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Superfluid β phase of ^{3}He

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It is known that in low magnetic fields the superfluid transition of ^{3}He in nematic aerogel occurs into the polar phase. Using a vibrating aerogel resonator, we observe that in high magnetic fields this transition splits into two discrete transitions, occurring at different temperatures. According to theoretical models, a new superfluid phase—the β phase—should be realized between these two transitions. The temperature range of existence of the new phase is measured as a function of magnetic field. The results are well consistent with theoretical expectations for the β phase.

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Introduction.—Superfluid ^{3}He is an ideal system for experimental studies of Fermi systems with p-wave spin-triplet Cooper pairing: it is intrinsically pure, its Fermi surface is an ideal sphere, and the superfluid coherence length can be easily varied in the range of 20–80 nm by changing pressure [1]. Similar Cooper pairing may occur also in unconventional superconductors [2], in some of quantum gases [3], and in neutron stars [4].

In isotropic space in bulk superfluid ^{3}He the free energy and the superfluid transition temperature are degenerate with respect to spin and orbital momentum projections. This allows a variety of superfluid phases with the same transition temperature, but in zero magnetic field only two phases (A and B) with the lowest energy are realized. However, anisotropy of the space may lift the degeneracy and other phases can be stabilized.

The degeneracy with respect to orbital angular momentum projections can be lifted in ^{3}He confined by globally anisotropic aerogels where a scattering of ^{3}He quasiparticles is anisotropic. Theory predicts that if the effective mean free path of ^{3}He quasiparticles along a certain direction is substantially greater than along others, then new phases of ^{3}He—polar, polar-distorted A, and polar-distorted B phases—may become favorable [5–12]. However, experiments with weakly anisotropic silica aerogels show that the observed A-like and B-like superfluid phases have the same order parameters as bulk A and B phases of ^{3}He correspondingly, although the anisotropy affects the superfluid phase diagram and a spatial distribution of the order parameter [13–20]. Yet polar, polar-distorted A, and polar-distorted B phases were observed and investigated in ^{3}He confined in so-called nematic aerogel [21–30]. Nematic aerogels consist of nearly parallel strands which result in a strongly anisotropic scattering of ^{3}He quasiparticles inside the aerogel [31,32]. If the anisotropy is large enough, the superfluid transition of ^{3}He occurs to the polar phase, and, on further cooling, transitions to polar-distorted A, and polar-distorted B phases may occur. Both polar and A phases are equal spin pairing phases and contain Cooper pairs with only ± 1 spin projections on a specific direction ($\uparrow\uparrow$ and $\downarrow\downarrow$ pairs), but, in contrast to the A phase, the polar phase is not chiral and has a Dirac nodal line in the energy spectrum of Bogoliubov quasiparticles in the plane perpendicular to aerogel strands.

The degeneracy with respect to spin projections may be lifted by magnetic field. In bulk ^{3}He in strong magnetic fields the transition temperature for different spin components is split leading to the formation of new (A_1 and A_2) phases instead of the A phase. Then, instead of the second-order superfluid transition at zero field at $T = T_c$, there are two second-order transitions: to the A_1 phase at $T = T_{A1} > T_c$ and to the A_2 phase at $T = T_{A2} < T_c$. The A_1 phase contains only $\uparrow\uparrow$ pairs and exists in a narrow range of temperatures ($\sim 0.02 T_c$ in field of 10 kOe) which increases proportionally to the field [33–37]. The A_2 phase contains also $\downarrow\downarrow$ pairs, which fraction rapidly grows with cooling, and this phase is continuously transformed to the A phase, where fractions of both spin components are equal to each other. Similar splitting should also occur in the polar phase in a strong magnetic field [38,39]. On cooling, the superfluid transition should occur to the so-called β phase [1] (or P_1 phase in notation of Refs. [38,39]) instead of the pure polar phase. On further cooling, the second-order transition to the distorted β (or P_2) phase is expected.

The order parameters of the β and distorted β phases are

$$A^{P1}_{\mu j} = \frac{\Delta_1}{\sqrt{2}}(d_\mu + i e_\mu) m_j, \tag{1}$$

$$A^{P2}_{\mu j} = \frac{\Delta_1}{\sqrt{2}}(d_\mu + i e_\mu) m_j + \frac{\Delta_2}{\sqrt{2}} e^{i\varphi}(d_\mu - i e_\mu) m_j, \tag{2}$$

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correspondingly, where Δ_1 and Δ_2 are gap parameters, $e^{i\varphi}$ is a phase factor, **d** and **e** are mutually orthogonal unit vectors in spin space (which are perpendicular to the magnetization), and **m** is a unit vector in orbital space aligned along the direction of nematic aerogel strands [5]. From Eqs. (1), (2) it follows that orbital parts of order parameters of β and distorted β phases are the same as in the polar phase, but the β phase contains only $\uparrow\uparrow$ Cooper pairs, while the distorted β phase is a condensate of $\uparrow\uparrow$ [the first term in Eq. (2)] and $\downarrow\downarrow$ [the second term in Eq. (2)] pairs. On cooling, the distorted β phase is continuously transformed to the pure polar phase, that is, Δ_2 in Eq. (2) becomes equal to Δ_1.

In this Letter, we present results of high magnetic field experiments in superfluid ^{3}He in nematic aerogel, wherein the solid ^{3}He layers on the aerogel strands have been replaced by ^{4}He. The ^{4}He coverage is necessary in order to stabilize the polar phase in low magnetic fields [27]. Indeed, previous NMR experiments have shown that in this case in low magnetic field the superfluid transition of ^{3}He in such aerogel occurs to the polar phase [28]. In the present experiments we used a vibrating wire (VW) resonator with nematic aerogel attached to it, as in VW experiments with ^{3}He in silica aerogel [40,41]. We measured temperature dependencies of resonance properties of the resonator and observed the splitting of the superfluid transition, which we attribute to the appearance of the β phase.

Theory.—In ^{3}He in nematic aerogel, on cooling from the normal phase, a superfluid transition to the β phase should occur at the temperature

$$T_{P1} = T_{ca} + T_c \eta H, \tag{3}$$

where H is the magnetic field, T_{ca} is a superfluid transition temperature of ^{3}He in nematic aerogel for $H = 0$, and $\eta \sim 10^{-3}$ kOe^{-1} [38]. On further cooling, the transition to the distorted β phase is expected at the temperature

$$T_{P2} = T_{ca} - T_c \eta H \frac{\beta_{12345}}{-\beta_{15}}, \tag{4}$$

where $\beta_{15} = \beta_1 + \beta_5$, and so on, β_i, $i \in \{1, \ldots, 5\}$ are coefficients in the Ginzburg-Landau free energy functional [1], or beta parameters.

From Eqs. (3) and (4) we obtain that the temperature range of existence of the β phase $(T_{P1} - T_{P2} = T_c \eta H \beta_{234}/(-\beta_{15}))$ is proportional to H, and

$$\frac{T_{P1} - T_{ca}}{T_{ca} - T_{P2}} = \frac{-\beta_{15}}{\beta_{12345}}. \tag{5}$$

Unfortunately, beta parameters of ^{3}He in nematic aerogel are unknown. Assuming bulk ^{3}He beta parameters [42], the fraction in Eq. (5) equals 1.36 at 15.4 bar.

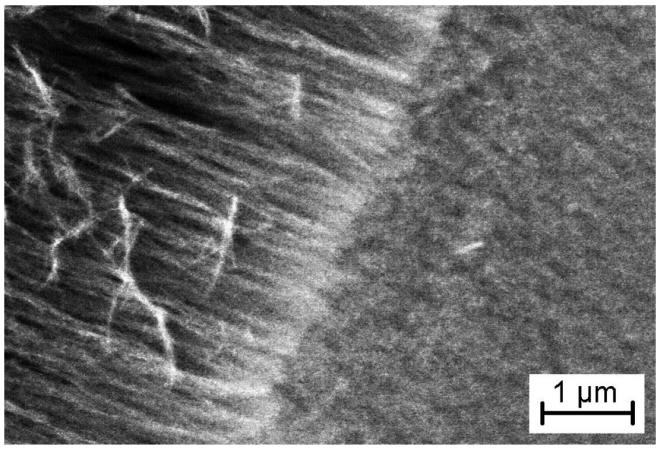

FIG. 1. Scanning electron microscope image of the free surface of mullite aerogel sample.

Methods.—We used an original sample of mullite nematic aerogel (Metallurg Engineering Ltd.) with density of 150 mg/cm^3, porosity of 95.2%, and with a size along strands ≈2.6 mm. The aerogel strands have a diameter of ≤ 14 nm (from scanning transmission electron microscope images) and a characteristic separation of 60 nm. Effective mean free paths of ^{3}He quasiparticles in directions parallel and transverse to the strands in the limit of $T = 0$ are 900 and 235 nm correspondingly [29]. The sample for the experiments was cut along the strands from the original sample, so the edges, where the strands begin and end, are not damaged and are perfectly flat: the irregularities are about 100 nm (Fig. 1). Sizes of the obtained rectangular parallelepiped in direction transverse to the strands is ≈2 × 3 mm.

The experimental sample is glued using a very small amount of Stycast-1266 epoxy resin to 240 μm NbTi wire, bent into a shape of an arch with total height of 10 mm and distance between legs of 4 mm. Strands of the aerogel were oriented along the oscillatory motion. The aerogel wire is mounted in a cylindrical experimental cell (of internal diameter 6 mm) made from Stycast-1266 surrounded by a main superconducting solenoid, so that the sample is located at the maximum of the magnetic field (with homogeneity of 0.1% at distances ±3 mm). A sketch of the cell is shown in Fig. 2.

In order to measure the temperature, a quartz tuning fork was used, the resonance linewidth of which in ^{3}He depends on temperature (see the Supplemental Material [43]). The experiments were carried out at a pressure of 15.4 bar in magnetic fields 0.5–10.25 kOe generated by the main solenoid. In order to avoid atomic layers of solid ^{3}He on the aerogel strands and to stabilize the polar phase in low fields [27], we had added 1.55 mmole of ^{4}He into the empty cell at $T \leq 100$ mK and then filled it with ^{3}He. This amount of ^{4}He is enough to completely remove solid ^{3}He from aerogel strands and, according to our estimations, corresponds to 2.5–3.2 atomic layers of ^{4}He coverage [48].

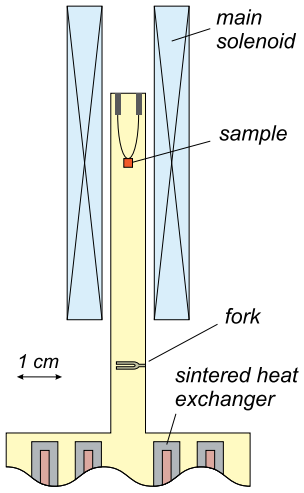

FIG. 2. The sketch of the experimental cell.

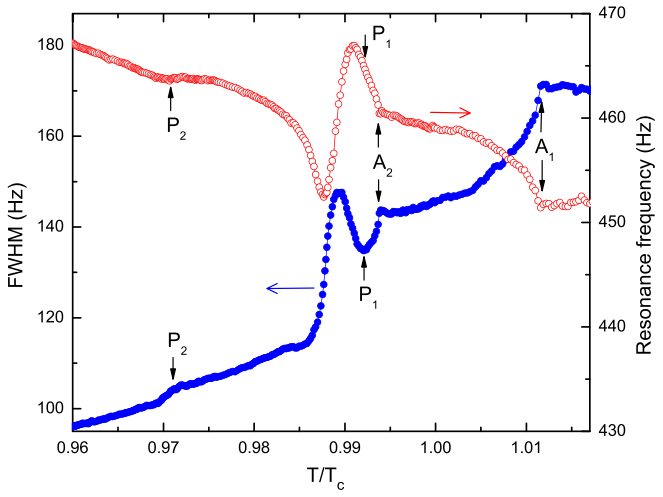

FIG. 3. Temperature dependencies of FWHM (solid circles) and frequency (open circles) of the main resonance of the VW resonator measured in magnetic field of 10.25 kOe at excitation current of 0.4 mA. Arrows indicate the features we associate with T_{P2}, T_{P1}, T_{A2}, and T_{A1}. T_c is a superfluid transition temperature of bulk ^{3}He in zero magnetic field. $P = 15.4$ bar.

The necessary temperatures were obtained by a nuclear demagnetization cryostat. A measurement procedure of the aerogel resonator is similar to that of a conventional wire resonator [49]. An alternating current, with amplitude varying from 0.4 to 8.9 mA (depending on H and being set to keep the amplitude of oscillations field independent), is passed through the VW. The Lorentz force sets the wire into oscillations. At $T \sim 1$ K the resonance frequency and the full width at half-maximum (FWHM) of our VW resonator in vacuum are 621 and 0.3 Hz, respectively. Motions of the VW in the magnetic field generates a Faraday voltage. This voltage was amplified by a room-temperature step-up transformer 1:30 and measured with a lock-in amplifier. In-phase and quadrature signals (obtained by sweeping the frequency of the driving current) were joint fitted to Lorentz curves in order to extract the FWHM and the resonance frequency. In liquid ^{3}He the maximum velocity of the VW in the used temperature range did not exceed 0.2 mm/s. In a given field additional experiments with 2 times smaller excitation current were also done and showed the same results.

Similar samples (cut from the same original mullite nematic aerogel sample) have been used in NMR experiments [28] and in VW experiments in low magnetic fields [50]. NMR experiments show that at 15.4 bar in magnetic fields $\lesssim$300 Oe the superfluid transition of ^{3}He in such aerogel occurs into the polar phase at $T_{ca} \approx 0.985T_c$ which then exists down to $\approx 0.9T_c$. The temperature width of the superfluid transition is found to be about $0.002\,T_{ca}$. We can expect that the present sample should have nearly the same T_{ca} and the width of the superfluid transition. We also note that in VW experiments described in Ref. [50] an additional (the second) resonance mode had been observed, existing only below T_{ca}. This second mode is an analog of the second-sound-like mode (called also as slow sound mode) observed in silica aerogel in superfluid helium [51,52] and corresponds to motions in opposite directions of the superfluid component inside the aerogel and the normal component (together with the aerogel strands). On cooling from $T = T_{ca}$, the resonant frequency of this additional mode very rapidly increases from 0 up to $\sim$1.6 kHz, and in a narrow temperature range below (but very close to) T_{ca} becomes close to the resonance frequency of the main mechanical VW resonance resulting in an interaction of these modes (see Ref. [50] for details). In the present experiments we focused on measurements of the main resonance, which intensity is significantly greater.

Results.—Most of the experiments were done on a slow (0.002–0.004 T_c per hour) warming of the cell. In Fig. 3 we show results obtained in magnetic field of 10.25 kOe, where we measured the FWHM and the frequency of the main resonance of the VW. In Fig. 3 we mark features (A_1, A_2, P_1, and P_2) which we ascribe to transitions at temperatures T_{A1}, T_{A2}, T_{P1}, and T_{P2}.

Let us consider these features with decreasing temperature. At $T > T_{A1}$ both bulk ^{3}He and ^{3}He in aerogel are in the normal state: on cooling, the FWHM slowly increases and the frequency decreases. The superfluid transition to the A_1 phase in bulk ^{3}He occurs at $T = T_{A1}$. The accuracy of determination of T_{A1} is only $\pm 0.003T_c$ because at $T = T_{A1}$ the fork is in the normal phase where it is less sensitive to temperature variations. Below T_{A1} the FWHM decreases and at $T = T_{A2}$ the transition to the A_2 phase takes place. According to our temperature calibration, T_{A2} is slightly higher (by $0.0015T_c$) than it follows from Ref. [35]. On further cooling, the FWHM decreases more rapidly but below $T = T_{P1}$ it starts to increase that can be due to only the superfluid transition of ^{3}He in aerogel. We assume that in the given magnetic field this transition occurs to the β phase.

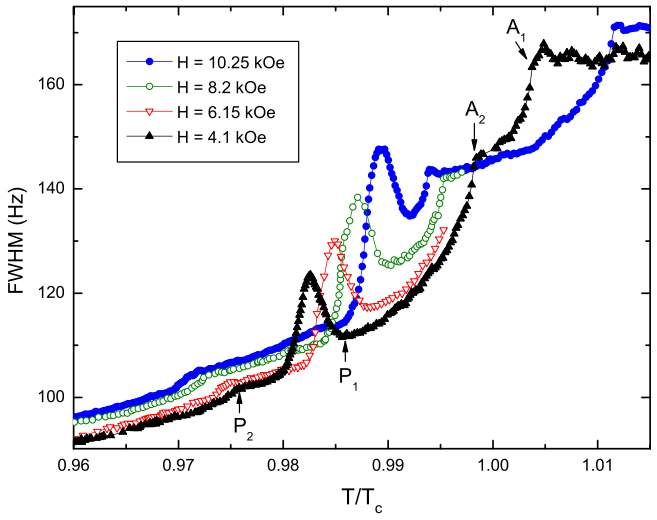

FIG. 4. Temperature dependencies of the FWHM of the main resonance of the VW resonator measured in magnetic fields of 10.25 (solid circles), 8.2 (open circles), 6.15 (open triangles), and 4.1 kOe (solid triangles) at corresponding excitation currents of 0.4, 0.5, 0.67, and 1 mA. For a better view, the arrows mark P_1, P_2, A_1, and A_2 features only for $H = 4.1$ kOe. P = 15.4 bar.

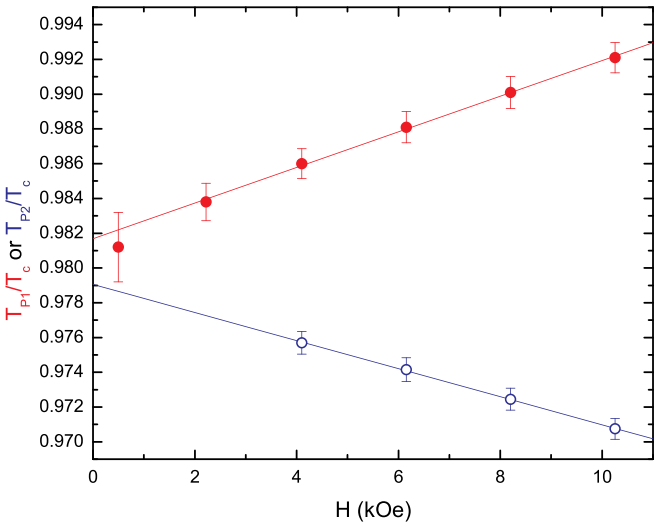

FIG. 5. P_1–P_2 splitting of the superfluid transition of ^{3}He in nematic aerogel in magnetic field. Open and solid circles indicate transitions between distorted β and β, β, and normal phases, respectively. Lines are linear approximations of experimental data.

At lower temperature (at $T = T_{P2}$) we observe "step" on the FWHM plot or "kink" on the resonance frequency plot, which we refer to the transition between the β phase and the distorted β phase existing at $T < T_{P2}$. We note that on cooling below $T = T_{P2}$ the intensity of the second resonance mode starts to grow rapidly, but at $T_{P2} < T < T_{P1}$ its intensity is very small. We assume that in this temperature range (i.e., in the expected β phase) this mode is less excited and, in comparison with experiments described in Ref. [50], we did not observe a clear repulsion between the main and the second resonance modes at $T \approx T_{P1}$. However, the interaction between these modes remains, and just below $T = T_{P1}$ on the main resonance we observe a peaklike change of the linewidth as well as the rapid change of the resonance frequency.

In Fig. 4 we show temperature dependencies of the FWHM of the main VW resonance obtained in different magnetic fields. As it was expected, the temperature range of existence of the β phase ($T_{P1} - T_{P2}$) is decreased with the decrease of H.

In Fig. 5 we summarize results of our experiments and show the measured at 15.4 bar dependencies of T_{P1} and T_{P2} on H. The results are well fitted by linear functions as it follows from the theory. The ratio of slopes of the fit lines $[(dT_{P1}/dH)/(-dT_{P2}/dH)]$ equals 1.27. From Eq. (5) this ratio is expected to be equal to 1.36 if we consider the beta parameters of bulk ^{3}He [42]. We note that the linear fits do not match at $H = 0$. This discrepancy may be due to a finite width ($\sim 0.002 T_c$) of the superfluid transition of ^{3}He in aerogel. It may result in a systematic error in determination of T_{P1} of the same order. In any case, it can be seen that the temperature range of existence of the β phase is nearly proportional to H and the value of the splitting is of the same order as it was observed in the bulk A phase [35,36].

Conclusions.—Using the VW techniques, we have observed the splitting of a superfluid transition temperature of ^{3}He in nematic aerogel in high magnetic field, that is, instead of one transition at $T = T_{ca}$, there are two transitions: the first at $T = T_{P1} > T_{ca}$ and the second at $T = T_{P2} < T_{ca}$. We ascribe such splitting to the appearance at $T_{P2} < T < T_{P1}$ of a new superfluid phase of ^{3}He—the β phase. This conclusion is based on the following: (i) In low magnetic field in such aerogel, the superfluid transition occurs to the polar phase. In this case high magnetic field should result in the splitting of a superfluid transition due to the appearance of the β phase. (ii) The value of the temperature range of existence of this new phase and its linear dependence on the magnetic field are well consistent with theoretical expectations for the β phase [38,39]. Despite that, our experimental method does not allow us to directly determine the structure of the order parameter of the observed superfluid phase in high magnetic field. One of other possible ways to prove the existence of the β phase is to measure NMR frequency shift from the Larmor value and to compare the results with theoretical predictions [39]. The maximal expected absolute value of this shift in the β phase is very small (about 1 Hz), but seems to be measurable if a sufficiently homogeneous magnetic field is used.

Worthy to mark that, although the superfluid A-like phase of ^{3}He in silica aerogel corresponds to the A phase of bulk ^{3}He, the $A_1 - A_2$ splitting in pure ^{3}He in silica aerogel was not observed [53]. Theory explains this by suppression

of the splitting due to the presence of solid ^{3}He atomic layers on aerogel strands [54]. Similar phenomena can take place in ^{3}He in a nematic aerogel, which is a subject for future work. Another interesting direction of research is studies of topological defects in the β phase and their evolution during a transition to the polar phase.

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