Model of Persistent Breaking of Discrete Symmetry

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We show there exist UV-complete field-theoretic models in general dimension, including 2 + 1, with the spontaneous breaking of a global symmetry, which persists to the arbitrarily high temperatures. Our example is a conformal vector model with the $O(N) \times \mathbb{Z}_2$ symmetry at zero temperature. Using conformal perturbation theory we establish \mathbb{Z}_2 symmetry is broken at finite temperature for N > 17. Similar to recent constructions of [N. Chai *et al.*, Phys. Rev. D **102**, 065014 (2020)., N. Chai *et al.*, Phys. Rev. Lett. **125**, 131603 (2020).], in the infinite N limit our model has a nontrivial conformal manifold, a moduli space of vacua, which gets deformed at finite temperature. Furthermore, in this regime the model admits a persistent breaking of O(N) in 2 + 1 dimensions, therefore providing another example where the Coleman-Hohenberg-Mermin-Wagner theorem can be bypassed.

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Introduction.-The phenomenon of spontaneous symmetry breaking is ubiquitous: many real systems as well as field theoretic models exhibit spontaneous breaking of both discrete and continuous symmetries at zero or sufficiently small temperature. The conventional picture suggests the full symmetry is restored for sufficiently high temperatures. There are also situations when the low-temperature phase is symmetric but symmetry is broken as the temperature increases. In this case too one normally expects the symmetry to be eventually restored for even higher temperatures, and there are seemingly many theoretical results supporting such a conclusion. Yet as we discuss below they rely on the stringent assumptions that can be evaded. Which raises the question-is spontaneous breaking that persists to arbitrarily high temperatures possible? In this Letter we answer this question by constructing UV-complete field-theoretical models in diverse dimensions which exhibit persistent breaking of both discrete and continuous symmetries.

For lattice systems it is well appreciated the symmetry is restored for temperates large in comparison with the lattice spacing [1]. This suggests in effective field theory spontaneous breaking is possible up to UV scale, as illustrated by the UV-incomplete example of Ref. [2]. Yet the latticebased arguments are not applicable to UV-complete field theoretic models, which we focus on. There is also the famous Coleman-Hohenberg-Mermin-Wagner theorem [3,4] and its generalizations, which rule out the possibility of the longer range order at $T \neq 0$ in a wide class of twodimensional d = 2 + 1 systems. This seemingly prohibits spontaneous breaking of a continuous symmetry, but here too there are many assumptions and exceptions, starting from the example of Ref. [5]. Importantly to what follows, the original work [4] already notes the argument may break down if the lattice model exhibits long-range interactions. Thus, there is a phase transition in a Heisenberg magnet with suitably adjusted long-range interactions, see, e.g., Ref. [6] for a recent review on the Coleman-Hohenberg-Mermin-Wagner theorem and its limitations. This suggests nonlocal field theories, which may result from such lattice models in the continuous limit may be immune to various no-go results.

To better illustrate this idea we briefly mention Coleman's no-go theorem [7], which in d = 1 + 1 excludes spontaneous symmetry breaking because the corresponding Goldstone bosons, being massless, would have infrared divergences. This argument works for the short-range interactions, whereas as above introduction of the long-range forces allows phase-transitions in the one-dimensional systems [8–12]. The no-go results in various dimensions are related and can be evaded simultaneously: Our model exhibits persistent breaking for 1 < d < 3 + 1.

The discussion above mostly applies to continuous symmetries. For discrete symmetries even less is known. As for continuous symmetries, there is no universal theoretical argument requiring discrete symmetry to be restored at high temperatures. At the same time to the best of our knowledge there were no examples of persistent symmetry breaking in 2 + 1 dimensions. In higher

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dimensions there are CFTs in $d = 4 - \epsilon$ [13–15] and d = 4[16,17] which have some of their internal symmetries broken at arbitrary finite temperature. Yet these models are not free of criticism. In the former case ϵ cannot be taken to one and therefore theories in question are not necessarily unitary [18]. And the latter case of [16] is inconclusive because of the possible impact of 1/Ncorrections. We also refer the reader to Ref. [19], where asymptotically safe theories [20] in d = 3 + 1 were considered in the context of persistent symmetry breaking. Another idea explored in the literature is placing the theory in a curved spacetime. Thus, the O(N) model in AdS evades the Coleman-Hohenberg-Mermin-Wagner theorem [21] but at high temperatures the symmetry is restored. There are other AdS/CFT candidates of persistent order [22–28] which are perturbatively stable, but the symmetric phase has smaller free energy. (See Ref. [29] where the holographic conformal order is studied on S^3 .) Other nonunitary models of persistent breaking in the presence of chemical potential include [30-35]. We should also mention the renowned Berezinskii-Kosterlitz-Thouless transition [36–38] as an example evading the Coleman-Hohenberg-Mermin-Wagner theorem yet not leading to persistent symmetry breaking.

The main goal of this Letter is to construct an example which would be free of aforementioned deficiencies. Our model is a conformal, and hence UV-complete vector theory which breaks discrete symmetry at finite temperature T. Because of scale invariance this breaking persists to arbitrary high T. The result holds true in 2+1 and extends to other 1 < d < 3 + 1 where the model is manifestly stable (Stability suggests the model is unitary; it would be interesting to confirm this by calculating anomalous dimensions at the IR fixed point. We also expect the fixed point to be a CFT, but since our model is nonlocal strictly speaking this has to be verified along the lines [39].) Generalizations of our model exhibit persistent breaking of continuous symmetries in d = 2 + 1 and beyond [40]. Our construction bypasses the Coleman-Hohenberg-Mermin-Wagner theorem due to its nonlocal nature. It can be viewed as two weakly interacting copies of the long-range Ising (LRI) model [8,41,42] (For original and recent studies of the LRI model see Refs. [39,43-51].)

The model.—To begin with, we introduce our model. Consider the following Gaussian action in $1 \le d < 4$ dimensions,

$$S_{0} = \mathcal{N}_{\phi} \int d^{d}x_{1} \int d^{d}x_{2} \frac{\vec{\phi}(x_{1}) \cdot \vec{\phi}(x_{2})}{|x_{1} - x_{2}|^{2(d - \Delta_{\phi})}} + \mathcal{N}_{\sigma} \int d^{d}x_{1} \int d^{d}x_{2} \frac{\sigma(x_{1})\sigma(x_{2})}{|x_{1} - x_{2}|^{2(d - \Delta_{\sigma})}}.$$
 (1)

The fundamental fields include scalars $\vec{\phi}$ and σ transforming in vector and singlet representations of O(N). The

model also admits a \mathbb{Z}_2 symmetry that flips the sign of σ . The coefficients \mathcal{N}_{ϕ} and \mathcal{N}_{σ} are fixed so that the two-point functions of $\vec{\phi}$ and σ are canonically normalized. For brevity we suppress vector indices in what follows. The scaling dimensions of the generalized free fields are chosen to be

$$\Delta_{\phi} = \frac{d - \epsilon_1}{4}, \qquad \Delta_{\sigma} = \frac{d - \epsilon_3}{4}, \tag{2}$$

with $\epsilon_{1,3} \ll 1$, so that the following quartic operators become weakly relevant

$$\mathcal{O}_1 = (\phi^2)^2, \qquad \mathcal{O}_2 = \phi^2 \sigma^2, \qquad \mathcal{O}_3 = \sigma^4, \quad (3)$$

with scaling dimensions $\Delta_1 = 4\Delta_{\phi}$, $\Delta_2 = 2(\Delta_{\phi} + \Delta_{\sigma})$, and $\Delta_3 = 4\Delta_{\sigma}$. This model is conformal, we list OPE coefficients C_{ii}^k and other technical details in the Appendix.

Next, we consider the following deformation

$$S = S_0 + \sum_{i=1}^3 \frac{g_i \mu^{\epsilon_i}}{N} \int d^d x \mathcal{O}_i(x), \tag{4}$$

where $\epsilon_i \ll 1$. [Note that $\epsilon_2 = (\epsilon_1 + \epsilon_3)/2$.] It induces an RG flow of the form

$$\mu \frac{dg_i}{d\mu} = -\epsilon_i g_i + \frac{\pi^{d/2}}{N\Gamma(\frac{d}{2})} \sum_{j,k} C^i_{jk} g_j g_k + \dots$$
(5)

As we argue below, for N > 10 there is a fixed point with $g_2 < 0$. Moreover, the IR CFTs with negative g_2 are stable. They define a class of theories with the persistent symmetry breaking.

To understand the unbroken symmetries of the IR critical point at finite temperature we consider the effective potential V_{eff} for the zero mode. To leading order in ϵ_i , thermal fluctuations simply induce quadratic terms in addition to the quartic potential (4),

$$V_{\text{eff}}(\phi,\sigma;\beta) = \mathcal{M}_{\phi}(\beta)\phi^{2} + \mathcal{M}_{\sigma}(\beta)\sigma^{2} + \frac{g_{1}\mu^{\epsilon_{1}}}{N}\mathcal{O}_{1} + \frac{g_{2}\mu^{\epsilon_{2}}}{N}\mathcal{O}_{2} + \frac{g_{3}\mu^{\epsilon_{3}}}{N}\mathcal{O}_{3} + \mathcal{O}(\epsilon_{i}^{2}),$$
(6)

where

$$\mathcal{M}_{\phi}(\beta) = 2 \frac{g_{1} \mu^{\epsilon_{1}}}{N} \left(1 + \frac{2}{N}\right) \langle \phi^{2} \rangle_{\beta} + \frac{g_{2} \mu^{\epsilon_{2}}}{N} \langle \sigma^{2} \rangle_{\beta},$$
$$\mathcal{M}_{\sigma}(\beta) = \frac{g_{2} \mu^{\epsilon_{2}}}{N} \langle \phi^{2} \rangle_{\beta} + 6 \frac{g_{3} \mu^{\epsilon_{3}}}{N} \langle \sigma^{2} \rangle_{\beta}.$$
(7)

As shown in the Appendix, thermal expectation values of the generalized free fields are given by

$$\langle \phi^2 \rangle_{\beta} = N \frac{2\zeta(2\Delta_{\phi})}{\beta^{2\Delta_{\phi}}}, \qquad \langle \sigma^2 \rangle_{\beta} = \frac{2\zeta(2\Delta_{\sigma})}{\beta^{2\Delta_{\sigma}}}, \qquad (8)$$

which follow straightforwardly from the thermal Green's function. Obviously, full $O(N) \times \mathbb{Z}_2$ symmetry is preserved by the minimum of the potential at zero temperature. However, the finite temperature effects might break it if $\mathcal{M}_{\phi} < 0$ or $\mathcal{M}_{\sigma} < 0$. If this happens, the higher order perturbative corrections cannot restore the symmetry, because the higher loop contributions to \mathcal{M}_{ϕ} , \mathcal{M}_{σ} are suppressed by higher powers of ϵ_i , whereas the terms with higher powers of fundamental fields are subdominant in the vicinity of the origin. To establish symmetry breaking at finite temperature, it is therefore sufficient to show that the model admits a fixed point where one of the \mathcal{M} 's becomes negative.

To ensure stability of the model it is necessary to satisfy $g_1, g_3 \ge 0$, while g_2 could be negative provided $4g_1g_3 \ge g_2^2$. If all g_i are positive, $\mathcal{M}_{\phi}, \mathcal{M}_{\sigma}$ are positive as well, the potential is minimized at the origin $\phi = \sigma = 0$ and the symmetry is restored. The only scenario of symmetry breaking is therefore when $g_2 < 0$. The RG flow (5) terminates at a weakly interacting IR fixed point in the vicinity of the original Gaussian theory. At the critical point at leading order in ϵ_i the couplings satisfy

$$\epsilon_{1}g_{1} = \frac{\pi^{d/2}}{N\Gamma(\frac{d}{2})} \left(C_{11}^{1}g_{1}^{2} + C_{22}^{1}g_{2}^{2}\right),$$

$$\epsilon_{2}g_{2} = \frac{\pi^{d/2}}{N\Gamma(\frac{d}{2})} \left(2C_{12}^{2}g_{1}g_{2} + C_{22}^{2}g_{2}^{2} + 2C_{23}^{2}g_{2}g_{3}\right),$$

$$\epsilon_{3}g_{3} = \frac{\pi^{d/2}}{N\Gamma(\frac{d}{2})} \left(C_{22}^{3}g_{2}^{2} + C_{33}^{3}g_{3}^{2}\right).$$
(9)

There is always a trivial fixed point with $g_1 = (\Gamma(d/2)/8\pi^{d/2})(N/N+8)\epsilon_1$, $g_2 = 0$ and $g_3 = (N\Gamma(d/2)/72\pi^{d/2})\epsilon_3$. It represents two decoupled theories: the so-called long range Ising model [39] and its O(N) generalization. This fixed point was recently studied in, e.g., Ref. [51].

To simplify the analysis and illustrate the main idea in what follows we consider only a particular case of equal $\epsilon_i = \epsilon$. The case of nonequal ϵ_i is similar and also admits persistent symmetry breaking. It is also convenient to rescale the couplings $g_i = \tilde{g}_i (\Gamma(d/2)/\pi^{d/2})\epsilon_i$. Before proceeding with the case of finite N we take the infinite N limit. In this case the equations (9) drastically simplify, yielding the conformal manifold—a one-parameter family of fixed points

$$\tilde{g}_1 = \frac{1}{8}, \qquad \tilde{g}_3 = 2\tilde{g}_2^2,$$
(10)

depicted in Fig. 1. For the negative branch $\tilde{g}_2 = -2\sqrt{\tilde{g}_1\tilde{g}_3}$ the effective potential degenerates into



FIG. 1. Fixed points in the $g_2 - g_3$ plane. Orange line shows a continuous family of solutions (10) emerging in the infinite *N* approximation. Black dots are the asymptotic locations of fixed points (14) in the limit of large *N*. Blue points are the solutions of Eq. (9) for $N \le 200$.

$$V_{\rm eff}(\phi,\sigma;\beta) = \frac{\mu^{\epsilon}}{N} (2\sqrt{g_1} \langle \phi^2 \rangle_{\beta} x + x^2), \qquad (11)$$

$$x = \sqrt{g_1}\phi^2 - \sqrt{g_3}\sigma^2. \tag{12}$$

The minimum is reached at

$$\sigma^2 = \sqrt{\frac{g_1}{g_3}} (\phi^2 + \langle \phi^2 \rangle_\beta). \tag{13}$$

This is a one-dimensional family of vacua with the nonzero expectation value of σ , signaling spontaneous symmetry breaking. For positive g_2 and $\langle \phi^2 \rangle_\beta$ only $\sigma = 0$ is admitted.

From the discussion above it is clear negative g_2 is necessary for symmetry to be broken. Hence the crucial question is if the fixed point(s) with $g_2 < 0$ survive in the finite *N* regime. Before proceeding with an arbitrary *N* we employ 1/N expansion to find, in addition to Eq. (10), the consistency condition

$$4\tilde{g}_2^2 + 2\tilde{g}_2 - \frac{3}{4} = 0. \tag{14}$$

Hence at large but finite *N* the continuous family (10) collapses into two solutions, one with positive and one with negative g_2 . Upon taking finite *N* corrections into account, degeneracy $4g_1g_3 = g_2^2$ is lifted. Minimization of the potential (6) yields

$$\begin{pmatrix} \phi^2 \\ \sigma^2 \end{pmatrix} = \frac{-N\mu^{-\epsilon}}{4g_1g_3 - g_2^2} \begin{pmatrix} 2g_3 & -g_2 \\ -g_2 & 2g_1 \end{pmatrix} \begin{pmatrix} \mathcal{M}_{\phi} \\ \mathcal{M}_{\sigma} \end{pmatrix}, \quad (15)$$

provided resulting ϕ^2 and σ^2 both are positive. Yet this is never the case for any solutions of Eq. (9). The true

minimum of the potential is therefore achieved either at $\phi^2 = 0$ or $\sigma^2 = 0$. For all solutions of Eq. (9) $\mathcal{M}_{\phi} > 0$ but \mathcal{M}_{σ} become negative for the branch with negative g_2 and N > 17. In these cases the minimum is achieved at

$$\binom{\phi^2}{\sigma^2} = \frac{-N\mu^{-\epsilon}}{2g_3} \binom{0}{\mathcal{M}_{\sigma}}.$$
 (16)

At leading order in ϵ this value is correct at all temperatures, as follows from the dimensional analysis. Clearly, σ^2 is strictly positive, indicating symmetry breaking.

Conclusions.—To summarize, the model (1) with the choice $\epsilon_i = \epsilon \ll 1$ for finite *N* has two different IR fixed points, and for N > 10 one of them has negative g_2 . The fixed points with $g_2 < 0$ and N > 17 exhibit symmetry breaking at arbitrary nonzero temperature

$$O(N) \times \mathbb{Z}_2 \to O(N). \tag{17}$$

The behavior for nonequal small ϵ_i is qualitatively similar.

The model (1) admits a straightforward generalization to two Gaussian free fields in the vector representations of $O(N_1)$ and $O(N_2)$. The example in this note corresponds to the $N_2 = 1$ case. Deforming this theory by weakly relevant quartic operators and following the same steps as above, one finds IR fixed points which exhibit spontaneous breaking of the global continuous symmetry at finite temperature. In 2 + 1 dimensions it is therefore an example of persistent breaking of a continuous global symmetry, which bypasses the Coleman-Hohenberg-Mermin-Wagner theorem [3,4,7] by virtue of being a CFT with long-range interactions. We will discuss this case in detail in Ref. [40]. Our nonlocal $O(N) \times \mathbb{Z}_2$ model is similar to that one studied in Refs. [13,14]. It would be also interesting to explore if they are related in the large N limit.

Our findings clearly show persistent breaking is possible in the UV complete yet nonlocal models. This raises the question if nonlocality is truly necessary, i.e., if there could be UV-complete unitary *local* field theoretic models in d =2 + 1 exhibiting persistent breaking of discrete symmetries. It is an important open question to construct such an example or rule out this possibility.

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Appendix: Supplemental Material.—The action defines a conformal model with the canonically normalized two-point function of fundamental fields $\vec{\phi}$ and σ ,

$$\langle \phi_a \phi_b \rangle = \frac{\delta_{ab}}{|x_{12}|^{2\Delta_{\phi}}}, \qquad \langle \sigma \sigma \rangle = \frac{1}{|x_{12}|^{2\Delta_{\sigma}}}.$$
 (A1)

The two-point functions of the operators O_i (3)

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \delta_{ij} \frac{N_i}{|x|^{2\Delta_i}},$$
 (A2)

where

$$N_1 = 8N^2 \left(1 + \frac{2}{N}\right), \qquad N_2 = 4N, \qquad N_3 = 24.$$
 (A3)

Similarly, the three-point functions

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ij}^k N_k}{|x_{12}|^{\Delta - 2\Delta_k} |x_{23}|^{\Delta - 2\Delta_i} |x_{13}|^{\Delta - 2\Delta_j}},$$

$$\Delta = \Delta_i + \Delta_j + \Delta_k, \qquad (A4)$$

are fixed by the OPE coefficients (we list only nonzero ones)

$$C_{11}^{1} = 8(N+8), \quad C_{22}^{1} = 2, \quad C_{12}^{2} = 4(N+2),$$

 $C_{22}^{2} = 16, \quad C_{23}^{2} = 12, \quad C_{32}^{3} = 2N, \quad C_{33}^{3} = 72.$ (A5)

They are related by

$$C_{ij}^k = C_{ik}^j N_j / N_k. \tag{A6}$$

At finite temperature two-point function takes the form

$$\langle \phi_a \phi_b \rangle = \sum_{m=-\infty}^{\infty} \frac{\delta_{ab}}{[(\tau + m\beta)^2 + \vec{x}^2]^{\Delta_{\phi}}},$$
 (A7)

and similarly for σ . From here one trivially finds Eq. (8).

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