Thermodynamic Stability Implies Causality

L. Gavassino, M. Antonelli, and B. Haskell
Nicolaus Copernicus Astronomical Center, Polish Academy of Sciences, Ulica Bartycka 18, 00-716 Warsaw, Poland

(Received 9 June 2021; revised 19 November 2021; accepted 20 December 2021; published 6 January 2022)

The stability conditions of a relativistic hydrodynamic theory can be derived directly from the requirement that the entropy should be maximized in equilibrium. Here, we use a simple geometrical argument to prove that, if the hydrodynamic theory is stable according to this entropic criterion, then localized perturbations to the equilibrium state cannot propagate outside their future light cone. In other words, within relativistic hydrodynamics, acausal theories must be thermodynamically unstable, at least close to equilibrium. We show that the physical origin of this deep connection between stability and causality lies in the relationship between entropy and information. Our result may be interpreted as an "equilibrium conservation theorem," which generalizes the Hawking-Ellis vacuum conservation theorem to finite temperature and chemical potential.

DOI: 10.1103/PhysRevLett.128.010606

Introduction.—A hydrodynamic theory is said to be stable if small deviations from the state of global thermodynamic equilibrium do not have the tendency to grow indefinitely, but remain bounded over time. It is said to be causal if signals do not propagate faster than light. Every hydrodynamic theory should guarantee the validity of these two principles, the former arising from the definition of equilibrium as the state toward which dissipative systems evolve as $t \to +\infty$, the latter arising from the principle of relativity (if signals were superluminal, there would be a reference frame in which the effect precedes the cause). Whenever a new theory is proposed, it needs to pass these two tests, to be considered reliable. To date, these properties have been mostly studied as two distinct, disconnected features of the equations of the theory, to be discussed separately. Intuitively, this approach seems natural, as stability and causality are two principles which pertain to two complementary branches of physics: thermodynamics [1] and field theory [2].

However, in reality these two features appear to be strongly correlated. Divergence-type theories are causal if and only if they are stable [3] while Israel-Stewart theories are causal if they are stable [4,5]. Geroch and Lindblom [6] analyzed a wide class of causal theories for dissipation and found that many causality conditions have an important stabilizing effect. Finally, Bemfica *et al.* [7] recently proved a theorem, according to which, if a strongly hyperbolic theory is stable in the fluid rest frame, and it is causal, then it is stable in every reference frame, formalizing a widespread intuition [8,9]. All these results suggest the existence of an underlying physical mechanism connecting causality and stability. Discovering it would lead to a complete change of paradigm. In fact, it would provide a new insight into the physical meaning of the (usually complicated) mathematical structure which ensures causality. Furthermore, it would importantly simplify the (usually tedious) job of testing both causality and stability, maybe reducing one to the other.

To date, a "fully explanatory" mechanism connecting causality and stability has never been proposed. In fact, such a connection is usually found *a posteriori*, by direct comparison between the two distinct sets of conditions (as in [4]), or through complicated mathematical proofs, as in [7]. The goal of this Letter is to finally explain simply the relationship between causality and stability. We prove, with a geometrical argument, that if a theory is *thermodynamically* stable, namely, if the entropy is maximized at equilibrium (see Gavassino [10]), it is also causal, close to equilibrium [11]. We show that the key to understanding this result from a physical perspective is the underlying relationship between entropy and information. Furthermore, we explain why causality alone does *not* imply stability (see, e.g., [12,13]), but one needs at least to prove stability in a particular reference frame (in agreement with [7]).

We adopt the signature $(-,+,+,+)$ and we work in natural units $c = k_B = 1$.

Thermodynamic stability.—Under which conditions is a relativistic fluid thermodynamically stable? Consider a fluid "F" that is in contact with a heat-particle bath "H". Assume that the total system "fluid + bath" is isolated and evolves spontaneously from a state 1 to a state 2. Then the total entropy should not decrease (given a quantity A, we call $\Delta A \coloneqq A_2 - A_1$):

$$\Delta S_{\text{tot}} = \Delta S_F + \Delta S_H = \Delta S_F + \int_1^2 dS_H \geq 0. \qquad (1)$$

If Q^I are the relevant conserved charges of the system, e.g., baryon number and four momentum [2], we can write $dS_H = -\alpha_I^H dQ_H^I$, where α_I are the thermodynamic conjugates of Q^I and we are adopting Einstein's convention for the index I.

Considering that $dQ_H^I = -dQ_F^I$ (charge conservation), and that the bath is *defined* as a system that is so large that $\alpha_I^H = \text{const} =: \alpha_I^\star$ in any interaction with F [14–16], we find that ($\alpha_I^\star$ are constants)

$$\Delta S_{\text{tot}} = \Delta(S_F + \alpha_I^\star Q_F^I) \geq 0. \quad (2)$$

This implies that the equilibrium state of F is the state that maximizes the functional $\Phi = S_F + \alpha_I^\star Q_F^I$ for unconstrained variations [14–22]. Hence, for an arbitrary spacelike 3D surface Σ which extends over the support of F, we need to require that

$$E[\Sigma] := -\delta\Phi[\Sigma] = \int_\Sigma E^a n_a d\Sigma \geq 0,$$
$$\text{with} \quad E^a = -\delta(s^a + \alpha_I^\star J^{Ia}) = -\delta s^a - \alpha_I^\star \delta J^{Ia}, \quad (3)$$

where s^a is the fluid's entropy current, J^{Ia} are the currents whose fluxes are Q_F^I, and "δ" is an arbitrary *finite* perturbation from the equilibrium state. In most applications, E^a may be truncated to second order in the perturbations $\delta\varphi_i$ to the hydrodynamic fields (like the fluid four velocity and the temperature field).

Let us list the most important properties of E^a: (i) For any unit vector n^a, timelike and past directed ($n^a n_a = -1$, $n^0 < 0$), we have

$$E^a n_a \geq 0. \quad (4)$$

(ii) For the same n^a as in (i), $E^a n_a = 0$ on any point where the perturbation to every observable is zero, and only on these points. (iii) The four divergence of E^a is nonpositive

$$\nabla_a E^a \leq 0. \quad (5)$$

The first property follows from $E[\Sigma] \geq 0$, which must hold for any spacelike 3D surface Σ covering F [23]. Note that the vector $n^a = n^a[\Sigma]$ appearing in (3) is the unit normal to Σ, which is timelike past directed [24]. The second property follows from the definition of E^a, and from the assumption that the equilibrium state is unique. The third property follows from (2). Conditions (i)–(iii) imply that E is a nonincreasing "square-integral norm" of the perturbation $\delta\varphi_i$, enforcing the Lyapunov stability of the equilibrium state [25–27]. In Supplemental Material [28] we show that (i)–(iii) are mathematically equivalent to the Gibbs stability criterion [10].

The criterion for *thermodynamic stability* described above is a sufficient condition for *hydrodynamic stability*, but contains more information than a hydrodynamic stability analysis: while the latter is a dynamical property of the field equations (an on-shell criterion [29]), the former is a property of the constitutive relations (it must be respected also off shell). In fact, thermodynamic stability also implies stability to thermodynamic fluctuations, whose probability distribution [14,19],

$$\mathcal{P}[\delta\varphi_i] \propto e^{\delta\Phi[\Sigma,\delta\varphi_i]} = e^{-E[\Sigma,\delta\varphi_i]}, \quad (6)$$

must be peaked at $\delta\varphi_i = 0$, leading to conditions (i) and (ii).

To see the difference between hydrodynamic and thermodynamic stability, consider the case of a perfect fluid, whose current E^a is [4,10,20]

$$TE^a = \frac{u^a}{2}(\rho + p)\delta u^b \delta u_b + \delta u^a \delta p + \frac{u^a}{2}\left[\frac{1}{c_s^2}\frac{(\delta p)^2}{\rho + p} + \frac{nT}{c_p}(\delta\sigma)^2\right] + \mathcal{O}[(\delta\varphi_i)^3], \quad (7)$$

where u^a, n, T, ρ, p, σ, c_s, and c_p are fluid velocity, particle density, temperature, energy density, pressure, entropy per particle, speed of sound, and specific heat at constant pressure (quantities without "δ" are evaluated at equilibrium). Conditions (i) and (ii) produce the thermodynamic inequalities (assuming $n, T > 0$)

$$0 < c_s^2 \leq 1, \qquad \rho + p > 0, \qquad c_p > 0. \quad (8)$$

A positive c_p guarantees stability to heat transfer. However, since a perfect fluid does not conduce heat, the inequality $c_p > 0$ is invisible to a hydrodynamic stability analysis. On the other hand, thermodynamic stability implies stability also to virtual processes [17], which become real when thermal fluctuations are included in the description [30,31], or when we couple the fluid with other fluids [32] or heat baths [15,16].

Finally, it is also relevant to mention that, in ideal-gas kinetic theory, E^a *always* obeys conditions (i)–(iii), and is given by [10,33] (ε is $+1$ for bosons and -1 for fermions)

$$E^a = \int \frac{(\delta f)^2 p^a}{2f(1 + \varepsilon f)} \frac{d^3 p}{p^0} + \mathcal{O}[(\delta f)^3], \quad (9)$$

where $f = f(x, p)$ is the invariant distribution function, counting the number of particles in a small phase-space volume centered on (x, p) [34]. Hence, for ideal gases, the conditions of thermodynamic stability (i)–(iii) are also a criterion of consistency with the kinetic description.

The argument for causality.—Our goal is to show that conditions (i)–(iii) imply causality. We work, for clarity, in $1 + 1$ dimensions, on a scale that is assumed sufficiently small that we can neglect the gravitational field. The generalization to $3 + 1$ dimensions (and curved space-time) is presented in Supplemental Material [28]. Working in an inertial coordinate system (t, x), we consider a perturbation $\delta\varphi_i$ that is initially confined on the semiaxis $x \leq 0$, namely,

$$\delta\varphi_i(0, x) = 0, \quad \forall \ x > 0. \quad (10)$$

We apply the Gauss theorem to the triangle ABC shown in Fig. 1 and use condition (iii):

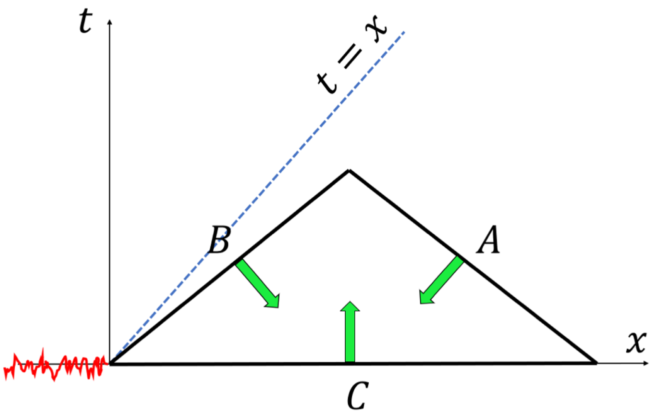

FIG. 1. Visualization of the geometric argument. The initial perturbation (in red) is located to the left of the origin. Causality requires it to stay confined in the $t \geq x$ half-plane. We build the triangle ABC in a way that all its edges are spacelike. B and C intersect at the origin. We can regulate B to be arbitrarily close to the line $t = x$. The green arrows are a Euclidean representation of the unit normal vectors to the edges and are taken inward pointing, consistently with our choice of metric signature [35].

$$E[A] + E[B] + E[C] = \int_{(\text{triangle})} \nabla_a E^a dtdx \leq 0. \tag{11}$$

The 1D surfaces A, B, and C are all spacelike, so that their unit normal vector must be taken inward pointing [35–37]. Combining (10) with condition (ii) we obtain $E[C] = 0$. Furthermore, since the unit normals to A and B are timelike past directed, we can use (i) to show that $E[A]$ and $E[B]$ are non-negative, so that (11) implies

$$E[A] = E[B] = 0. \tag{12}$$

But this implies, recalling (ii), that $\delta\varphi_i$ must be zero on all the sides of the triangle. Since we can make the triangle arbitrarily long (A and C may extend to $x = +\infty$) and the side B may be arbitrarily close to the line $t = x$ (without crossing it, because B must be spacelike), we finally obtain

$$\delta\varphi_i(t, x) = 0 \quad \text{for } x > t. \tag{13}$$

This shows that no perturbation can propagate outside the light cone, hence linear causality [38].

Physical interpretation.—To be able to understand the physical meaning of the argument above, we need first to have an intuitive interpretation of E^a.

Within the usual interpretation of entropy as uncertainty, in the sense that S_{tot} reflects our ignorance, interpreted as a lack of information [39], about the exact system's microstate (recall Boltzmann's formula $S_{\text{tot}} = \ln \Gamma$, where Γ is the number of microscopic realizations of a given macrostate), Eq. (2) implies

$$E = \begin{pmatrix} \text{ignorance at} \\ \text{equilibrium} \end{pmatrix} - \begin{pmatrix} \text{ignorance in the} \\ \text{perturbed state} \end{pmatrix}. \tag{14}$$

Hence, E is the net information carried by the perturbation. The Gibbs stability criterion ($E \geq 0$), then, is the statement that any perturbation increases our knowledge about the microstate. Now, if we look at Eq. (3) and invoke condition (ii), it follows that we can identify E^a with the current of information transported by the perturbation (see Supplemental Material [28] for a direct proof). In fact, if $E^a = 0$ in a given region of space $\mathcal{R}$, then the average value of any observable on $\mathcal{R}$ coincides with the microcanonical average (i.e., the equilibrium value). Since the microcanonical ensemble assigns equal *a priori* probability to every microstate, there is no information in $\mathcal{R}$.

Now that we have an interpretation of E^a, let us examine conditions (i) and (iii). The latter is the second law of thermodynamics, as seen from the point of view of information theory: our initial information about the microstate of the system can only be lost (or transported from one place to another) in time, but never created, because all the initial conditions tend, as $t \to +\infty$, to the same final macrostate (the equilibrium). However, the most interesting condition for us is (i): it is easy to show that imposing (i), namely that the density of information is non-negative in any frame, is equivalent to requiring that E^a is timelike (or lightlike) future directed, namely

$$E^a E_a \leq 0 \qquad E^0 \geq 0. \tag{15}$$

This is where the contact with causality is established. In fact, if information is transported by a nonspacelike four-current, it propagates along causal trajectories and cannot exit the light cone (namely, no perturbation can transport information faster than light). This result may be seen as the finite-temperature analog of the Hawking-Ellis vacuum conservation theorem [35,40]. It establishes that *information* (in their case *energy*) is not spontaneously formed in an *equilibrium* (in their case *empty*) region and cannot enter it from outside its causal past. In this analogy, the Gibbs stability criterion plays the role of the dominant energy condition.

The inverse argument.—It is natural to ask whether we can reverse the argument and show that causality implies stability. This is in general not true (see, e.g., [12,13]). In fact, let us assume that we still have an information current E^a, defined by Eq. (3), and that conditions (ii) and (iii) are valid (they are typically ensured by construction when there is an entropy current). The causality requirement reduces to imposing that E^a is timelike and lightlike, but this does not specify its orientation. It might be the case that E^a, for some configurations, is past directed, generating instability. Thus, in general

$$(\text{causality}) \not\Rightarrow (\text{i}).$$

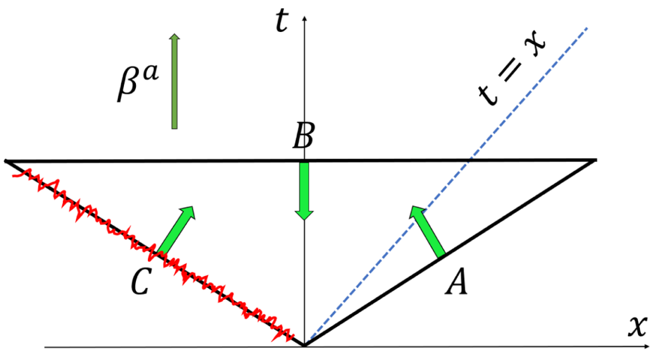

FIG. 2. Visualization of the geometric argument for the theorem of Bemfica *et al.* [7]. All the edges of the triangle ABC are spacelike. We create an arbitrary initial perturbation (in red) on the side C. Since A is outside the causal future of C, we are free to set the perturbation to zero on A. The inverse temperature four-vector β^a (dark green) is aligned with the t axis. The light green arrows are a Euclidean representation of the unit normals to the edges.

However, to fix the orientation we only need to assume that there is a preferred reference frame in which $E^0 \geq 0 \; \forall \; \delta\varphi_i$. It is natural, and it usually simplifies the calculations, to take this reference frame to be aligned with the equilibrium inverse-temperature four-vector β^a, which always exists, is unique, and is timelike future directed [16,41,42]. Hence, we can conclude that

$$(\text{causality}) + (E^a \beta_a \leq 0) \Rightarrow (\text{i}),$$

which is consistent with the more general theorem of Bemfica *et al.* [7].

We can give a more rigorous geometrical proof of this result, considering the triangle in Fig. 2, assuming causality and that $E^a \beta_a \leq 0$. The setting is similar to that of the previous geometric argument, however, note that now (t, x) is not an arbitrary inertial frame, but it has been chosen is such a way that $\beta^a \propto \delta^a_t$. Furthermore, the arbitrary initial perturbation has now been imposed on the side C of the triangle and not on the x axis. Again we can apply the Gauss theorem, to obtain

$$E[A] + E[B] + E[C] \leq 0. \qquad (16)$$

Since there is no perturbation on A, we know that $E[A] = 0$. Furthermore, given that the normal to B is

$$n^a[B] = -\frac{\beta^a}{\sqrt{-\beta^b \beta_b}}, \qquad (17)$$

we can use the condition $E^a \beta_a \leq 0$ to show that $E[B] \geq 0$. Hence, we have

$$-E[C] \geq E[B] \geq 0. \qquad (18)$$

Noting that $E[C]$ is computed taking the normal to C future directed, as in Fig. 2, we conclude that $-E[C]$ quantifies the information contained in C. Its positiveness, for any possible choice of initial perturbation on C and for any possible triangle (having the properties described in Fig. 2), leads to (i) and hence to stability.

Example 1: perfect fluids.—We conclude the Letter with a couple of examples. Consider the information current of a perfect fluid (7), assuming that $\delta\sigma = 0$ to first order. Then, the condition of stability in the fluid rest frame reduces to (note that $u^a = T\beta^a$)

$$-TE^a u_a = (\rho + p)\frac{\delta u^b \delta u_b}{2} + \frac{(\delta p)^2}{2(\rho + p)c_s^2} \geq 0. \qquad (19)$$

This produces the conditions $\rho + p > 0$ (positive inertial mass [24]) and $c_s^2 \geq 0$ (stability of the fluid against compression), which exist also in the Newtonian theory. The causality requirement $E^a E_a \leq 0$ reads

$$(\rho + p)\delta u^b \delta u_b + \frac{(\delta p)^2}{(\rho + p)c_s^2} \pm 2\delta p \sqrt{\delta u^b \delta u_b} \geq 0, \qquad (20)$$

which produces the well-known condition $c_s^2 \leq 1$ (subluminal speed of sound). The reader might be surprised that $c_s^2 \leq 1$ is also a stability condition. After all, a sound wave that propagates faster than light is still governed by a wave equation, hence its amplitude should remain bounded over time. However, again we need to remember that a system is *thermodynamically* stable if it is stable also to virtual processes. One can verify that a virtual process in which the amplitude of a sound wave grows with time increases the entropy of the fluid in those reference frames in which the sound wave moves backward in time, generating instability [43]. Indeed, it is well known that if a causal microscopic Lagrangian produces an effective macroscopic fluid theory with $c_s^2 > 1$, then the equilibrium state is unstable and the perfect fluid description is not applicable, because some high frequency modes must grow [12,44,45].

Example 2: Cattaneo equation.—As a second example we consider a rigid infinite solid bar ($1 + 1$ dimensions in flat spacetime), with uniform density, and we model the heat propagation within extended irreversible thermodynamics [46,47]. We take the fields $(\varphi_i) = (T, q)$, representing temperature and heat flux, as degrees of freedom and impose, in the rest-frame of the solid, the conservation law

$$nc_p \partial_t T + \partial_x q = 0. \qquad (21)$$

The (t, x) components of the entropy current are postulated to be

$$s^a = \left(s - \frac{1}{2}\chi q^2, \frac{q}{T}\right), \qquad (22)$$

where $s = s(T)$ is the equilibrium entropy density. Combining the conservation law (21) and the constitutive relation (22), one can show (just apply the technique of [10]) that the information current is

$$E^a = \left(\frac{nc_p(\delta T)^2}{2T^2} + \frac{1}{2}\chi(\delta q)^2, \frac{\delta q \delta T}{T^2} \right). \quad (23)$$

The requirement $E^0 > 0$ for $\delta\varphi_i \neq 0$ immediately produces the stability conditions

$$c_p > 0 \qquad \chi > 0, \quad (24)$$

the first ensuring stability to heat diffusion [1], the second to fluctuations of q. The requirement that E^a should not be spacelike ($E^a E_a \leq 0$) produces

$$\frac{1}{\chi n c_p T^2} \leq 1. \quad (25)$$

This is, indeed, the causality condition of the model (but it is also an important stability condition, see [27], Appendices 3 and 4). In fact, if we postulate an information annihilation rate $\nabla_a E^a = -(\delta q)^2/(\kappa T^2) \leq 0$ ($\kappa > 0$ is the heat conductivity coefficient), the resulting linearized field equation is the Cattaneo equation

$$\chi n c_p T^2 \partial_t^2 T - \partial_x^2 T + \frac{nc_p}{\kappa}\partial_t T = 0, \quad (26)$$

whose characteristic maximum signal propagation speed is $(\chi n c_p T^2)^{-1/2}$ [48,49]. Again, we see that the causality condition is merely thermodynamic (it involves only thermodynamic coefficients) and is unaffected by the value of the kinetic coefficient κ. In fact, while causality is a geometric constraint on the direction of the information current, κ only quantifies the rate at which information is destroyed. In the limit in which $\kappa \to +\infty$, heat does not propagate infinitely fast. Instead, information becomes a conserved quantity, and (26) becomes a nondissipative causal wave equation.

Conclusions.—On the practical side, our work shows that the entropy-based stability criterion developed in [10] is enough also to ensure linear causality, simplifying the job of testing the reliability of a theory. On the theoretical side, it reveals the central importance of the information current E^a in relativistic hydrodynamics, shedding new light on the role of information theory in a relativistic context. The reason why it took so long to achieve this understanding is that the focus has been up to now on trying to connect causality with *hydrodynamic* stability, while the real connection is with *thermodynamic* stability, which is a much more complete reliability criterion.

This work was supported by the Polish National Science Centre Grants No. SONATA BIS 2015/18/E/ST9/00577 and No. OPUS 2019/33/B/ST9/00942. Partial support comes from PHAROS, COST Action CA16214. L. G. thanks M. Shokri and G. Torrieri for stimulating discussions. We also thank M. M. Disconzi and the anonymous referees for providing useful comments, which helped us improve the clarity of the Letter.

[1] D. Kondepudi and I. Prigogine, *Modern Thermodynamics* (John Wiley and Sons, Ltd., New York, 2014).
[2] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, USA, 1995).
[3] R. Geroch and L. Lindblom, Phys. Rev. D **41**, 1855 (1990).
[4] W. A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) **151**, 466 (1983).
[5] T. S. Olson, Ann. Phys. (N.Y.) **199**, 18 (1990).
[6] R. Geroch and L. Lindblom, Ann. Phys. (N.Y.) **207**, 394 (1991).
[7] F. S. Bemfica, M. M. Disconzi, and J. Noronha, arXiv:2009.11388.
[8] G. S. Denicol, T. Kodama, T. Koide, and P. Mota, J. Phys. G Nucl. Phys. **35**, 115102 (2008).
[9] S. Pu, T. Koide, and D. H. Rischke, Phys. Rev. D **81**, 114039 (2010).
[10] L. Gavassino, Classical Quantum Gravity **38**, 21LT02 (2021).
[11] We restrict our attention to linear causality, namely to the requirement that the retarded Green's function of the linearized problem should vanish outside the future light cone [12].
[12] Y. Aharonov, A. Komar, and L. Susskind, Phys. Rev. **182**, 1400 (1969).
[13] M. Kiamari, M. Rahbardar, M. Shokri, and N. Sadooghi, Phys. Rev. D **104**, 076023 (2021).
[14] R. Pathria and P. D. Beale, in *Statistical Mechanics*, 3rd ed., edited by R. Pathria and P. D. Beale (Academic Press, Boston, 2011), pp. 583–635.
[15] L. Gavassino and M. Antonelli, Classical Quantum Gravity **37**, 025014 (2020).
[16] L. Gavassino, Found. Phys. **50**, 1554 (2020).
[17] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed. (Wiley, New York, NY, 1985).
[18] K. Huang, *Statistical Mechanics*, 2nd ed. (John Wiley & Sons, New York, 1987).
[19] L. Landau and E. Lifshitz, *Statistical Physics* (Elsevier Science, New York, 2013), Vol. 5.
[20] E. Stueckelberg, Helv. Phys. Acta **35**, 568 (1962).
[21] W. Israel, in *E.C.G. Stueckelberg, An Unconventional Figure of Twentieth Century Physics: Selected Scientific Papers with Commentaries*, edited by J. Lacki, H. Ruegg, and G. Wanders (Birkhäuser Basel, Basel, 2009), pp. 101–113.
[22] M. Grmela and H. C. Öttinger, Phys. Rev. E **56**, 6620 (1997).
[23] Note that, in Eq. (3), Σ does not necessarily cover all the space. For example, if we define F and H to be two portions of a same fluid (with H infinitely larger than F), Σ must cover *only* the support of F, and not that of H. Since the

distinction between F and H is ultimately a convention, we need to require $E \geq 0$ for any Σ spacelike, leading to condition (i). See also [19], Sec. 21 "Thermodynamic inequalities" for a similar argument.

[24] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Co., San Francisco, 1973).

[25] J. LaSalle and S. Lefschetz, *Stability by Liapunov's Direct Method: With Applications*, Mathematics in Science and Engineering Vol. 4 (Academic Press, New York, 1961).

[26] I. Prigogine, Science **201**, 777 (1978).

[27] L. Gavassino, M. Antonelli, and B. Haskell, Phys. Rev. D **102**, 043018 (2020).

[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.010606 for more details. In Part 1, we explore in greater detail the bridge with non-equilibrium thermodynamics, statistical mechanics, and information theory; in Part 2, we generalize the stability-causality relations to $3+1$ dimensions, in curved spacetime.

[29] By *on shell* we mean "along solutions of the field equations"; by *off shell* we mean "independently from the field equations".

[30] P. Kovtun, G. D. Moore, and P. Romatschke, Phys. Rev. D **84**, 025006 (2011).

[31] G. Torrieri, J. High Energy Phys. 02 (2021) 175.

[32] L. Gavassino, M. Antonelli, and B. Haskell, Symmetry **12**, 1543 (2020).

[33] W. Israel and J. Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979).

[34] Equation (9) is written local inertial coordinates, with units such that $h^3 = g_s$, where h is Planck's constant and g_s the spin degeneracy.

[35] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2011).

[36] T. Dray and C. Hellaby, J. Math. Phys. (N.Y.) **35**, 5922 (1994).

[37] J. C. Feng, Phys. Rev. D **98**, 104035 (2018).

[38] Our argument tells us nothing about how signals propagate inside a region where $\delta\varphi_i$ is *already* different from zero. For this reason, our argument can serve only to prove *linear* causality, as defined in [12].

[39] E. T. Jaynes, Phys. Rev. **106**, 620 (1957).

[40] B. Carter, in The Future of Theoretical Physics and Cosmology, edited by G. W. Gibbons, E. P. S. Shellard, and S. J. Rankin (Cambridge University Press, 2003), pp. 177–184.

[41] F. Becattini, Phys. Rev. Lett. **108**, 244502 (2012).

[42] F. Becattini, Acta Phys. Pol. B **47**, 1819 (2016).

[43] L. Gavassino, arXiv:2111.05254.

[44] M. Ruderman, Phys. Rev. **172**, 1286 (1968).

[45] S. A. Bludman and M. A. Ruderman, Phys. Rev. D **1**, 3243 (1970).

[46] D. Jou, J. Casas-Vázquez, and G. Lebon, Rep. Prog. Phys. **51**, 1105 (1999).

[47] L. Gavassino and M. Antonelli, Front. Astron. Space Sci. **8**, 92 (2021).

[48] P. Kostädt and M. Liu, Phys. Rev. D **62**, 023003 (2000).

[49] R. Courant and D. Hilbert, *Methods of Mathematical Physics* (John Wiley & Sons, New York, 1953), Vols. 1 and 2.