Finite-Time Quantum Landauer Principle and Quantum Coherence

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The Landauer principle states that any logically irreversible information processing must be accompanied by dissipation into the environment. In this Letter, we investigate the heat dissipation associated with finite-time information erasure and the effect of quantum coherence in such processes. By considering a scenario wherein information is encoded in an open quantum system whose dynamics are described by the Markovian Lindblad equation, we show that the dissipated heat is lower bounded by the conventional Landauer cost, as well as a correction term inversely proportional to the operational time. To clarify the relation between quantum coherence and dissipation, we derive a lower bound for heat dissipation in terms of quantum coherence. This bound quantitatively implies that the creation of quantum coherence in the energy eigenbasis during the erasure process inevitably leads to additional heat costs. The obtained bounds hold for arbitrary operational time and control protocol. By following an optimal control theory, we numerically present an optimal protocol and illustrate our findings by using a single-qubit system.

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Introduction.—Any irreversible information processing unavoidably incurs a thermodynamic cost. This fundamental relationship between information and thermodynamics is embodied in the Landauer principle [1]. The principle states that the amount of heat dissipation Q required to erase information is lower bounded by the entropy change ΔS of information-bearing degrees of freedom, $\beta Q \geq \Delta S$. Here, β is the inverse temperature of the environment. This inequality—referred to as the Landauer bound or limit—lays a foundation for the thermodynamics of information [2–4] and computation [5,6] and provides a resolution to the paradox of Maxwell's demon [7]. The attainability of the Landauer bound in the slow quasistatic limit has been experimentally verified in various systems [8–13]. Nonetheless, modern computing requires fast memory erasure, which generally comes with a thermodynamic cost far beyond the Landauer limit. Thus, improving our understanding of heat dissipation in finite-time information processing is relevant to the development of efficient computing devices.

In recent years, the Landauer principle has been extensively studied in the framework of stochastic thermodynamics from the classical to the quantum regime [14–33]. One central issue is investigating how much heat needs to be dissipated to erase information in far-from-equilibrium situations. When the erasure fidelity is predetermined, a trade-off between dissipation and operation time apparently occurs, as indicated in slow-driving protocols [34–36]. Recently, Proesmans *et al.* have derived a trade-off relation for arbitrary driving speed in classical bits modeled by a double-well potential [37,38]. They showed that the minimum dissipation for erasing a classical bit is bounded from below by the Landauer cost, as well as a term inversely proportional to the operational time, which is somewhat reminiscent of a speed limit [39–42]. In the short-time limit, the finite-time correction term is dominant over the Landauer cost. From this development, quantum extensions relevant to information erasure in qubits are strongly desired.

In addition, in the quantum regime, quantum coherence is one of the crucial aspects. Recently, the role of quantum coherence has been intensively discussed in the context of finite-time thermodynamics [43–50]. In the case of conventional heat engines driven by heat baths with different temperatures, quantum coherence hinders the thermodynamic performance in the linear response regime with respect to small driving amplitudes [51] or small driving speeds [52]. On the other hand, power outputs can be enhanced by coherence in several far-from-equilibrium models [44,53–55]. Thus, the role of quantum coherence in nonequilibrium thermodynamics is highly elusive. Concerning the Landauer principle, Miller *et al.* have examined the slow-driving case and have found that quantum coherence generates additional dissipation [56]. This property is consistent with the aforementioned slow-driving case in heat engines. Hence, as the next step toward a complete understanding of the quantum Landauer principle, determining the effect of an arbitrary driving speed on the relation between quantum coherence and dissipation is clearly important.

From these two backgrounds, in this Letter, we address the following questions: (i) What is the effect of a finite-time protocol with a finite erasure error on heat dissipation? (ii) What is the role of quantum coherence in heat dissipation for an arbitrary driving speed? These two are clearly fundamental for the in-depth understanding of the quantum Landauer principle. To answer these questions, we consider a dynamical class described by the Markovian Lindblad equations, which guarantee thermodynamically consistent dynamics. We then rigorously provide quantitative answers to these questions. The result for the first question is presented below in inequality (3), which is identified as a quantum extension of the classical version obtained in Ref. [37]. For the second question, we show the inevitable heat dissipation caused by quantum coherence as below in inequality (10), which explicitly shows that quantum coherence in the qudit always induces additional heat dissipation compared to classical protocols. The obtained results hold for an arbitrary control protocol and the driving speed. By using a single-qubit system, we present an optimal control protocol obtained numerically by optimizing the dissipation and erasure fidelity, which supports our findings.

Erasure setup and first main result.—We consider an information erasure process realized by using a controllable open qudit system with an arbitrary dimension d and an infinite heat bath at the inverse temperature $\beta = 1/(k_B T)$. The former and latter are referred to as information-bearing and non-information-bearing degrees of freedom, respectively. The information content we want to erase is encoded in the density matrix of the system, which is typically set to a maximally mixed state $\varrho_0 = \mathbb{I}/d$. This mixed state is regarded as an ensemble of many initial pure states that are subject to reset. The maximally mixed state is sufficient to understand the average dissipated heat of the erasure process for all initial pure states, and if an erasure protocol can reliably reset the qudit from the maximally mixed state, then it does so for an arbitrary pure state (see Supplemental Material [57] for details). The information is erased in a finite time τ by varying the control Hamiltonian H_t. Here, we focus on a dynamical class in which the system is always weakly attached to the heat bath, and its dynamics are described by the Lindblad master equation [68,69]. Let ϱ_t be the density matrix of the system at time t; then, the dynamics are described as

$$\dot{\varrho}_t = \mathcal{L}_t(\varrho_t),$$
$$\mathcal{L}_t(\varrho) \coloneqq -i\hbar^{-1}[H_t, \varrho] + \sum_k \mathcal{D}[L_k(t)]\varrho, \qquad (1)$$

with the dissipator given by $\mathcal{D}[L]\varrho \coloneqq L\varrho L^\dagger - \{L^\dagger L, \varrho\}/2$. Here, $L_k(t)$ are time-dependent jump operators that account for transitions between different energy eigenstates. The dot indicates the time derivative, and $[\circ, \star]$ and $\{\circ, \star\}$ denote the commutator and anticommutator of the two operators, respectively. Assume that the generator $\mathcal{L}_t$ obeys the quantum detailed balance with respect to the Hamiltonian H_t at all times [70]. This condition ensures that the thermal state $\pi_t \coloneqq e^{-\beta H_t}/\mathrm{tr}\{e^{-\beta H_t}\}$ is the instantaneous stationary state of the Lindblad equation, $\mathcal{L}_t(\pi_t) = 0$. Hereinafter, both the Planck constant and the Boltzmann constant are set to unity: $\hbar = k_B = 1$.

Resetting the system state to a specific state results in a change in the system entropy ΔS, which is quantified by the von Neumann entropy. The entropy decrease in the information-bearing degrees of freedom is compensated by the amount of heat transferred to the environment Q. The change in the system entropy and the average dissipated heat are, respectively, written as follows [71]:

$$\Delta S \coloneqq -\mathrm{tr}\{\varrho_0 \ln \varrho_0\} + \mathrm{tr}\{\varrho_\tau \ln \varrho_\tau\},$$
$$Q \coloneqq -\int_0^\tau \mathrm{tr}\{H_t \dot{\varrho}_t\} dt. \qquad (2)$$

The Landauer bound can be immediately derived from the second law of thermodynamics, which states that the irreversible entropy production Σ_τ during period τ is always non-negative [72], $\Sigma_\tau = -\Delta S + \beta Q \geq 0$.

Under the given setup, we now show the first main result, leaving the details of the proof in the Supplemental Material [57],

$$\beta Q \geq \Delta S + \frac{\|\varrho_0 - \varrho_\tau\|_1^2}{2\tau\bar{\gamma}_\tau} \geq \Delta S + \frac{[2(1-1/d) - \epsilon]^2}{2\tau\bar{\gamma}_\tau}. \qquad (3)$$

Here, $\|\ldots\|_1$ is the trace norm, $\epsilon \coloneqq \|\varrho_\tau - |0\rangle\langle 0|\|_1$ quantifies the distance error to the ground state, and $\bar{\gamma}_\tau$ is the time-averaged dynamical activity that characterizes the timescale of the thermal relaxation, defined as $\bar{\gamma}_\tau \coloneqq \tau^{-1}\int_0^\tau \sum_k \mathrm{tr}\{L_k(t)\varrho_t L_k(t)^\dagger\} dt$ [39,73]. The obtained bound implies that the cost of erasing the information within a finite time is at least the Landauer cost plus a distance term proportional to τ^{-1}. The result holds for *arbitrary* operational time, driving speed, and the final state, which can be far from the ground state.

Physically important aspects of this relation are now in order. First, suppose that we use a protocol in which the stored information is fully erased—that is, ϱ_τ is equal to $|0\rangle\langle 0|$. Then, the bound inequality (3) becomes

$$\beta Q \geq \ln d + \frac{2(1-1/d)^2}{\tau\bar{\gamma}_\tau}. \qquad (4)$$

For a fast erasure $\tau\bar{\gamma}_\tau \ll 1$, the second term in the lower bound becomes dominant. On the other hand, the lower bound reduces exactly to the Landauer cost in the slow-erasure limit $\tau\bar{\gamma}_\tau \gg 1$. The inequality (4) is the quantum extension of the classical finite-time Landauer bound derived in Ref. [37] [see inequality (9) therein]. Next, consider the case of imperfect information erasure; that is, ϱ_τ is not equal to $|0\rangle\langle 0|$. In this case, we can discuss how quantum coherence remaining at the final time is related to dissipation, thus revealing an intrinsic difference between classical and quantum protocols. To this end, let $\Lambda(\circ) \coloneqq \sum_n \Pi_n(\circ)\Pi_n$ be the dephasing map in the energy eigenbasis of the ending Hamiltonian H_τ, in which Π_n is the projection operator onto the nth eigenspace. Because the trace norm is contractive under completely positive trace-preserving maps, we can decompose $\|\varrho_0 - \varrho_\tau\|_1^2$ into classical and quantum parts as

$$\|\varrho_0 - \varrho_\tau\|_1^2 = \|\Lambda(\varrho_0) - \Lambda(\varrho_\tau)\|_1^2 + C_{\rm res}, \tag{5}$$

where $C_{\rm res} \geq 0$ is a quantum term quantifying the residual coherence in the final state with respect to the energy eigenbasis $\{|n_\tau\rangle\}$. Analogously, the von Neumann entropy production can also be decomposed as

$$\Delta S = \Delta S_{\rm cl} + C_{\rm rel}, \tag{6}$$

where $\Delta S_{\rm cl} \coloneqq S[\Lambda(\varrho_0)] - S[\Lambda(\varrho_\tau)]$ is the classical entropy change in terms of the population distribution and $C_{\rm rel} \coloneqq S[\Lambda(\varrho_\tau)] - S(\varrho_\tau) \geq 0$ is the relative entropy of coherence in the final state [74]. Consequently, the lower bound in inequality (3) can be separated into classical and quantum terms as

$$\beta Q \geq \underbrace{\Delta S_{\rm cl} + \frac{\|\Lambda(\varrho_0) - \Lambda(\varrho_\tau)\|_1^2}{2\tau\bar{\gamma}_\tau}}_{\text{classical}} + \underbrace{C_{\rm rel} + \frac{C_{\rm res}}{2\tau\bar{\gamma}_\tau}}_{\text{quantum}}. \tag{7}$$

The classical term is a bound that can also be derived by using a classical probabilistic process. The above expression indicates that the quantum coherence left in the final state contributes a non-negative term in the lower bound.

Second main result.—The relation (7) describes the quantum coherence in the final state. However, this is not enough to capture the role of coherence because quantum coherence is generated during the time it takes to reach the final state. Therefore, in addition to the above argument, we discuss the general relation between quantum coherence and heat dissipation during a finite time τ. To achieve a general result, we assume a general initial density matrix and consider the time evolution obeying the dynamics (1). Quantum coherence in the energy eigenbasis is typically generated because of the presence of noncommuting terms in the Hamiltonian. For the present aim, using the following intuitive ℓ_1 norm of coherence [74] conveniently helps quantify the amount of coherence contained in the state ϱ_t:

$$C_{\ell_1}(\varrho_t) \coloneqq \sum_{m\neq n} |\langle m_t|\varrho_t|n_t\rangle|. \tag{8}$$

Here, $\{|n_t\rangle\}$ is the instantaneous energy eigenbasis of the Hamiltonian H_t [75]. Mathematically, C_{ℓ_1} is the sum of the absolute values of all the off-diagonal elements with respect to the energy eigenbasis. This quantifier was shown to be a suitable measure of coherence [74] and is widely used in literature [76]. The amount of coherence accumulated during period τ can thus be defined naturally,

$$\mathcal{C}_\tau \coloneqq \int_0^\tau C_{\ell_1}(\varrho_t) dt. \tag{9}$$

The second main result of this Letter is that the average dissipated heat in a finite time is lower bounded by the Landauer cost plus a non-negative coherence term (see the Supplemental Material [57]),

$$\beta Q \geq \Delta S + \frac{\bar{\gamma}_\tau^R \mathcal{C}_\tau^2}{2\tau}. \tag{10}$$

In this equation, $\bar{\gamma}_\tau^R$ is the time average of the characteristic relaxation rate, given by $(\bar{\gamma}_\tau^R)^{-1} = \tau^{-1}\int_0^\tau [\sum_k \|L_k(t)\|_\infty^2]^{-1} dt$ in the $d = 2$ case (see the Supplemental Material [57] for the form of $\bar{\gamma}_\tau^R$ in the generic case). Here, $\|\ldots\|_\infty$ is the spectral norm. We emphasize that the result holds for the *arbitrary* operational time and driving speed. The obtained relation sets a lower bound on dissipation in terms of information-theoretic entropy and quantum coherence. The inequality (10) implies that the quantum coherence produced during information erasure must be accompanied by additional heat. The greater the generation of coherence, the more heat is dissipated. The relation (10) is valid for the entire driving speed regime, thus covering the slow-driving limit [56]. Combining this with the first main result (3), the Landauer bound can be strengthened as $\beta Q \geq \Delta S + \max\{\|\varrho_0 - \varrho_\tau\|_1^2/\bar{\gamma}_\tau, \bar{\gamma}_\tau^R \mathcal{C}_\tau^2\}/(2\tau)$, when we use the maximally mixed state for the initial state.

Numerical demonstration with the optimal control theory.—We exemplify our findings with a simple model of information erasure by using a single qubit. Two-level qubit systems are relevant in quantum computation and are commonly used to store memory in measurement-driven engines [77,78]. The qubit can be viewed as a spin-1/2 particle weakly coupled to a large bath of bosonic harmonic oscillators [79], evolving according to the Hamiltonian

$$H_t = \frac{\epsilon_t}{2}[\cos(\theta_t)\sigma_z + \sin(\theta_t)\sigma_x] \tag{11}$$

and two jump operators $L_1(t) = \sqrt{\alpha\epsilon_t(N_t + 1)}|0_t\rangle\langle 1_t|$, $L_2(t) = \sqrt{\alpha\epsilon_t N_t}|1_t\rangle\langle 0_t|$, in which $\sigma_{x,y,z}$ are the Pauli matrices, α is the coupling strength, $N_t \coloneqq 1/(e^{\beta\epsilon_t} - 1)$ is the Planck distribution, and ϵ_t and θ_t are time-dependent control parameters. The quantity ϵ_t is the energy gap between the instantaneous energy eigenstates, whereas θ_t controls the relative strength of coherent tunneling to energy bias [79]. If θ_t is fixed at all times, it corresponds to a classical protocol. Otherwise, quantum coherence in the energy eigenbasis is generated, implying a genuine quantum protocol.

Resetting the qubit to the ground state $|0\rangle$ with a probability close to 1 can be achieved with various control protocols. For example, we can either gradually increase the energy gap ϵ_t from an initial value $\epsilon_0 \approx 0$ to a final value $\beta\epsilon_\tau \gg 1$ while also changing θ_t [56], or we can quench the Hamiltonian at $t = 0$ and let the system relax to an equilibrium state close to $|0\rangle\langle 0|$. Here we particularly consider the Pareto-optimal protocols [80], which optimize two incompatible objectives: the success probability and the average dissipated heat. Specifically, we solve the optimization problem of minimizing the following multi-objective functional:

$$\mathcal{F}[\{\epsilon_t, \theta_t\}] \coloneqq \lambda\tau\beta Q - (1-\lambda)F(\varrho_\tau, |0\rangle\langle 0|). \quad (12)$$

Here, $\lambda \in [0, 1)$ is a weighting factor, and $F(\varrho, \sigma) = \mathrm{tr}\{\sqrt{\sqrt{\varrho}\sigma\sqrt{\varrho}}\}^2$ is the fidelity of the two quantum states ϱ and σ [81]. The first and second terms of the functional correspond to the average dissipated heat and erasure fidelity, respectively. When $\lambda = 0$, the reliability of the information erasure takes precedence over dissipation, and the final state is optimized to be as close to the ground state as possible. The $\lambda > 0$ case indicates that the dissipation is also minimized under a given allowable error of the final state. Because of physical limitations, imposing constraints on the control parameters is reasonable. Hereinafter, we set the following lower and upper bounds on the parameters $\beta\epsilon_t \in [0.4, 10]$ and $\theta_t \in [-\pi, \pi]$.

Obtaining the analytical solution for the optimization problem in Eq. (12) under the given constraints is a daunting task. Hence, we numerically solve the optimal protocol by discretizing the protocol and minimizing the functional $\mathcal{F}$ by using a nonlinear programming method [57]. To demonstrate the effect of quantum coherence on information erasure, we consider $\varrho_0 = \mathbb{I}/2$. The initial state thus does not contain any amount of coherence. We examine two control protocols: the "optimal" protocol found through the nonlinear programming method for $\lambda > 0$ satisfying $(1-\lambda)/\lambda = 10^4$ and the "nonoptimal" protocol used in Ref. [56], in which $\epsilon_t = \epsilon_0 + (\epsilon_\tau - \epsilon_0)\sin(\pi t/2\tau)^2$ and $\theta_t = \pi(t/\tau - 1)$. Both protocols drive the system to the ground state with the same order of error at the final time. Figure 1(a) shows the time variation of the control parameters ϵ_t and θ_t for each protocol. The energy gap ϵ_t in each protocol increases gradually in different ways and eventually reaches the same value. Interestingly, in the optimal protocol, θ_t is always fixed at 0; hence, no coherence is created during the period τ. On the other hand, the nonoptimal protocol constantly changes θ_t and generates quantum coherence. We express the density matrix in the Bloch representation and plot the time evolution of the Bloch vector in Fig. 1(b). Notice that the qubit evolves through completely different paths toward the ground state for each protocol. Figure 1(c) shows the average dissipated heat for each protocol at each time. Note also that the optimal protocol clearly dissipates less heat than the nonoptimal protocol, which is consistent with our finding that the generation of coherence incurs additional heat costs.

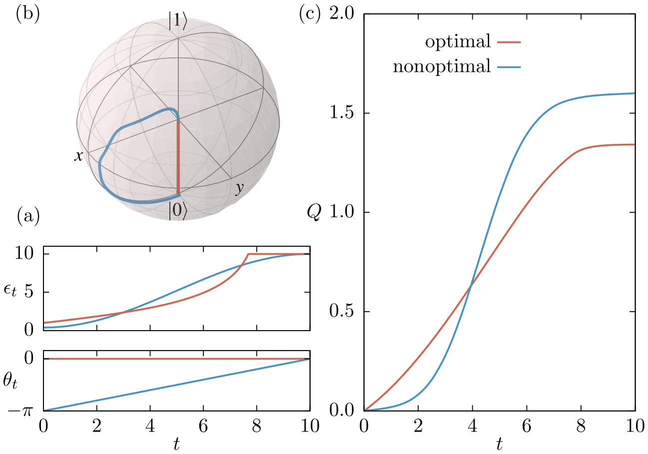

FIG. 1. Numerical results obtained with the optimal and nonoptimal protocols, depicted by the red and blue lines, respectively. (a) Time variations of the control parameters ϵ_t and θ_t. (b) Geometrical representation of the time evolution of the qubit on the Bloch sphere. (c) Average dissipated heat over time. Other parameters are given by $\alpha = 0.2$, $\beta = 1$, $\epsilon_0 = 0.4$, $\epsilon_\tau = 10$, and $\tau = 10$.

From an energetic point of view, quantum coherence has been shown to be detrimental to the erasure of information. To optimize dissipation, the qubit should behave as a classical bit. Further, we numerically find that this is the case even when dissipation is not minimized—that is, when $\lambda = 0$. In other words, quenching the Hamiltonian to $H_0 = \epsilon_\tau\sigma_z/2$ at time $t = 0$ and relaxing the system to equilibrium is the best protocol to bring the qubit as close as possible to the ground state. We conjecture that the quench protocol, namely, thermal relaxation, is the optimal protocol in terms of erasure reliability. Note that the conjecture is restricted to the $\varrho_0 = \mathbb{I}/2$ case because the shortcut-to-equilibration protocol [82] or an optimal protocol [57] may outperform the quench protocol for the $\varrho_0 \neq \mathbb{I}/2$ case. When the initial state is not described by the maximally mixed state and contains some coherence, we also find that the optimal protocol hardly produces quantum coherence as compared to the nonoptimal protocol (see the Supplemental Material [57] for details). From the analytical and numerical evidence, we can conclude that the creation of quantum coherence should be avoided when erasing information.

Next, we investigate the performance of the bound inequality (3) with optimal and nonoptimal protocols. We vary the operational time and plot the dissipated heat βQ, derived bound $\Delta S + \|\varrho_0 - \varrho_\tau\|_1^2/(2\tau\bar{\gamma}_\tau)$, Landauer cost ΔS, and coherence term $\bar{\gamma}_\tau^R \mathcal{C}_\tau^2/(2\tau)$ as functions of τ in Fig. 2. Notice that the dissipated heat is always bounded from below by the bound inequality (3) and is far beyond

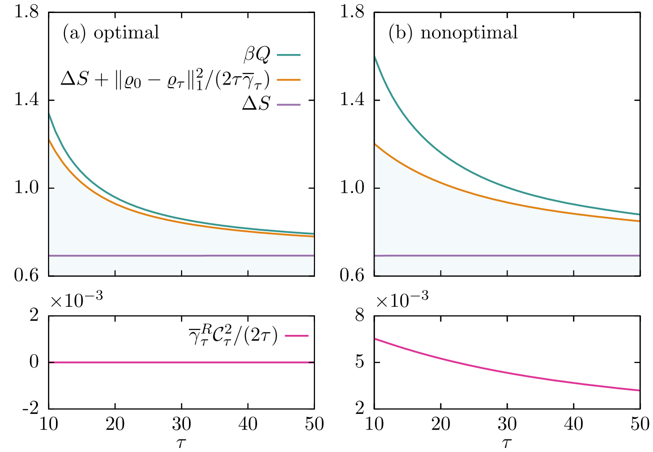

FIG. 2. Numerical illustration of the bound inequality (3) with the (a) optimal and (b) nonoptimal protocols. Upper: shows the average dissipated heat βQ, derived bound $\Delta S + \|\varrho_0 - \varrho_\tau\|_1^2/(2\tau\overline{\gamma}_\tau)$, and Landauer cost ΔS for each operational time. Lower: shows the amount of quantum coherence generated in the energy eigenbasis. The operational time τ is varied, while the other parameters are set to $\alpha = 0.2$, $\beta = 1$, $\epsilon_0 = 0.4$, and $\epsilon_\tau = 10$.

the Landauer cost. Particularly, in the case of the optimal protocol, the derived bound is tight and asymptotically saturated as τ increases. The optimal protocol is also less dissipative than the nonoptimal protocol for all operational times. Simultaneously, the optimal protocol does not create coherence, whereas a positive amount of coherence is generated in the nonoptimal protocol. Regarding the tightness of the derived bounds, inequality (3) can be tighter or looser than inequality (10). If little or no coherence is produced, the former is generally tighter than the latter. Conversely, when a large amount of coherence is generated—that is, when $\mathcal{C}_\tau \gg 1$—the latter is stronger than the former (see the Supplemental Material [57] for numerical illustrations).

Summary.—We derived the lower bound on the thermodynamic cost associated with finite-time information erasure for Markovian open quantum dynamics. The bound is far beyond the Landauer cost for fast control protocols. We also revealed the relation between quantum coherence and heat dissipation for the entire driving speed regime, stating that the creation of quantum coherence inevitably causes additional heat costs. In the context of the Landauer principle, this relation implies that quantum coherence is detrimental to erasing information from an energetic viewpoint. We confirmed the results with both optimal and nonoptimal protocols. Our findings are not only fundamentally critical but also helpful in establishing a design principle for efficient memory erasure. The generalization of the results obtained here to other cases, such as the finite-size environments and the non-Markovian regime [83], is a future study.

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[1] R. Landauer, Irreversibility and heat generation in the computing process, IBM J. Res. Dev. **5**, 183 (1961).
[2] T. Sagawa, Thermodynamics of information processing in small systems, Prog. Theor. Phys. **127**, 1 (2012).
[3] J. M. Parrondo, J. M. Horowitz, and T. Sagawa, Thermodynamics of information, Nat. Phys. **11**, 131 (2015).
[4] J. Goold, M. Huber, A. Riera, L. del Rio, and P. Skrzypczyk, The role of quantum information in thermodynamics—a topical review, J. Phys. A **49**, 143001 (2016).
[5] C. H. Bennett, The thermodynamics of computation—a review, Int. J. Theor. Phys. **21**, 905 (1982).
[6] D. H. Wolpert, The stochastic thermodynamics of computation, J. Phys. A **52**, 193001 (2019).
[7] K. Maruyama, F. Nori, and V. Vedral, Colloquium: The physics of Maxwell's demon and information, Rev. Mod. Phys. **81**, 1 (2009).
[8] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, Experimental verification of Landauer's principle linking information and thermodynamics, Nature (London) **483**, 187 (2012).
[9] Y. Jun, M. c. v. Gavrilov, and J. Bechhoefer, High-Precision Test of Landauer's Principle in a Feedback Trap, Phys. Rev. Lett. **113**, 190601 (2014).
[10] L. L. Yan, T. P. Xiong, K. Rehan, F. Zhou, D. F. Liang, L. Chen, J. Q. Zhang, W. L. Yang, Z. H. Ma, and M. Feng, Single-Atom Demonstration of the Quantum Landauer Principle, Phys. Rev. Lett. **120**, 210601 (2018).
[11] J. Hong, B. Lambson, S. Dhuey, and J. Bokor, Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits, Sci. Adv. **2**, e1501492 (2016).
[12] O.-P. Saira, M. H. Matheny, R. Katti, W. Fon, G. Wimsatt, J. P. Crutchfield, S. Han, and M. L. Roukes, Nonequilibrium thermodynamics of erasure with superconducting flux logic, Phys. Rev. Research **2**, 013249 (2020).
[13] S. Dago, J. Pereda, N. Barros, S. Ciliberto, and L. Bellon, Information and Thermodynamics: Fast and Precise Approach to Landauer's Bound in an Underdamped Micromechanical Oscillator, Phys. Rev. Lett. **126**, 170601 (2021).
[14] T. Sagawa and M. Ueda, Minimal Energy Cost for Thermodynamic Information Processing: Measurement and Information Erasure, Phys. Rev. Lett. **102**, 250602 (2009).
[15] M. Esposito and C. V. den Broeck, Second law and Landauer principle far from equilibrium, Europhys. Lett. **95**, 40004 (2011).
[16] S. Hilt, S. Shabbir, J. Anders, and E. Lutz, Landauer's principle in the quantum regime, Phys. Rev. E **83**, 030102 (R) (2011).
[17] S. Deffner and C. Jarzynski, Information Processing and the Second Law of Thermodynamics: An Inclusive, Hamiltonian Approach, Phys. Rev. X **3**, 041003 (2013).

[18] D. Reeb and M. M. Wolf, An improved Landauer principle with finite-size corrections, New J. Phys. **16**, 103011 (2014).

[19] C. Browne, A. J. P. Garner, O. C. O. Dahlsten, and V. Vedral, Guaranteed Energy-Efficient Bit Reset in Finite Time, Phys. Rev. Lett. **113**, 100603 (2014).

[20] S. Lorenzo, R. McCloskey, F. Ciccarello, M. Paternostro, and G. M. Palma, Landauer's Principle in Multipartite Open Quantum System Dynamics, Phys. Rev. Lett. **115**, 120403 (2015).

[21] J. Goold, M. Paternostro, and K. Modi, Nonequilibrium Quantum Landauer Principle, Phys. Rev. Lett. **114**, 060602 (2015).

[22] A. M. Alhambra and M. P. Woods, Dynamical maps, quantum detailed balance, and the Petz recovery map, Phys. Rev. A **96**, 022118 (2017).

[23] S. Campbell, G. Guarnieri, M. Paternostro, and B. Vacchini, Nonequilibrium quantum bounds to Landauer's principle: Tightness and effectiveness, Phys. Rev. A **96**, 042109 (2017).

[24] G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B. Vacchini, and M. Paternostro, Full counting statistics approach to the quantum non-equilibrium Landauer bound, New J. Phys. **19**, 103038 (2017).

[25] A. B. Boyd, D. Mandal, and J. P. Crutchfield, Thermodynamics of Modularity: Structural Costs beyond the Landauer Bound, Phys. Rev. X **8**, 031036 (2018).

[26] J. Klaers, Landauer's Erasure Principle in a Squeezed Thermal Memory, Phys. Rev. Lett. **122**, 040602 (2019).

[27] N. Shiraishi and K. Saito, Information-Theoretical Bound of the Irreversibility in Thermal Relaxation Processes, Phys. Rev. Lett. **123**, 110603 (2019).

[28] A. Dechant and Y. Sakurai, Thermodynamic interpretation of Wasserstein distance, arXiv:1912.08405.

[29] F. Buscemi, D. Fujiwara, N. Mitsui, and M. Rotondo, Thermodynamic reverse bounds for general open quantum processes, Phys. Rev. A **102**, 032210 (2020).

[30] A. M. Timpanaro, J. P. Santos, and G. T. Landi, Landauer's Principle at Zero Temperature, Phys. Rev. Lett. **124**, 240601 (2020).

[31] D. H. Wolpert, Strengthened Landauer bound for composite systems, arXiv:2007.10950.

[32] T. Van Vu and Y. Hasegawa, Lower Bound on Irreversibility in Thermal Relaxation of Open Quantum Systems, Phys. Rev. Lett. **127**, 190601 (2021).

[33] P. M. Riechers and M. Gu, Impossibility of achieving Landauer's bound for almost every quantum state, Phys. Rev. A **104**, 012214 (2021).

[34] P. R. Zulkowski and M. R. DeWeese, Optimal finite-time erasure of a classical bit, Phys. Rev. E **89**, 052140 (2014).

[35] P. R. Zulkowski and M. R. DeWeese, Optimal control of overdamped systems, Phys. Rev. E **92**, 032117 (2015).

[36] P. R. Zulkowski and M. R. DeWeese, Optimal protocols for slowly driven quantum systems, Phys. Rev. E **92**, 032113 (2015).

[37] K. Proesmans, J. Ehrich, and J. Bechhoefer, Finite-Time Landauer Principle, Phys. Rev. Lett. **125**, 100602 (2020).

[38] K. Proesmans, J. Ehrich, and J. Bechhoefer, Optimal finite-time bit erasure under full control, Phys. Rev. E **102**, 032105 (2020).

[39] N. Shiraishi, K. Funo, and K. Saito, Speed Limit for Classical Stochastic Processes, Phys. Rev. Lett. **121**, 070601 (2018).

[40] K. Funo, N. Shiraishi, and K. Saito, Speed limit for open quantum systems, New J. Phys. **21**, 013006 (2019).

[41] V. T. Vo, T. Van Vu, and Y. Hasegawa, Unified approach to classical speed limit and thermodynamic uncertainty relation, Phys. Rev. E **102**, 062132 (2020).

[42] T. Van Vu and Y. Hasegawa, Geometrical Bounds of the Irreversibility in Markovian Systems, Phys. Rev. Lett. **126**, 010601 (2021).

[43] M. Horodecki and J. Oppenheim, Fundamental limitations for quantum and nanoscale thermodynamics, Nat. Commun. **4**, 2059 (2013).

[44] R. Uzdin, A. Levy, and R. Kosloff, Equivalence of Quantum Heat Machines, and Quantum-Thermodynamic Signatures, Phys. Rev. X **5**, 031044 (2015).

[45] M. Lostaglio, K. Korzekwa, D. Jennings, and T. Rudolph, Quantum Coherence, Time-Translation Symmetry, and Thermodynamics, Phys. Rev. X **5**, 021001 (2015).

[46] K. Korzekwa, M. Lostaglio, J. Oppenheim, and D. Jennings, The extraction of work from quantum coherence, New J. Phys. **18**, 023045 (2016).

[47] G. Francica, J. Goold, and F. Plastina, Role of coherence in the nonequilibrium thermodynamics of quantum systems, Phys. Rev. E **99**, 042105 (2019).

[48] J. P. Santos, L. C. Céleri, G. T. Landi, and M. Paternostro, The role of quantum coherence in non-equilibrium entropy production, npj Quantum Inf. **5**, 23 (2019).

[49] G. Francica, F. C. Binder, G. Guarnieri, M. T. Mitchison, J. Goold, and F. Plastina, Quantum Coherence and Ergotropy, Phys. Rev. Lett. **125**, 180603 (2020).

[50] H. Tajima and K. Funo, Superconducting-like Heat Current: Effective Cancellation of Current-Dissipation Trade-off by Quantum Coherence, Phys. Rev. Lett. **127**, 190604 (2021).

[51] K. Brandner, M. Bauer, and U. Seifert, Universal Coherence-Induced Power Losses of Quantum Heat Engines in Linear Response, Phys. Rev. Lett. **119**, 170602 (2017).

[52] K. Brandner and K. Saito, Thermodynamic Geometry of Microscopic Heat Engines, Phys. Rev. Lett. **124**, 040602 (2020).

[53] M. O. Scully, K. R. Chapin, K. E. Dorfman, M. B. Kim, and A. Svidzinsky, Quantum heat engine power can be increased by noise-induced coherence, Proc. Natl. Acad. Sci. U.S.A. **108**, 15097 (2011).

[54] G. Watanabe, B. P. Venkatesh, P. Talkner, and A. del Campo, Quantum Performance of Thermal Machines over Many Cycles, Phys. Rev. Lett. **118**, 050601 (2017).

[55] P. Menczel, C. Flindt, and K. Brandner, Thermodynamics of cyclic quantum amplifiers, Phys. Rev. A **101**, 052106 (2020).

[56] H. J. D. Miller, G. Guarnieri, M. T. Mitchison, and J. Goold, Quantum Fluctuations Hinder Finite-Time Information Erasure near the Landauer Limit, Phys. Rev. Lett. **125**, 160602 (2020).

[57] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.010602 for details of analytical and numerical calculations, which includes Refs. [58–67].

[58] J. Maas, Gradient flows of the entropy for finite Markov chains, J. Funct. Anal. **261**, 2250 (2011).

[59] J. P. Garrahan, Simple bounds on fluctuations and uncertainty relations for first-passage times of counting observables, Phys. Rev. E **95**, 032134 (2017).

[60] T. Van Vu and Y. Hasegawa, Uncertainty relations for underdamped Langevin dynamics, Phys. Rev. E **100**, 032130 (2019).

[61] V. Cavina, A. Mari, and V. Giovannetti, Slow Dynamics and Thermodynamics of Open Quantum Systems, Phys. Rev. Lett. **119**, 050601 (2017).

[62] P. N. Shivakumar and K. H. Chew, A sufficient condition for nonvanishing of determinants, Proc. Am. Math. Soc. **43**, 63 (1974).

[63] J. Koenemann, G. Licitra, M. Alp, and M. Diehl, OpenOCL—Open Optimal Control Library, https://openocl.github.io/ (2019).

[64] T. Schmiedl and U. Seifert, Optimal Finite-Time Processes in Stochastic Thermodynamics, Phys. Rev. Lett. **98**, 108301 (2007).

[65] H. Then and A. Engel, Computing the optimal protocol for finite-time processes in stochastic thermodynamics, Phys. Rev. E **77**, 041105 (2008).

[66] E. Aurell, C. Mejía-Monasterio, and P. Muratore-Ginanneschi, Optimal Protocols and Optimal Transport in Stochastic Thermodynamics, Phys. Rev. Lett. **106**, 250601 (2011).

[67] A. P. Solon and J. M. Horowitz, Phase Transition in Protocols Minimizing Work Fluctuations, Phys. Rev. Lett. **120**, 180605 (2018).

[68] G. Lindblad, On the generators of quantum dynamical semigroups, Commun. Math. Phys. **48**, 119 (1976).

[69] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of *N*-level systems, J. Math. Phys. (N.Y.) **17**, 821 (1976).

[70] R. Alicki, On the detailed balance condition for non-Hamiltonian systems, Rep. Math. Phys. **10**, 249 (1976).

[71] R. Alicki, The quantum open system as a model of the heat engine, J. Phys. A **12**, L103 (1979).

[72] H. Spohn, Entropy production for quantum dynamical semigroups, J. Math. Phys. (N.Y.) **19**, 1227 (1978).

[73] C. Maes, Frenesy: Time-symmetric dynamical activity in nonequilibria, Phys. Rep. **850**, 1 (2020).

[74] T. Baumgratz, M. Cramer, and M. B. Plenio, Quantifying Coherence, Phys. Rev. Lett. **113**, 140401 (2014).

[75] In the case of energy degeneracy, essentially the same result can be obtained by using quantum coherence between energy eigenstates with different energies [57].

[76] A. Streltsov, G. Adesso, and M. B. Plenio, Colloquium: Quantum coherence as a resource, Rev. Mod. Phys. **89**, 041003 (2017).

[77] C. Elouard, D. Herrera-Martí, B. Huard, and A. Auffèves, Extracting Work from Quantum Measurement in Maxwell's Demon Engines, Phys. Rev. Lett. **118**, 260603 (2017).

[78] L. Bresque, P. A. Camati, S. Rogers, K. Murch, A. N. Jordan, and A. Auffèves, Two-Qubit Engine Fueled by Entanglement and Local Measurements, Phys. Rev. Lett. **126**, 120605 (2021).

[79] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Dynamics of the dissipative two-state system, Rev. Mod. Phys. **59**, 1 (1987).

[80] K. Miettinen, *Nonlinear Multiobjective Optimization* (Springer, New York, 1999), Vol. 12.

[81] R. Jozsa, Fidelity for mixed quantum states, J. Mod. Opt. **41**, 2315 (1994).

[82] R. Dann, A. Tobalina, and R. Kosloff, Shortcut to Equilibration of an Open Quantum System, Phys. Rev. Lett. **122**, 250402 (2019).

[83] A. Rivas, Strong Coupling Thermodynamics of Open Quantum Systems, Phys. Rev. Lett. **124**, 160601 (2020).