastic scattering wave vector is parallel to B_\parallel . (b)(c) Different DRILS processes between valence bands and the SLL in (b) LOR and (c) HOR. A plasmon mode $\omega(q)$ is generated in the second step. (d) Schematic plot of domain textures probed by DRILS in nematic-FQHE fluids. Large (small) nematic domains are active in LOR (HOR). Smaller domains are more disordered.

inserted in the bore of a 16T superconducting magnet. All measurements were performed at $T \leq 45$ mK. The electron density under illumination is directly determined in each cool down by RILS measurements of spin waves at $\nu = 3$ (Fig. S2) and yields $n = 2.8 \times 10^{15} \text{ m}^{-2}$ [43]. The stability of electron density and sample quality against illumination are confirmed by photoluminescence (PL) and RILS measurements under zero magnetic field [43]. Figure 1(a) describes the backscattering geometry at a small tilt angle $\theta = 20^\circ$. The finite wave vector transfer in backscattering is $k = |\mathbf{k}_I - \mathbf{k}_o| \sin \theta$, where \mathbf{k}_I and \mathbf{k}_o are wave vectors of the incoming and outgoing photons. DRILS spectra are excited with the linearly polarized tunable emission from a Ti:sapphire laser that is finely tuned to match the ω_I excitation. The incident power density was kept well below 10^{-4} W/cm^2 . The outgoing photons are dispersed by a triple grating spectrometer and recorded by a CCD camera.

Optical excitons contributing to the DRILS matrix element are built from transitions between quasiparticles in conduction states and holes in valence states shown in steps 1 and 3 in Figs. 1(b) and 1(c). These resonant channels are identified from PL and resonant Rayleigh scattering spectra at $\nu = 7/3$ as shown in Supplemental Material, Fig. S1 [43]. At $\nu = 7/3$ excitons participating in DRILS are extremely sharp, indicating that Landau levels in the bulk of nematic-FQHE fluid system support well-defined states even under nonuniform conditions that must

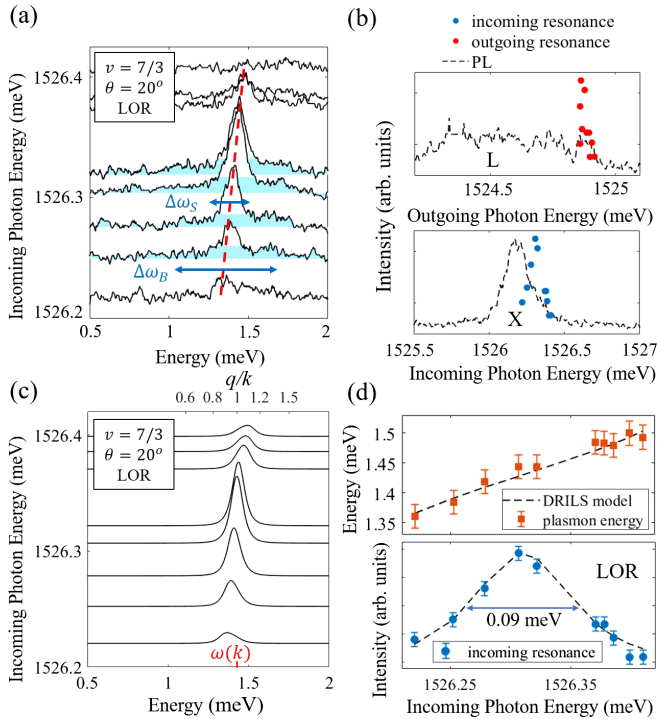


FIG. 2. (a) DRILS LOR spectra of nematic plasmon modes measured at $\nu = 7/3$. The dashed line marks their blueshift. Areas filled in cyan indicate the underlying continuum. Arrows mark the different energy ranges of plasmons and the continuum. (b) Outgoing (top panel) and incoming (bottom panel) resonances of plasmons. (c) LOR plasmon spectra generated by DRILS model, where $(\omega_1, \gamma_1) = (1527.31 \text{ meV}, 0.06 \text{ meV})$, $(\omega_2, \gamma_2) = (1524.88 \text{ meV}, 0.06 \text{ meV})$, $\omega(k) = 1.43 \text{ meV}$, and $\Delta\omega_S = 0.16 \text{ meV}$. The corresponding nonconserved wave vector q/k is labeled on the top x axis. (d) DRILS model fits (dashed lines) of plasmon energies (top panel) and incoming resonance (bottom panel) using the parameters above. The width of incoming resonance is reduced from $2\gamma_1$ due to the impact of plasmon wave vector distribution.

prevail due to coexistence of two distinct phases (the FQHE fluid and the nematic liquid). FQHE fluid at $\nu = 7/3$ is characterized by the determination of low energy magnetoroton excitations, which is suppressed by increasing temperatures or deviation of filling factors (Fig. S3) [43,46].

Two types of DRILS spectra of plasmons are reported. They are defined by the outgoing resonances in steps 3 shown in Figs. 1(b) and 1(c). In low outgoing resonance (LOR) mode the outgoing photon resonates with L transitions that involve states of correlated quantum fluids in the SLL [50]. In high outgoing resonance (HOR) mode the outgoing photon resonates with the excitonic X transition involving empty states in the partially populated SLL [50]. Figures 2 and 3 report DRILS results by nematic plasmons at $\nu = 7/3$.

In results for DRILS LOR shown in Fig. 2(a) there is a sharp nematic plasmon peak that slightly blueshifts with higher incoming photon energies within the small energy

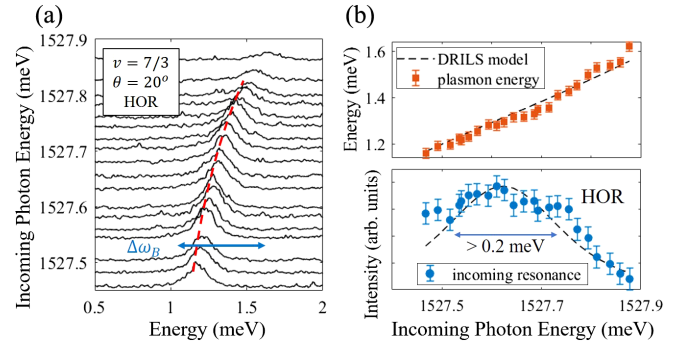


FIG. 3. (a) DRILS HOR spectra of nematic plasmon modes within the energy range $\Delta\omega_B$ measured at $\nu = 7/3$. The dashed line marks their blueshift. (b) DRILS model fits (dashed lines) of plasmon energies (top panel) and incoming resonance (bottom panel) in HOR, where $(\omega_1, \gamma_1) = (1527.61 \text{ meV}, 0.25 \text{ meV})$, $(\omega_2, \gamma_2) = (1526.31 \text{ meV}, 0.06 \text{ meV})$, $\omega(k) = 1.32 \text{ meV}$, and $\Delta\omega_B = 0.35 \text{ meV}$ (lower limit). The incoming resonance overlaps X^+ transitions in PL, which may be a superposition of several optical excitons.

range $\Delta\omega_S$. The small blueshift is due to minor impact of the breakdown of wave vector conservation. Figure 2(b) plots the intensities of plasmons as a function of outgoing (red dots) or incoming (blue dots) photon energies. The blue dots indicate a clear incoming resonance with the X exciton, and the red dots indicate an outgoing resonance associated with the Fermi level of correlated phases [near the high energy edge of L emission in Fig. 2(b)]. We expect the DRILS matrix element to be prominent when symmetry of the final state in L is connected to X exciton through plasmon-electron interactions. The sharp exciton in the outgoing resonance in LOR provides critical spectroscopic insight into links of large nematic domains with the Fermi level of 2DES. The results reveal interplays of topological order in the FQHE with nematic order that could impact anisotropic transport by edge modes [41].

In LOR, an additional broad continuum with energy range $\Delta\omega_B$ shown in Fig. 2(a) is weakly excited without double resonance. We find that in HOR this continuum of modes is DRILS active and gives rise to a series of plasmon modes within $\Delta\omega_B$ shown in Fig. 3(a). Different energy ranges of double resonant plasmons in LOR and HOR spectra reveal a rich set of domain sizes. As domain sizes increase ($D \gg \lambda$), the wave vector dispersions of elementary excitations would tend to be similar to those of a uniform electron fluid.

Modeling DRILS spectra with Eqs. (1) and (2) yields nematic domain sizes $D_S \approx 5.3 \mu\text{m}$ in LOR and $D_B < 2.2 \mu\text{m}$ in HOR [43]. Figure 2(c) displays nematic plasmons generated by DRILS model in LOR. Figures 2(d) and 3(b) show DRILS model fits of plasmon energies and intensities as a function of incoming photon energies in LOR and HOR, which are the two factors directly capturing the resonance evolution of plasmons. The fits successfully

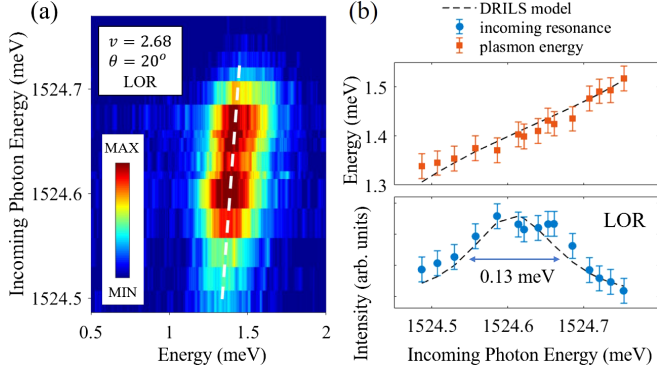


FIG. 4. (a) DRILS spectra of nematic plasmon modes at $\nu = 2.68$. The dashed line marks their blueshift. (b) Reproduction of plasmon energy (top panel) and incoming resonance (bottom panel) by DRILS model, where $(\omega_1, \gamma_1) = (1524.6 \text{ meV}, 0.08 \text{ meV})$, $(\omega_2, \gamma_2) = (1523.21 \text{ meV}, 0.08 \text{ meV})$, $\omega(k) = 1.41 \text{ meV}$, and $\Delta\omega = 0.24 \text{ meV}$. The color code is linear with intensity.

reproduce DRILS-LOR results. It supports the interpretation that LOR spectra are from large nematic domains with small impact of breakdown of wave vector conservation and thus represent well-defined bulklike collective modes. In contrast, the HOR incoming resonance [Fig. 3(b)] has a flatter top with a few outliers. When domain sizes are comparable to the inelastic scattering wavelength $\lambda = 1.2 \mu\text{m}$, wave vectors are no longer good quantum numbers and modeling of DRILS spectra may have to be modified.

It is highly significant that DRILS LOR involves larger domains where quantum fluids with a sharp optical transition at the Fermi level are better defined. In contrast, DRILS HOR involves intermediate states of electron-hole pairs forming X and X^+ excitons which are associated with empty states in the partially populated SLL. As a result, LOR and HOR effectively probe plasmons in two different ranges of nematic domain sizes: larger domains close to the Fermi level in LOR, and smaller domains producing stronger impact of breakdown of wave vector conservation in HOR [Fig. 1(d)]. FQHE-nematic fluids at $\nu = 7/3$ with DRILS-active plasmons thus clearly reveal distinct impacts of domain textures depending on their sizes.

Domain textures at other filling factors can also be resolved by DRILS. Figure 4(a) shows strong nematic plasmons of a non-FQHE state at $\nu = 2.68$. The DRILS model reproduces main resonance features, as shown in Fig. 4(b). Compared to $\nu = 7/3$ in LOR, the broader resonance reveals smaller domain sizes $D \approx 3.5 \mu\text{m}$, which exerts a larger impact of breakdown of wave vector conservation and is consistent with a higher disorder level in the upper half of the SLL.

Large domains of FQHE fluids support neutral collective excitations with well-defined wave vector dispersion that could be observed in RILS and DRILS spectra which

access bulk states. Extremely sharp long-wavelength magnetoroton excitations at $\nu = 1/3$ were observed [13,51]. Single resonance is considered here due to the lack of a pair of well-understood resonant channels with energy difference matching the magnetoroton energy. The narrow linewidth can be regarded as a consequence of very small breakdown of wave vector conservation in RILS spectra from large domains of the bulk FQHE fluid and the small curvature of the magnetoroton dispersion near zero momentum. The low-lying neutral collective excitations from bulklike FQHE fluid in large domains could be interpreted in terms of theories for uniform FQHE phases. With increasing wave vectors, dispersive long-wavelength modes observed in RILS at $\nu = 1/3$ split in a manner consistent with a two-roton bound state [13].

Long-wavelength magnetoroton excitations play key roles in recent geometrical theories of electron correlation in bulk FQHE phases [2,3]. Magnetoroton modes occur here as spin 2 chiral gravitons, corresponding to the long-wavelength quantum fluctuations of an internal dynamic metric [4–7]. At $\nu = 1/3$ only the mode with spin $S = -2$ is predicted to be active in RILS experiments in the $k \rightarrow 0$ limit [4–6]. The results in Ref. [13] that reveal a magnetoroton doublet offer a key experimental insight that needs to be interpreted. In the framework of geometrical theories, a doublet may occur in RILS spectra at finite k because of coupling between magnetorotons with $S = -2$ and $+2$ [4,6].

In summary, RILS and DRILS methods investigate impacts of domain textures on low-lying collective excitations in the nonuniform bulk of FQHE of 2DES in GaAs quantum structures. For domains larger than the inelastic light scattering wavelength ($\lambda = 1.2 \mu\text{m}$) there is nearly full conservation of wave vector in the light scattering events. Magnetoroton excitations have been identified in RILS spectra from $\nu = 5/2$ FQHE fluids [45]. RILS measurements with higher resolution in large domains of FQHE fluids at $\nu = 5/2$ could identify narrow long-wavelength neutral collective excitations with well-defined wave vector dispersion. Such experiments may reveal features of topological domain textures involving Pfaffian and anti-Pfaffian orders [15–20]. Suitable design of GaAs quantum structures [28], enhanced growth protocols [52], and characterization by RILS and DRILS may lead to the creation and identification of large domains of quantum fluids likely to host novel electron correlation effects.

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