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Subdiffusive Phases in Open Clean Long-Range Systems

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We show that a one-dimensional ordered fermionic lattice system with power-law-decaying hopping, when connected to two baths at its two ends with different chemical potentials at zero temperature, features two phases showing subdiffusive scaling of conductance with system size. These phases have no analogues in the isolated system (i.e., in absence of the baths) where the transport is perfectly ballistic. In the open system scenario, interestingly, there occurs two chemical-potential-driven subdiffusive to ballistic phase transitions at zero temperature. We discuss how these phase transitions, to our knowledge, are different from all the known nonequilibrium quantum phase transitions. We provide a clear understanding of the microscopic origin of these phases and argue that the subdiffusive phases are robust against the presence of arbitrary number-conserving many-body interactions in the system. These phases showing subdiffusive scaling of conductance with system size in a two-terminal setup are therefore universal properties of all ordered one-dimensional number-conserving fermionic systems with power-law-decaying hopping at zero temperature.

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Introduction.—Normal metals (conductors) have their own well-defined conductivity at a given temperature. For a metal wire of cross-sectional area A, length N, conductivity σ, and connected to two terminals at its two ends, the conductance (i.e., inverse of resistance) G is given as $G = \sigma A/N$. Importantly, since conductivity is independent of the dimension of the metal used, if the length of the wire is changed keeping the cross-sectional area fixed, the conductance scales inversely with N, i.e., $G \sim N^{-1}$. This corresponds to normal diffusive transport. In the absence of this behavior, conductivity no longer remains as a property of the material, but rather depends on the dimension of the wire in a nontrivial way. Two simple examples of these are perfect insulators (no transport) with $G \sim e^{-\lambda N}$ and perfect conductors (ballistic transport) with G independent of N.

It has been well established that transport behavior may deviate from the ones described above, especially for low-dimensional systems [1–4]. Such transport behavior, where $G \sim N^{-\delta}$, with $0 < \delta \neq 1$, is often called anomalous. Rapid miniaturization of devices has taken technology to limits where realizing such low-dimensional systems have become a real possibility, and thus understanding their transport properties has become imperative [5–8]. One of the most intriguing behaviors among the anomalous transport is the so-called subdiffusive transport, which corresponds to $\delta > 1$ (as opposed to superdiffusive transport for $0 < \delta < 1$). In this case, even though the conductivity of the wire goes to zero as $N \to \infty$, for any finite length, its conductance is exponentially larger than what one would expect for a perfect insulator. For finite-size systems, relevant in quantum technology, quantum chemistry and mesoscopic physics, this particular feature can make a significant difference. Subdiffusive behavior is often observed for systems residing at the critical regions separating localization-delocalization transitions [9–27]. It is usually associated with the presence of correlated or uncorrelated disorder in the system, although a complete microscopic understanding is largely missing. To the contrary, in this work, we reveal, and microscopically explain, a completely different way in which subdiffusive scaling of conductance with system length can occur in a large class of systems, even in the complete absence of disorder.

Specifically, we show that, a one-dimensional fermionic wire with long-range power-law-decaying hopping connected to two terminals at the two ends, surprisingly features two phases at zero temperature, showing subdiffusive scaling of conductance with N in the absence of any disorder. These unique subdiffusive phases arise due to an interplay between the long-range hopping and the dissipation governed by the two terminals, and have no analog in absence of either. We observe two chemical-potential-driven dissipative quantum phase transitions between phases featuring subdiffusive and ballistic transport [see Fig. 1]. We discuss how these phase transitions are different from all the previously well-known dissipative quantum phase transitions. Furthermore, we provide a clear microscopic understanding of the subdiffusive transport by

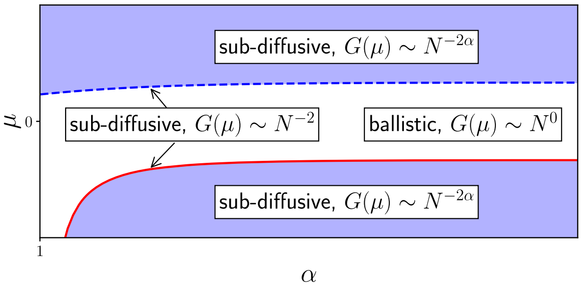

FIG. 1. Nonequilibrium phase diagram obtained from system size (N) scaling of zero-temperature conductance $G(\mu)$ as a function of chemical potential μ and long-range hopping exponent α for a one-dimensional open clean long-range system [Eq. (1)]. The critical lines correspond to the system band edges $\mu = 2\eta(\alpha)$ (blue dashed line) and $\mu = -2\zeta(\alpha)$ (red line), where $\eta(\alpha)$ is Dirichlet-eta function and $\zeta(\alpha)$ is the Riemann-zeta function.

connecting the corresponding scaling exponent to a nonanalyticity in the dispersion relation. We also argue that these subdiffusive phases are immune to the presence of many-body interactions in the system as long as the net number of particles within the system is conserved. The subdiffusive phases are therefore universal properties of all clean number-conserving one-dimensional fermionic systems with power-law-decaying hopping in the two-terminal setup.

Low-dimensional long-range lattice systems have been realized in various controlled experimental platforms [28–47], and have been reported to show exotic physics like time crystals [38,46], prethermalization [34], dynamical phase transitions [33,37,44,48], environment assisted transport [40], etc. This has lead to a large number of studies in quantum transport which is so far limited mostly to isolated (nondissipative) systems in the presence and absence of disorder [49–57], and a very recent study on dissipative spin chains at infinite temperature [58]. But, interestingly, the physics of dissipative long-range fermionic systems at low temperatures, a class of which reveals the unique universal physics described here, has remained entirely unexplored previously.

The clean long-range hopping model.—We consider the following one-dimensional lattice model of fermions with long-range hopping decaying as a power law

$$\hat{\mathcal{H}}_S = -\sum_{m=1}^{N}\left[\sum_{r=1}^{N-m}\frac{1}{m^{\alpha}}\left(\hat{c}_r^{\dagger}\hat{c}_{r+m} + \hat{c}_{r+m}^{\dagger}\hat{c}_r\right)\right], \tag{1}$$

where $\hat{c}_r$ is the fermionic annihilation operator at the rth site of the system. Interestingly, this long-range model Hamiltonian has recently been realized using Floquet engineering technique in superconducting qubits [47]. The above system Hamiltonian can be written as, $\hat{\mathcal{H}}_S = \sum_{\ell,m=1}^{N}\mathbf{H}_{\ell m}\hat{c}_{\ell}^{\dagger}\hat{c}_m$, where the matrix $\mathbf{H}$ is a Toeplitz matrix with elements given by $\mathbf{H}_{\ell m} = |\ell - m|^{-\alpha}$, and $\mathbf{H}_{\ell\ell} = 0$. The eigenspectrum of this matrix, which corresponds to the single-particle eigenvalues and eigenvectors of $\hat{\mathcal{H}}_S$, are difficult to find analytically for arbitrary N. But, in the thermodynamic limit, $N \to \infty$, the single particle eigenvalues can be obtained via a Fourier transform, and correspond to the dispersion relation [59], $\epsilon(k,\alpha) = -2\sum_{m=1}^{\infty} m^{-\alpha}\cos(mk)$. The infinite series summation in the dispersion relation is absolutely convergent for all k if $\alpha > 1$. It is in this case that the thermodynamic limit ($N \to \infty$) is well defined. We will therefore always consider $\alpha > 1$. It can be numerically verified that the eigenvalues of $\mathbf{H}$ tend to this dispersion relation in the large N limit and the corresponding single-particle eigenvectors of the system are completely delocalized. This property indicates that there should be ballistic transport in the system [51]. On the contrary, as we will show below, in the open system scenario, there is a surprising subdiffusive to ballistic phase transition as a function of chemical potential for all $\alpha > 1$ at zero temperature. We note that the band edges of the dispersion relation correspond to $\varepsilon(0,\alpha) = -2\zeta(\alpha)$, where $\zeta(\alpha) = \sum_{m=1}^{\infty} m^{-\alpha}$ is the Riemann-zeta function, and $\varepsilon(\pm\pi,\alpha) = 2\eta(\alpha)$ with $\eta(\alpha) = \sum_{m=1}^{\infty}(-1)^{m-1}m^{-\alpha}$ being the Dirichlet-eta function.

Open system conductance at zero temperature.—To calculate the conductance in an open quantum system setting, we consider the two terminal transport setup, where the system is connected to two baths at its two ends, i.e., the first and the Nth sites. Such open system setup is exactly what is used for realizing autonomous (continuous) quantum heat engines, refrigerators, thermoelectric generators etc [4,60,61]. The left (right) bath is modeled by a noninteracting Hamiltonian with an infinite number of modes $\hat{\mathcal{H}}_{B_1} = \sum_{r=1}^{\infty}\Omega_{r1}\hat{B}_{r1}^{\dagger}\hat{B}_{r1}$ ($\hat{\mathcal{H}}_{B_N} = \sum_{r=1}^{\infty}\Omega_{rN}\hat{B}_{rN}^{\dagger}\hat{B}_{rN}$), where $\hat{B}_{r1}$ ($\hat{B}_{rN}$) is the fermionic annhilation operator of the rth mode of the left (right) bath. The baths are connected to the system with the system-bath coupling Hamiltonian $\hat{\mathcal{H}}_{SB} = \sum_{\ell=1,N}\sum_{r=1}^{\infty}(\kappa_{r\ell}\hat{c}_{\ell}^{\dagger}\hat{B}_{r\ell} + \kappa_{r\ell}^{*}\hat{B}_{r\ell}^{\dagger}\hat{c}_{\ell})$. Initially, the baths are assumed to be at their own thermal states with their own temperatures and chemical potentials (μ_1, μ_N), while the system's initial state is arbitrary. We are specifically interested in the nonequilibrium steady state (NESS) in the zero temperature limit and linear response regime, $\mu_1 = \mu, \mu_N = \mu - \Delta\mu, \Delta\mu \to 0$.

It is possible to obtain the exact NESS properties of the system using the nonequilibrium Green's function (NEGF) approach [62–64]. The retarded NEGF for such a setup is given by $\mathbf{G}^{+}(\omega) = [\omega\mathbb{I} - \mathbf{H} - \Sigma^{(1)}(\omega) - \Sigma^{(N)}(\omega)]^{-1}$, where $\mathbb{I}$ is the N-dimensional identity matrix, and $\Sigma^{(1)}(\omega)$ [$\Sigma^{(N)}(\omega)$] is the self-energy matrix due to the left (right) bath. The only nonzero element in the $N \times N$ left (right) bath self-energy matrix is the top left (bottom right) corner element, $\Sigma_{\ell\ell}^{(\ell)}(\omega) = -i\{[\mathfrak{J}_{\ell}(\omega)]/[2]\} - \mathcal{P}\int[(d\omega')/(2\pi)]\{[\mathfrak{J}_{\ell}(\omega')]/[\omega - \omega']\}$, $\ell = \{1,N\}$ [59]. Here $\mathcal{P}$ denotes principal value, and $\mathfrak{J}_{\ell}(\omega)$ is the bath spectral function, defined as $\mathfrak{J}_\ell(\omega) = 2\pi \sum_{r=1}^{\infty} |\kappa_{r\ell}|^2 \delta(\omega - \Omega_{r\ell})$. The zero temperature particle conductance $G(\mu)$ is given in terms of the NEGF as

$$G(\mu) = \lim_{\Delta\mu \to 0} \frac{I}{\Delta\mu} = \frac{1}{2\pi} \mathcal{T}(\mu) = \frac{\mathfrak{J}_1(\mu) \mathfrak{J}_N(\mu) |\mathbf{G}^+_{1N}(\mu)|^2}{2\pi}, \tag{2}$$

where $\mathbf{G}^+_{1N}(\mu)$ denotes the $(1, N)$th element of the matrix $\mathbf{G}^+(\mu)$, $\mathcal{T}(\omega)$ is the transmission function and $I = \int_{\mu_1}^{\mu_N} [(d\omega)/(2\pi)] \mathcal{T}(\omega)$ is the NESS particle current. The scaling of conductance with system-size N is used to classify transport properties as described in the introduction. Note that, for anomalous transport, the classification of different transport regimes in terms of conductance scaling with system length, which is standard in an open system setting, may not lead the to same results as its corresponding isolated system counterpart where the regimes are classified via time scaling of spread of correlations [17].

Chemical-potential-driven subdiffusive to ballistic phase transition.—We numerically calculate the exact transmission function, and study the conductance scaling with system size. Our central result is as follows. For $1 < \alpha < \infty$,

$$\begin{aligned} &G(\mu) \sim N^{-2\alpha}, \quad \forall \ \mu < -2\zeta(\alpha), \mu > 2\eta(\alpha), \\ &G(\mu) \sim N^{-2}, \quad \text{at } \mu = -2\zeta(\alpha), 2\eta(\alpha), \\ &G(\mu) \sim N^{0}, \quad \forall \ -2\zeta(\alpha) < \mu < 2\eta(\alpha), \end{aligned} \tag{3}$$

where, as mentioned before, $\zeta(\alpha)$ is the Riemann-zeta function and $\eta(\alpha)$ is the Dirichlet-eta function. In other words, when the chemical potential μ is within the band of the system, the transport is ballistic, as expected. But, surprisingly, when μ lies outside the band of the system, the transport is subdiffusive, with an exponent of 2α. Moreover, when μ is located exactly at the band edges, the transport is again subdiffusive but with an α independent exponent. Representative plots showing the above behavior are given in Fig. 2. Figure 2(a) shows the behavior $G(\mu)$ with μ for various system sizes at a chosen value of α ($\alpha = 1.75$). Clearly, within the band, i.e., $-2\zeta(\alpha) < \mu < 2\eta(\alpha)$, there is no scaling of $G(\mu)$ with N, confirming perfect ballistic behavior, whereas outside that regime $G(\mu)$ scales with system size. Figure 2(b) shows the same result as in Fig. 2(a) with the y axis now scaled by $N^{2\alpha}$. All data points outside the band of the system collapse perfectly, thereby confirming the subdiffusive scaling. Likewise, the α independent scaling at the band edges can also be checked (not shown in the figure for $\alpha = 1.75$) numerically. Interestingly, this behavior is seen at all values of $\alpha > 1$. Figure 2(c) shows conductance scaling with system size at values close to the system band edges for a different value of α ($\alpha = 2$). The behavior consistent with Eq. (3) is clearly observed here.

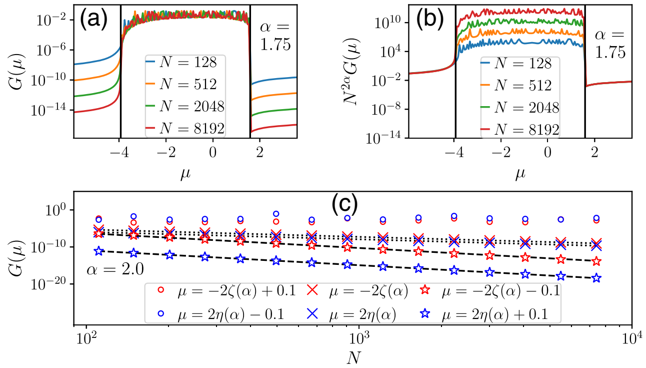

FIG. 2. (a) Zero-temperature conductance $G(\mu)$ as a function of chemical potential μ, at a chosen value of $\alpha = 1.75$ for various system sizes N. The two vertical lines correspond to band-edges $\mu = -2\zeta(\alpha)$ and $\mu = 2\eta(\alpha)$. (b) The same plot as in (a) but with the y-axis scaled by $N^{2\alpha}$. (c) The scaling of $G(\mu)$ with system size at various values of μ, for $\alpha = 2.0$. The black dashed lines are fits of $N^{-2\alpha}$. The black dotted lines are fits of N^{-2}. For the plots, the bath spectral functions are chosen to be $\mathfrak{J}_1(\omega) = \mathfrak{J}_N(\omega) = \Gamma\sqrt{1 - (\omega/\Lambda)^2}$, with $\Lambda = 8$, $\Gamma = 10$. All energy scales are in units of nearest-neighbor hopping strength.

Origin of the subdiffusive phases.—The origin of these surprising subdiffusive phases for chemical potentials outside the band of the system can be traced to the nonanalyticity property of the dispersion relation at its minimum value at $k = 0$. From Eqs. (2), it is evident that the system size scaling of conductance originates from that of $\mathbf{G}^+_{1N}(\mu)$. Since the baths are attached only to the first and the last sites, we conjecture that, for large N, system size scaling of $\mathbf{G}^+_{1N}(\mu)$ will be same as $\mathbf{g}^+_{1N}(\mu)$, where $\mathbf{g}^+(\mu) = [(\mu - i\epsilon)\mathbb{I} - \mathbf{H}]^{-1}$ is the retarded Green's function of the system in the absence of the baths. That is, $\mathbf{G}^+_{1N}(\mu) \propto \mathbf{g}^+_{1N}(\mu)$ with the proportionality constant being independent of N. Also, since the system is clean (ordered), in the $N \to \infty$ limit, one can obtain the bare retarded Green's function via a Fourier transform, $\mathbf{g}^+_{pq}(\mu) = \lim_{\epsilon \to 0} \int dk g^+(k, \mu) e^{-ik|p-q|}$, where $g^+(k, \mu) = [\mu - \varepsilon(k, \alpha) - i\epsilon]^{-1}$. Combining all of these, we have, for large N,

$$\mathbf{G}^+_{1N}(\mu) \propto \lim_{\epsilon \to 0} \int_{-\pi}^{\pi} dk \frac{e^{-ikN}}{\mu - \varepsilon(k, \alpha) - i\epsilon}. \tag{4}$$

The above heuristic expression, in combination with Eq. (2), relates the scaling of conductance with system size of an open system with spectral properties of the isolated system in the thermodynamic limit.

The major contribution to the above integral comes from the singularities of the integrand. It can be checked that $k = 0$ is always a singular point because $\varepsilon(k, \alpha)$ is nonanalytic at $k = 0$, $\lim_{k \to 0} \{[\partial^p \varepsilon(k, \alpha)]/[\partial k^p]\} \to \infty, \ \forall \ p > \alpha - 1$. To capture the effect of this nonanalyticity, we derive a nontrivial nonanalytic small k expansion of $\varepsilon(k,\alpha)$ for noninteger $\alpha > 1$ [59], $\varepsilon(k,\alpha) \simeq -2[\zeta(\alpha) - a_1|k|^{\alpha-1} - a_2k^2]$, $\forall\ |k| \ll 1$ where a_1 and a_2 are real numbers. The presence of $|k|^{\alpha-1}$ makes the above expression explicitly nonanalytic, clearly distinguishing it from a standard Taylor expansion. While evaluating the integral in Eq. (4) via contour integration, the noninteger value of α leads to a branch whose contribution to Eq. (4) can be shown to scale with system size as $N^{-\alpha}$ [59]. Though these results are obtained for noninteger values of α, integer values of α can be included by assuming an arbitrarily small fractional part.

Now, when μ is within the band of the system, $-2\zeta(\alpha) < \mu < 2\eta(\alpha)$, there are additional poles on the real line. It can be checked easily that such poles can at best generate an oscillatory behavior with N and thus cannot provide a scaling with N. These poles within the band therefore gives the leading behavior $\mathbf{G}^{+}_{1N}(\mu) \sim N^0$ implying ballistic transport. When μ is below the band of the system, i.e., $\mu < -2\zeta(\alpha)$, the additional poles on the real-line do not exist, and the main contribution to the integral comes from the nonanalytic point $k \to 0$. As mentioned above, in this case, the contour integration generates a scaling of the form $\mathbf{G}^{+}_{1N}(\mu) \sim N^{-\alpha}\ \forall\ \mu < -2\zeta(\alpha)$, leading to a subdiffusive exponent of 2α from Eq. (2). On the other hand, when $\mu > 2\eta(\alpha)$, one may argue that the main contribution to the integral comes from $k \sim \pm\pi$ corresponding to the upper band edge, $\varepsilon(\pm\pi,\alpha) = 2\eta(\alpha)$, where the denominator in Eq. (4) would be minimum. However, an expansion about this point, $\varepsilon(k \pm \pi,\alpha) \simeq 2\eta(\alpha) - 2a_2(1 - 2^{3-\alpha})k^2$, $|k| \ll 1$, shows that, unlike the lower band edge at $k = 0$, this point is analytic, and its contribution to the integral in Eq. (4) decays exponentially with N [59]. Consequently, for large enough N, the leading contribution once again stems from the singularity at $k = 0$, and giving $\mathbf{G}^{+}_{1N}(\mu) \sim N^{-\alpha}\ \forall\ \mu > 2\eta(\alpha)$, leading to the same subdiffusive exponent. However, interestingly, since the denominator in Eq. (4) is now large, the value of $\mathbf{G}^{+}_{1N}(\mu)$, and therefore the conductance, for $\mu > 2\eta(\alpha)$ is much smaller than that for $\mu < -2\zeta(\alpha)$, even though the system size scaling is the same. This is clearly seen in all the plots of Fig. 2.

A more careful analysis is required at the critical points $\mu = -2\zeta(\alpha), 2\eta(\alpha)$. At any finite N, the critical μ values always lie slightly outside the system band, but the minimum and maximum eigenvalues of $\mathbf{H}$ approach these values with increase in N. We find that it becomes difficult to use Eq. (4) to capture this behavior. Nevertheless, the conjecture $\mathbf{G}^{+}_{1N}(\mu) \propto \mathbf{g}^{+}_{1N}(\mu)$ still holds, and it can be directly numerically checked for finite N that $\mathbf{g}^{+}_{1N}[-2\zeta(\alpha)]$, $\mathbf{g}^{+}_{1N}[2\eta(\alpha)] \sim N^{-1}$, independent of α [59]. This therefore clearly gives the origin of N^{-2} scaling of conductance at $\mu = -2\zeta(\alpha), 2\eta(\alpha)$.

A different type dissipative quantum phase transition.— Since $G(\mu) \propto |\mathbf{G}^{+}_{1N}(\mu)|^2$, a non-analytic change in $G(\mu)$ corresponds to a non-analytic change in NESS, thereby pointing to a dissipative quantum phase transition. This type of dissipative quantum phase transition, to our knowledge, has not been discussed before. In existing examples of dissipative phase transitions in the literature that we know of (for example, Refs. [65–79]) the phase transition occurs on changing either a parameter in the system Hamiltonian, or the strength of the system-bath couplings. In contrast, here, the phase transition occurs as a function of the chemical potentials of the baths. These are not Hamiltonian parameters, either of the system or of the baths, but rather are the thermodynamic parameters fixed by the initial state of the baths. These control the zero temperature noise that originates from the baths, which, in turn control the NESS.

As is clear from the above results, this phase transition stems from the nonanalyticity of the dispersion relation of the system in the thermodynamic limit. It is therefore a property of the system in the large N limit, and is completely independent of details of the baths, as long as there is a unique NESS. In fact to guarantee a unique NESS, only two properties of the bath spectral functions are required: (a) the spectral functions for both the baths must be continuous, (b) the band of the baths must encompass the band of the system [64]. Notably, the strength of system-bath coupling, while determining the value of conductance, does not affect the system-size scaling of conductance. This is evident from validity of the conjecture $\mathbf{G}^{+}_{1N}(\mu) \propto \mathbf{g}^{+}_{1N}(\mu)$, and can also be verified numerically [59]. However, the presence of the baths are crucial to allow subdiffusive transport at the chemical potentials beyond the system bandwidth. The isolated system at such chemical potentials would either be completely empty or completely filled, thereby having no possibility of transport. Thus, the subdiffusive behavior observed here has no isolated system analog.

This phase transition is clearly a quantum phase transition, as it occurs strictly at zero temperature. At any finite temperature, at all values of chemical potentials of the baths, calculation of current or conductance will have finite contribution from energies within the system energy bands. At low temperatures, for chemical potentials outside the system band, this contribution will be small, but as system size is increased, will eventually be the leading contribution. So, at finite but low temperatures, for chemical potentials outside the system band, there will be a crossover from the subdiffusive to the ballistic behavior as a function of system size. Thus, like standard quantum phase transitions, this phase transition gives rise to a finite size crossover at finite but low temperatures.

It is important to note that the standard Lindblad equation approaches in local and global (eigenbasis) forms [4,80–95] cannot capture these subdiffusive phases. This is because such approaches, by construction, neglect contributions coming from bath energies which are away from system energy scales. Such an equation would therefore wrongly predict zero conductance for chemical potentials outside the system band at all system sizes [59]. Whether more refined quantum master equation approaches [96–101], including the Redfield equation [4,80,86,93], can capture the subdiffusive behavior remains to be seen and requires further investigation.

Universality of the subdiffusive phases.—When $\mu \leq -2\zeta(\alpha)$, it is intuitive and can be numerically checked [59] that there is a subextensive number of particles in the system. If a number-conserving many-body interaction term (i.e., higher than quadratic term, for example, $\hat{\mathcal{H}}_{\rm int} = \sum_{\ell,m} V_{\ell m}\hat{c}^{\dagger}_{\ell}\hat{c}_{\ell}\hat{c}^{\dagger}_{m}\hat{c}_{m}$, $\hat{\mathcal{H}}_S \to \hat{\mathcal{H}}_S + \hat{\mathcal{H}}_{\rm int}$) is now switched on, at large enough N, due to extremely low particle density in the system, it will play a negligible role for $\mu \leq -2\zeta(\alpha)$. A similar argument, in terms of holes rather than particles, can be made for $\mu \geq 2\eta(\alpha)$. Thus, the subdiffusive phases for chemical potential outside the system band, as well as the critical points, are robust against the presence of arbitrary number-conserving many-body interactions in the system. Therefore they are universal.

For $\mu < -2\zeta(\alpha)$ $[\mu > 2\eta(\alpha)]$, the intuitive picture that emerges is that few particles (holes) tunnel into the system from one bath due to quantum fluctuations, and then hop into the other bath with essentially a single long-range hop of amplitude $\sim |N^{-\alpha}|^2$. This is consistent with the scaling $G(\mu) \sim N^{-2\alpha}$. However, such a simple picture does not explain the $G(\mu) \sim N^{-2}$ scaling at the critical points $\mu = -2\zeta(\alpha), 2\eta(\alpha)$. Moreover, the scaling at the critical points is superuniversal since it is also independent of α, which controls the effective range of hopping. Conversely, for $-2\zeta(\alpha) < \mu < 2\eta(\alpha)$, there will be a finite particle density in the system and the many-body interactions can have a nontrivial effect that can change the nature of the transport, making this regime nonuniversal.

Direct demonstration of above statements in presence of many-body interactions is currently beyond the state-of-the-art numerical techniques. But, interestingly, long-range magnetization-conserving spin Hamiltonians with power-law-decaying interactions have been realized in several experimental platforms [39–44]. These can be mapped via Jordan-Wigner transformation into number-conserving fermionic Hamiltonians with power-law-decaying hopping and many-body interactions [59]. This makes experimental verification of the universal subdiffusive phases plausible. The effect of uncorrelated or correlated disorder [57] on such subdiffusive phases remains to be seen.

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