Editors' Suggestion

Contrarians Synchronize beyond the Limit of Pairwise Interactions

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We give evidence that a population of pure contrarian globally coupled D-dimensional Kuramoto oscillators reaches a collective synchronous state when the interplay between the units goes beyond the limit of pairwise interactions. Namely, we will show that the presence of higher-order interactions may induce the appearance of a coherent state even when the oscillators are coupled negatively to the mean field. An exact solution for the description of the microscopic dynamics for forward and backward transitions is provided, which entails imperfect symmetry breaking of the population into a frequency-locked state featuring two clusters of different instantaneous phases. Our results contribute to a better understanding of the powerful potential of group interactions entailing multidimensional choices and novel dynamical states in many circumstances, such as in social systems.

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When Kuramoto introduced his model of globally coupled phase oscillators [1,2], he certainly could have hardly fathomed that it would have acquired such a broad applicability over the decades [3–9] to systems of physical interest, such as Josephson junctions [10,11], laser arrays [12–14], oscillator glass [15,16], and charge density waves [17]. The model became soon a paradigm for the study of synchronization, i.e., the emergent property through which a system forms a collective, coherent, rhythm [3,8,18] via the interaction of its oscillatory units, which has been the object of a wide range of research in physics, biology, and engineering [19–21]. In the Kuramoto model, a necessary condition for synchronization is to have a rigorously non-negligible fraction (usually larger than a finite threshold) of oscillators which are acting as conformists, i.e., which are featuring *a strictly positive* coupling strength to the mean field. The emergence of order for repulsive strengths has indeed been observed so far only in specific, local, coupling arrangements [22,23].

In social science, various generalizations of the Kuramoto model were proposed for the study, for instance, of human crowd behavior during clapping [24,25] and crossing bridges [26,27] and of human decision making, in general. The inclusion of conformists and contrarians as positively and negatively coupled individuals to the mean field captures well the essential dichotomy of many social interactions and their relation to whatever the prevailing opinion might be [28,29]. It was also argued that the continuous spectrum of opinions as points on a circle reflect the political reality better than the traditionally considered linear continuum from the left to the right wing 30]]. Applications to social interactions of the Kuramoto model have not received, however, as much attention as it would have deserved. This may be due to two fundamental limitations of the original model. In the first place, social interactions typically unfold in more than two dimensions, simply because the spectrum of behavioral choices often goes beyond a circle representation. It has recently been noted that the quest for moral behavior in an evolutionary setting, for example, may entail choosing between many different strategies, all of which have highly nonlinear consequence for the individual and the social network as a whole [31,32]. Such a decision space, thus, surely requires more than a circle to be completely mapped out. And, second, social interactions inherently entail groups that are not accurately described by pairwise links in traditional oscillator networks [33]. Cooperation in large groups of unrelated individuals, for example, distinguishes us most from other mammals, and it is one of the central pillars of our evolutionary success [34,35]. But classical social

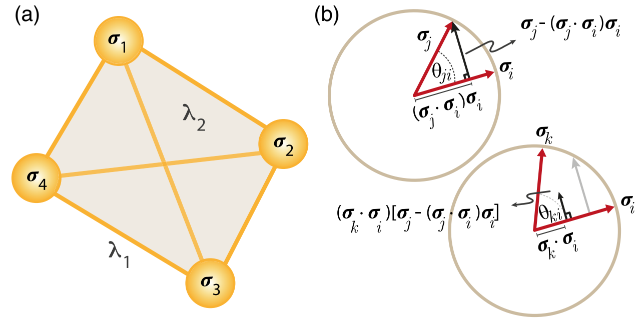

FIG. 1. (a) The setting of Eq. (2): D-dimensional oscillators are represented by yellow circles, and they interact via pairwise and triadwise interactions, with coupling strengths, respectively, given by λ_1 and λ_2. (b) $D=2$. The various trigonometric relationships which allow one to transform of the classical Kuramoto model [Eq. (1)] into Eq. (2).

networks with only pairwise links simply do not provide a unique procedure for defining a group [36].

Aiming to overcome these limitations, in this Letter, we consider a D-dimensional Kuramoto model with one- and two-simplex interactions and which includes both conformists and contrarians as positively and negatively coupled individuals to the mean field to capture the fact that some individuals—the conformists—prefer to go with the mainstream, while others—the contrarians—prefer to oppose it. We will show that these generalizations allow us to retain full analytical tractability of the model while also yielding fundamentally novel behavior. Notably, in addition to a rich plethora of behavior previously associated with various Kuramoto models [37–42], such as mono- and multistability, spontaneous symmetry breaking, and explosive (i.e., discontinuous) transitions to synchronization, we give evidence that synchronization may occur also in the complete absence of conformists, a result which is *inherently prohibited* when only pairwise interactions take place in the ensemble. Thus, even if everybody contests the prevailing attitude, consensus is possible due to higher-order structures in the population.

Let us then start by considering an ensemble of globally coupled N oscillators satisfying

$$\dot{\theta}_i = \omega_i + \frac{\lambda_1}{N}\sum_{j=1}^{N}\sin(\theta_j-\theta_i) + \frac{\lambda_2}{2N^2}\sum_{j=1}^{N}\sum_{k=1}^{N}\sin(\theta_j+\theta_k-2\theta_i), \qquad (1)$$

where θ_i is the phase and ω_i the natural frequency of each oscillator i ($i=1,2,\ldots,N$), while λ_1 and λ_2 are two real parameters accounting for the coupling strengths of pairwise and triadwise interactions, respectively, as schematically shown in Fig. 1(a). Now, one can consider the unit vector $\boldsymbol{\sigma}_i = [\cos(\theta_i), \sin(\theta_i)]$ and the antisymmetric matrix

$$\boldsymbol{W}_i = \begin{pmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{pmatrix}.$$

With the help of a few trigonometric relationships [which are schematically reported in Fig. 1(b)] and after denoting by $\boldsymbol{\rho}$ the mean vector of all $\boldsymbol{\sigma}_i$ (i.e., $\boldsymbol{\rho} = (1/N)\sum_{i=1}^{N}\boldsymbol{\sigma}_i$), Eq. (1) can be rewritten as

$$\dot{\boldsymbol{\sigma}}_i = \boldsymbol{W}_i\boldsymbol{\sigma}_i + \lambda_1[\boldsymbol{\rho}-(\boldsymbol{\rho}\cdot\boldsymbol{\sigma}_i)\boldsymbol{\sigma}_i] + \lambda_2(\boldsymbol{\rho}\cdot\boldsymbol{\sigma}_i)[\boldsymbol{\rho}-(\boldsymbol{\rho}\cdot\boldsymbol{\sigma}_i)\boldsymbol{\sigma}_i]. \qquad (2)$$

In fact, there is no reason for limiting Eq. (2) to the dynamics of two-dimensional vectors. On the contrary, the same equation can be adopted to describe the evolution of D-dimensional vectors $\boldsymbol{\sigma}_i$ (of norm one) whose trajectories lie on a $(D-1)$-dimensional unit sphere $\mathcal{S}^{D-1}$. In the special case of $\lambda_1=\lambda_2=0$ equal to zero (i.e., where there are no interactions among the oscillators), $\boldsymbol{\sigma}_i$ rotates independently along some trajectory on $\mathcal{S}^{D-1}$ dictated by a real antisymmetric matrix $\boldsymbol{W}_i \in \mathbb{R}^{D\times D}$ which, in what follows, is independently drawn at random for each node i. Specifically, each upper triangular element of $\boldsymbol{W}_i$ is sampled from a Gaussian distribution $\mathcal{N}(0,1)$, and the lower triangular elements are accordingly fixed to make $\boldsymbol{W}_i$ antisymmetric.

For $D=2$ and $D=3$, Eq. (2) admits a rigorous analytical treatment, which ultimately yields a self-consistent equation for the order parameter $R \equiv \langle|\boldsymbol{\rho}|\rangle_T$ (with $\langle\ldots\rangle_T$ indicating time average over a sufficiently long time span T and $|\ldots|$ indicating the norm). Such a self-consistent equation allows, on its turn, the computation of the stationary points (and of their stability) for all values of λ_1 and λ_2. The interested reader can find the complete details of the treatment in Supplemental Material [43]. On the other hand, we performed large-scale simulations of Eq. (2), at $D=2$ and $D=3$, in an ensemble of $N=5000$ oscillators, by the use of a fourth-order Runge-Kutta algorithm with integration time $h=10^{-3}$. In our simulations, we monitored both the forward and the backward transition to synchronization. In the forward (backward) transition, the system is initialized in the fully incoherent (fully coherent) state, which corresponds to $R(t=0)\sim 0$ $[R(t=0)=1]$, and, after a suitable transient time, the asymptotic state of the order parameter R is calculated. In practice, this is realized by selecting a random unitary vector $e_D \in \mathbb{R}^D$ and letting each oscillator $\boldsymbol{\sigma}_i$ be initially equal to e_D with probability $0\le\mu\le 1$, while setting $\boldsymbol{\sigma}_i(t=0) = -e_D$ with probability $1-\mu$. Then, the case $\mu=1$ ($\mu=0.5$) gives the proper initial conditions for inspection of the backward (forward) transition to synchronization.

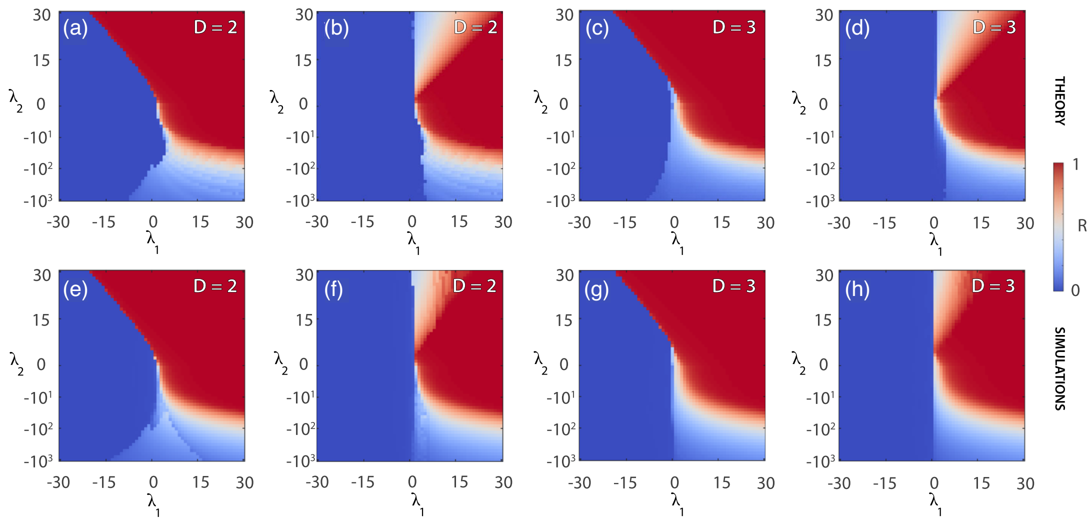

FIG. 2. Theoretical predictions [(a)–(d), calculated from Eq. (26) of Supplemental Material [43]] and numerical simulations [(e)–(h), calculated from Eq. (2)] for $D = 2$ and $D = 3$ of the order parameter R (see the text for definitions) as a function of the coupling constants λ_1 and λ_2. (a),(c),(e),(g) refer to the backward transition; (b),(d),(f),(h) refer to the forward transition. In (a)–(d), the values around the line $\lambda_1 = -\lambda_2$ of the quadrant defined by $\lambda_1 > 0$ and $\lambda_2 < 0$ are interpolated (see [43] for a full discussion on the theoretical limitations occurring at $\lambda_1 = -\lambda_2$ for the assessment of stability of the synchronous state). In each panel, the synchronized (incoherent) state is represented by the red (blue) color, and the values of R are coded according to the vertical color bar reported at the rightmost of the figure. Notice that, for a better representation of the results, a vertical logarithmic scale is adopted for $|\lambda_2|$ in the semiplane $\lambda_2 < 0$. See the main text for the details on the size and on the initialization of the system.

We start our discussion by reporting in Fig. 2 the theoretical predictions [panels of the first row, obtained by solving Eq. (26) of Supplemental Material [43]] and the numerical simulations [panels of the second row, obtained from Eq. (2)], at $D = 2$ (first two columns) and $D = 3$ (second two columns), for R in the parameter plane (λ_1, λ_2). Precisely, Figs. 2(a), 2(c), 2(e), and 2(g) refer to the backward transition, whereas Figs. 2(b), 2(d), 2(f), and 2(h) report the results of the forward transition to synchronization.

Figure 2 is informative on many relevant dynamical scenarios supported by system (2). Some of these scenarios confirm and actually extend findings already reported in the literature: for instance, the fact that triadwise interactions are detrimental for the forward transition, as in the limit of $\lambda_1 = 0$ the forward transition disappears, and, in general, the threshold in λ_1 needed to produce the synchronous state from incoherence increases monotonically with λ_2 [see Figs. 2(b), 2(d), 2(f), and 2(h)] [44,45]. Some other scenarios, instead, point to novel and relevant features of the system, which certainly deserve more detailed investigations: for instance, the fact that, despite the simplicity of the model, the simple combination between pure 1- and 2-simplices makes it possible for the system to exhibit monostability, bistability, and multistability regions, as well as the fact that in the region of negative λ_1 and positive λ_2 the system features an abrupt transition from the fully synchronized state to the unsynchronized one. Furthermore, the theoretical predictions (contained in Ref. [43]) and the numerical results are in very good agreement, and, therefore, the developed theory allows one to unveil the origin of bistability in the Kuramoto model, as reported also in various other settings and circumstances [44,45].

But the most remarkable evidence that Fig. 2 is communicating is the presence of synchronization features in the backward transition for negative values of the coupling strengths. Precisely, for $D = 2$ and $D = 3$ the theory [Figs. 2(a) and 2(c), respectively] predicts the emergence of synchronization even when λ_1 and λ_2 are both negative, a dynamical state which actually would be inherently prohibited if the interplay among the elements were limited to pairwise interactions. In other words, group interactions lead to the emergence of synchronization, or in social terms to the emergence of agreement, even if all individuals in the network are contrarians; i.e., they are opposing the mainstream at all times [28]. In simulations, we have observed such a state only for $D = 2$ (i.e., for the classical Kuramoto model), whereas a detailed analysis of the $D = 3$ case will be reported elsewhere.

For a better visualization of this latter macroscopic and generic effect, in the following we will fix $D = 2$ and focus on the microscopic details of the observed synchronization of *pure contrarians*. To that purpose, in Fig. 3, we report four time snapshots [Figs. 3(a)–3(d)] of the vectors $\{\boldsymbol{\sigma}_i \equiv \exp(i\theta_i)\}$ (with norms slightly adjusted around unity for better visibility) in the plane ($\sigma_x = \cos\theta$, $\sigma_y = \sin\theta$), together with the oscillators' instantaneous phases [Figs. 3(a1)–3(d1)] and frequencies [Figs. 3(a2)–3(d2)] as functions of their natural frequency ω_i. It is seen that the

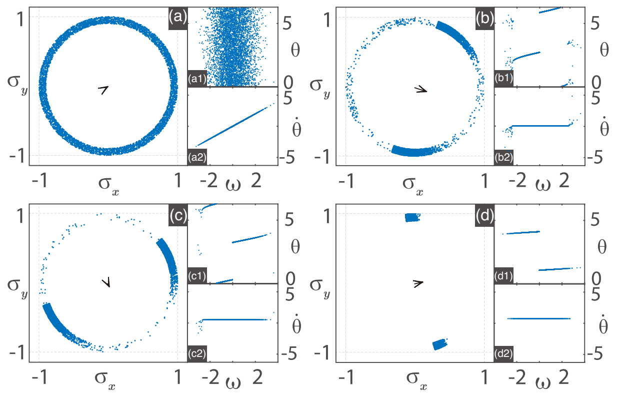

FIG. 3. The microscopic details behind the synchronization of contrarians. (a)–(d) report (with blue dots) a time snapshot of the set of vectors $\{\boldsymbol{\sigma}_i \equiv \exp(i\theta_i)\}$ (for better visibility, the norms of such vectors have been slightly adjusted around unity) in the plane ($\sigma_x = \cos\theta$, $\sigma_y = \sin\theta$). The black arrow stands for the instantaneous order parameter vector $\boldsymbol{\rho} \equiv \sum_{i=1}^{N} \exp(i\theta_i)$. (a) $\lambda_1 = -10$, $\lambda_2 = -10^2$; (b) $\lambda_1 = 0$, $\lambda_2 = -10^2$; (c) $\lambda_1 = -10$, $\lambda_2 = -10^3$; (d) $\lambda_1 = 10$, $\lambda_2 = -10^3$. In all cases, $\mu = 1$ for the initialization of the system. (a1)–(d1) [(a2)–(d2)] report the instantaneous phase θ_i (the instantaneous frequency $\dot{\theta}_i$) for each oscillator i in the ensemble, as a function of the oscillator's natural frequency ω_i.

system sets actually in a *collective state*: Starting from a fully incoherent dynamics ($\lambda_1 = -10$, $\lambda_2 = -10^2$), where the unit vectors are almost homogenously distributed around the unit circle [Fig. 3(a)], with unlocked phases [Fig. 3(a1)] and frequencies [Fig. 3(a2)], a progressive increase in λ_1 determines a symmetry-breaking scenario wherein two groups of oscillators form [Figs. 3(b), 3(b1), and 3(b2), obtained at $\lambda_1 = 0$, $\lambda_2 = -10^2$] and grow in correspondence with decreasing λ_2, up to the final organization of the population of contrarians [Figs. 3(c), 3(c1), and 3(c2), obtained for $\lambda_1 = -10$, $\lambda_2 = -10^3$]. In this latter state, almost all oscillators display instantaneous phases within either one of the two phase clusters, whose centers of mass are located at positions $\tilde{\phi}(t)$ and $\pi - \tilde{\phi}(t)$, respectively [as can be seen in Fig. 3(c1)] and rotate coherently with a fully locked frequency [see Fig. 3(c2)]. As a consequence of the two clusters, the value of the order parameter remains small, yet distinguishable from that corresponding to the fully incoherent dynamics. Moreover, it is very interesting to notice that, while the presence of only pairwise interactions would determine the total destruction of such a state, when pairwise and triadwise interactions are simultaneously present, positive values of λ_1 may actually enhance the dichotomic nature of the state [see Figs. 3(d), 3(d1), and 3(d2)]. We refer the interested reader to the discussion of our Supplemental Material [43], where we prove that for $D = 2$ and $\lambda_2 = 0$ (i.e., for the classical case of the Kuramoto model with only pairwise interactions), if λ_1 is negative, the only solution of the self-consistent equation is $R = 0$ (i.e., synchronization of contrarians is prevented in this case), while the presence of three-body interactions introduces a new solution of the self-consistent equation (the observed new state) even for $\lambda_1 < 0$ and $\lambda_2 < 0$.

Finally, we report that the emergence of such collective state does not necessarily require an all-to-all configuration, but, on the contrary, it can be observed also in sparser connectivity structures. To this purpose, we set $N = 10^2$ and simulate Eq. (1) for $\lambda_1 = -10$, $\lambda_2 = -1000$, starting from an all-to-all configuration and initializing the phases with $\mu = 1$. As a consequence, the system sets in the collective state reported in Fig. 4(a). Then, every 50 time steps, a fraction $1 - p$ of triangular interactions is randomly removed from the double sum of the second term of the right-hand side of Eq. (1). Figure 4(d) reports then the behavior of $R_1 = (1/N)\|\sum_{j=1}^{N} e^{i\theta_j}\|$ (red curve with squares) and $R_2 = (1/N)\|\sum_{j=1}^{N} e^{2i\theta_j}\|$ (blue curve with circles) vs the fraction p of remaining triangles. It is clearly seen that the new collective state is resilient up to the removal of about 20% of triadwise interactions [see Fig. 4(b)] and eventually disappears completely when $p = 1$, in full agreement with the rigorous result that a population of pure contrarians is strictly prevented from synchronizing in the limit of pairwise interactions.

In summary, we have reported, both analytically and numerically, on the emergence of synchronization even when λ_1 and λ_2 are both negative. In other words, group interactions may lead to the emergence of synchronization, or in social terms to the emergence of agreement, even if all individuals in the network are contrarians, i.e., opposing the mainstream at all times. Microscopically, we have shown

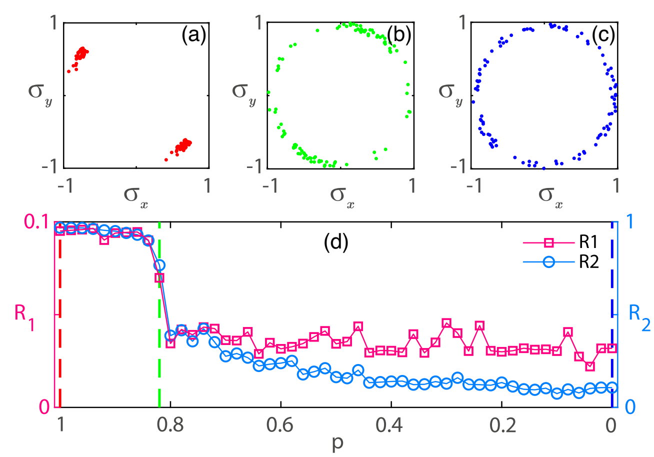

FIG. 4. The resilience of the new state in sparser structures. (d) reports R_1 (red curve with squares) and R_2 (blue curve with circles) vs the fraction p of remaining triangles (see the text for definitions). (a)–(c) show snapshots of the corresponding states of the oscillators in the plane ($\sigma_x = \cos\theta$, $\sigma_y = \sin\theta$) for three values of p ($p = 1, 0.84, 0$) marked by vertical dashed lines in (d) and colored with the same colors. The same stipulations for the phase representation as in the caption of Fig. 3.

that this is due to imperfect symmetry breaking that splits the population into two groups with different phases, but with frequency synchrony in one.

Our research should find applicability in better understanding decision making in human groups, especially where the decision space is multidimensional, such as in evolutionary settings involving the provisioning of public goods [35] or the emergence of moral behavior [32]. Swarming in three dimensions [46], in particular, where making swift decisions play a key role even if not involving humans, such as in fish schools or murmurations under predation [47,48], might also benefit from the insights reported in our research.

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