


Imprint of the Seesaw Mechanism on Feebly Interacting Dark Matter and the Baryon Asymmetry

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We show that the type-I seesaw, responsible for generating the light neutrino mass, itself is capable of accommodating one of the three right handed neutrinos as a freeze-in type of dark matter where the required smallness of the associated coupling is connected to the lightness of the (smallest) active neutrino mass. It turns out that (a) the nonthermal production of dark matter having mass $\lesssim \mathcal{O}(1)$ MeV (via decays of W , Z bosons and standard model Higgs) consistent with relic density and (b) its stability determine this smallest active neutrino mass uniquely $\sim \mathcal{O}(10^{-12})$ eV. On the other hand, the study of flavor leptogenesis in this scenario (taking into account the latest neutrino data and Higgs vacuum stability issue) fixes the scale of two other right handed neutrinos.

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Among the various unresolved issues of present day particle physics and cosmology, perhaps the most pressing ones are the origin of tiny neutrino mass [1–3], the nature of dark matter (DM) [4,5] and the observed matter-antimatter asymmetry of the Universe [6]. In order to address these issues one has to anyway go beyond the standard model (SM) of particle physics, hence it would be very pertinent to search for a single minimal framework that can accommodate all these three problems together. To start with, one notices that the type-I seesaw mechanism [7–11] of neutrino mass generation provides a very promising platform for this. In this mechanism, three additional heavy SM singlet right handed neutrinos (RHN) are added to the SM particle content. A handful of attempts have been made in identifying one of them as dark matter without including any further beyond the SM fields and symmetries. For example, in the original ν MSM model [12,13], the lightest RHN (say, N_1) is shown to be the DM having mass $\sim \mathcal{O}$ (keV). While the production of DM proceeds via the Dodelson-Widrow (DW) mechanism [14] incorporating the effective active-sterile neutrino mixing θ_1 , the Akhmedov-Rubakov-Smirnov mechanism [15] takes care of the observed baryon asymmetry via coherent oscillations of heavy RHNs. It turns out that the DW mechanism cannot make up the entire DM relic density taking into account the existing recent constraints on θ_1 [16–22]. However, a variant of this incorporating a resonant production of DM via Shi-Fuller mechanism [23] can still be operative [23,24].

Though it bypasses the constraint on the mixing angle θ_1 , the mechanism suffers from an unnatural level of degeneracy required between the two heavy RHNs $N_{2,3}$. Most of the other constructions with the aim of identifying the RHN sector serving as the origin of DM and baryon asymmetry require additional fields and/or enhanced symmetry [25,26].

In this Letter, we stick to the most minimal construction of type-I seesaw while identifying N_1 as the feebly interactive massive particle (FIMP) [27,28] type of DM and the rest of RHNs are mainly responsible for generating light neutrino mass and matter-antimatter asymmetry. Interestingly, we find that a sufficient production of N_1 can be obtained from the decays of W , Z , and SM Higgs h which are intricately related to the specific entries of neutrino Yukawa matrix Y_ν and in turn depend on the lightest active neutrino mass (m_1). These entries, being involved in generating the respective active-sterile mixing θ_1 associated with N_1 , also control possible decays of N_1 . It turns out that an interplay between the production and decays of N_1 (such that it remains stable over the cosmological time scale) fixes the allowed range of DM mass (M_1) consistent with the stringent limits on θ_1 .

The importance of our work lies in the fact that it provides perhaps the most minimal platform in the literature to address neutrino mass, dark matter, and matter-antimatter asymmetry where the small coupling usually required for a FIMP realization is connected to the smallness of the lightest active neutrino mass m_1 . Such a connection is presented here for the first time to the best of our knowledge. It indicates an upper limit on m_1 as $m_1 \lesssim \mathcal{O}(10^{-12})$ eV. Interestingly, we find the DM relic turns out to be effectively independent of the DM mass within its allowed range. This opens up the possibility that

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the scenario can be tested if the recent and future experiments can measure m_1 . As masses of $N_{2,3}$ ($M_{2,3}$) are unconstrained at this stage, we find that imposing an additional constraint on Y_ν as $\text{Tr}[Y_\nu^\dagger Y_\nu] < \mathcal{O}(1)$ (justified later) restricts the production of N_1 from the decays of $N_{2,3}$. This ambiguity of fixing $M_{2,3}$ is resolved once we incorporate flavor leptogenesis [29–32].

We start with the conventional type-I seesaw Lagrangian (in charged lepton diagonal basis) involving SM lepton (l_L) and Higgs (H) doublets by

$$-\mathcal{L}_{\text{Int}} = (Y_\nu)_{\alpha i} \bar{l}_{L\alpha} \tilde{H} N_i + \frac{1}{2} M_i \bar{N}_i^c N_i + \text{H.c.}, \quad (1)$$

with $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$. We assume the RHN mass matrix M as diagonal with hierarchical masses. As a result of the electroweak (EW) symmetry breaking, the light neutrino mass matrix is given by the seesaw formula, $m_\nu = -m_D M^{-1} m_D^T$ which is diagonalized by $U^\dagger m_\nu U^* = \text{diag}(m_1, m_2, m_3) \equiv m_\nu^d$, where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [33–35] and $(m_D)_{ij} = (Y_\nu)_{ij} v / \sqrt{2}$, where $v = 246$ GeV.

To begin with, we consider the mass of the DM to be lighter than the W boson mass (m_W) so that the possibility of its production from decays of the SM gauge bosons (via active-sterile neutrino mixing) and Higgs (via neutrino Yukawa interaction) remains plausible. On the other hand, masses of the remaining RHNs are assumed to be above the EW scale. Considering the fact that the decay of N_1 can even proceed via the relevant active-sterile neutrino mixing $m_{D1i}/M_1 \equiv V_{i1}$, we propose the following structure of neutrino Yukawa matrix at the leading order

$$Y_\nu = \begin{pmatrix} 0 & y_{e2} & y_{e3} \\ 0 & y_{\mu 2} & y_{\mu 3} \\ 0 & y_{\tau 2} & y_{\tau 3} \end{pmatrix}. \quad (2)$$

As a result of the vanishing left block (L_B), N_1 remains completely decoupled and hence absolutely stable while $N_{2,3}$ along with the right block (R_B) entries of Y_ν generate light neutrino mass via seesaw. This also ensures that the lightest active neutrino mass m_1 becomes zero and a vanishing V_{i1} results. The entries of Y_ν can be written using the Casas-Ibarra parametrization [36]:

$$m_D = -iUD_{\sqrt{m}} R^T D_{\sqrt{M}}, \quad (3)$$

where U is the PMNS [33–35] mixing matrix, D_m (D_M) is the diagonal active neutrino (RHN) mass matrix: m_ν^d (M) and R is a complex orthogonal matrix chosen to be of the form

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_R & \sin \theta_R \\ 0 & -\sin \theta_R & \cos \theta_R \end{pmatrix}, \quad (4)$$

where θ_R is a complex angle in general. We employ the best fitted values [33] of mixing angles, CP phase as well as mass-square differences to define the U and $D_{\sqrt{m}}$.

Under such a situation, N_1 being completely segregated cannot be produced by any interaction (except a gravitational one perhaps). This problem can be circumvented by perturbing m_D , i.e., introducing small but nonzero entries in $L_B = (\epsilon_1, \epsilon_2, \epsilon_3)^T$ with $\epsilon_{i=1,2,3} \ll 1$. The order of smallness will be determined from the relic satisfaction of DM as well as from the stability of N_1 . Note that origin of these ϵ_i can be associated [37] to a tiny m_1 or an additional angle of rotation (say φ) over R or including both. In this work, without any loss of generality, we would like to pursue our analysis with small m_1 as the same result can be obtained from the use of φ . In this case, following Eq. (3), it is seen that ϵ_i turns out to be proportional to $\sqrt{m_1 M_1}$. The DM phenomenology is almost independent to R_B of Y_ν .

With such a scenario in mind, the active neutrinos and $N_{2,3}$ remain in thermal equilibrium with other SM fields while N_1 is expected to be in out-of-equilibrium (due to its small coupling proportional to ϵ_i) in the early Universe having negligible abundance. Later, once the temperature goes down, the DM is expected to be produced non-thermally from decay of some massive particle or via annihilations. In this simplest seesaw setup, we find subsequent to the EW symmetry breaking, the DM N_1 can be produced from the following dominant decays:

$$W^\pm \rightarrow N_1 \ell_i^\pm, \quad Z \rightarrow N_1 \nu_i, \quad h \rightarrow N_1 \nu_i; \quad N_{i \neq 1} \rightarrow N_1 h(Z).$$

The relevant parts of the Lagrangian responsible for the above decays via the active sterile mixing $V = m_D M^{-1}$ are followed from the gauge interactions

$$\begin{aligned} -\mathcal{L}_G \subset & \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i,j=1}^3 [\bar{N}_i^c (V^\dagger)_{ij} \gamma^\mu P_L \ell_j] + \frac{g}{2C_{\theta_w}} Z_\mu \\ & \times \sum_{i,j=1}^3 [\bar{\nu}_i (U^\dagger V)_{ij} \gamma^\mu P_L N_j^c + \bar{N}_i^c (V^\dagger V)_{ij} \gamma^\mu P_L N_j^c] \\ & + \text{H.c.}, \end{aligned} \quad (5)$$

and Yukawa interactions

$$\begin{aligned} -\mathcal{L}_Y \subset & \frac{\sqrt{2}}{v} h \sum_{i,j=1}^3 [\bar{\nu}_i (U^\dagger V)_{ij} M_j N_j + \bar{N}_i^c (V^\dagger U)_{ij} m_j \nu_j^c] \\ & + \bar{N}_i^c (V^\dagger V)_{ij} M_j N_j + \text{H.c.}, \end{aligned} \quad (6)$$

where ν_i are active neutrino mass eigenstates. The origin of the most relevant mixing $V_{i1} = m_{D1i}/M_1$ ($\equiv \epsilon_i(v/\sqrt{2}M_1)$) is traced back to ϵ_i entries of Y_ν .

We then employ the coupled set of Boltzmann equations involving N_1 and $N_{2,3}$ separately to study the evolution of their abundance (Y_{N_i}) till the present time, as

$$\frac{dY_{N_1}}{dz} = \frac{2M_{pl}z g_p^{1/2}}{1.66m_h^2 g_s} \left[\sum_{i=2,3} \left(Y_{N_i} \sum_{x=Z,W} \langle \Gamma_{N_i \rightarrow N_1 x} \rangle \right) + \sum_{x=Z,h} Y_x^{eq} \langle \Gamma_{x \rightarrow N_1 \nu} \rangle + Y_W^{eq} \langle \Gamma_{W^\pm \rightarrow N_1 \ell^\pm} \rangle \right], \quad (7)$$

$$\frac{dY_{N_i}}{dz} = -\frac{2M_{pl}z g_p^{1/2}}{1.66m_h^2 g_s} [(Y_{N_i} - Y_{N_i}^{eq}) \langle \Gamma^D \rangle + Y_{N_i} \sum_{x=h,Z} \langle \Gamma_{N_i \rightarrow N_1 x} \rangle], \quad (i=2,3), \quad (8)$$

with $z = m_h/T$. Here, $\Gamma^D = \Gamma(N_i \rightarrow lH) + \Gamma(N_i \rightarrow \bar{l}\bar{H}) = (M_i/8\pi v^2)(m_D^\dagger m_D)_{ii}$ and $\langle \Gamma_{A \rightarrow BC} \rangle$ represents the thermally averaged decay width [38]. All relevant decay widths are obtained from Eqs. (5) and (6). Note that the annihilations producing N_1 are very much suppressed ($\sim \epsilon_i^4$) compared to decay ($\sim \epsilon_i^2$) and hence are not included. At this stage, we presume N_1 to be stable over the cosmological timescale which will be justified in a while. Back reactions involving N_1 are not included as N_1 number density is vanishingly small to start with and for the same reason, terms proportional to Y_{N_1} are also dropped. Substituting the abundance $Y_{N_1}(z_\infty)$ after freeze-in, the relic density is obtained from

$$\Omega_{N_1} h^2 = 2.755 \times 10^5 \left(\frac{M_1}{\text{MeV}} \right) Y_{N_1}(z_\infty). \quad (9)$$

The variation of the dark matter abundance Y_{N_1} as a function of z is shown in Fig. 1 where Y_{N_1} (combined contribution as denoted by the solid blue line) reaches an asymptotic value $Y_{N_1}(z_\infty)$, so as to obtain the correct relic, $\Omega_{N_1} h^2 = 0.12$ [39] via Eq. (9). Note that, we have parameters $m_1, M_{1,2,3}$, and θ_R . In generating this plot, we have fixed M_1 at 0.1 MeV while $M_{2(3)}$ are kept at $3.5(75) \times 10^9$ GeV as deemed fit for generating correct

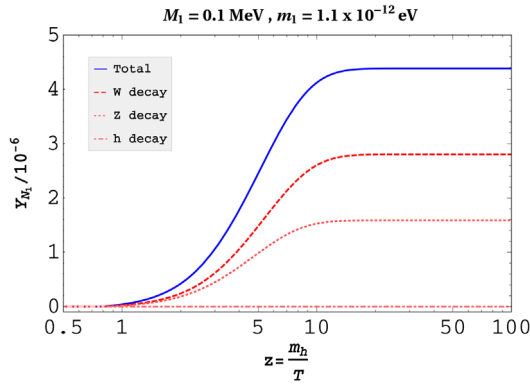


FIG. 1. Abundance plot of N_1 with individual contributions (explained in inset) from different decays; final abundance (solid blue line) corresponds to the correct DM relic.

baryon asymmetry via leptogenesis (discussed later). With such a choice of M_1 , $m_1 \sim 1.1 \times 10^{-12}$ eV is found to satisfy the relic implying $|\epsilon_i| \sim 10^{-15}$. We find that the production of N_1 is dominated by the decay of gauge bosons, in particular by W^\pm decay [40], as emphasized in Fig. 1. The reason is the following. The production of N_1 from gauge and Higgs bosons depends on ϵ_i elements of Y_ν only (via V) whereas N_1 production [43] from decay of $N_{2,3}$ involves a product of ϵ_i of L_B and elements of R_B (via $V^\dagger V$) as seen from Eqs. (5) and (6). While entries of L_B are generated from m_1 , elements of R_B are controlled by the magnitude of θ_R , mostly by $\text{Im}[\theta_R]$. We find that any value of $\text{Im}[\theta_R] \lesssim 5$ keeps R_B entries (or more precisely $\text{Tr}[Y_\nu^\dagger Y_\nu]$) below $\mathcal{O}(1)$. We also notice that with larger $\text{Im}[\theta_R]$, entries of R_B would increase significantly. Such a large Y_ν would be problematic not only from perturbativity but also due to the fact that EW vacuum becomes unstable [44]. Since DM production except from $N_{2,3}$ decays are anyway independent to entries of R_B , we refrain from quoting specific value for $\text{Re}[\theta_R]$ at this moment and reserve the related discussion for the leptogenesis part.

As the DM N_1 mixes with the SM fields via the active-sterile mixing angle V_{i1} , we need to look for the all possible decay channels of it. There are three body decays, (a) (via off shell W/Z): $N_1 \rightarrow l_1^- l_2^+ \nu_l$, $N_1 \rightarrow l^- q_1 \bar{q}_2$, $N_1 \rightarrow l^- l^+ \nu_l$, $N_1 \rightarrow \nu_l \bar{l} l'$, $N_1 \rightarrow \nu_l q \bar{q}$, $N_1 \rightarrow \nu_l \nu_l' \bar{\nu}_l'$, $N_1 \rightarrow \nu_l \nu_l \bar{\nu}_l$; (b) (via off-shell Higgs): $N_1 \rightarrow \nu_\ell \bar{\ell} \ell'$; as well as (c) (radiative decay of N_1): $N_1 \rightarrow \gamma \nu$. Keeping in mind that the expected lifetime of N_1 must be greater than the age of the Universe, it turns out that the most stringent constraint is obtained from (c), which can be translated on the active-sterile neutrino mixing V_{i1} as [45–47],

$$\theta_1^2 = \sum_{i=1,2,3} |V_{i1}|^2 \leq 2.8 \times 10^{-18} \left(\frac{\text{MeV}}{M_1} \right)^5. \quad (10)$$

Below in Fig. 2, we generate the relic contour plot in the $\theta_1^2 - M_1$ plane drawn as the solid purple line, while the region in light blue is excluded from the above constraint. So at this point, we find N_1 as a successful FIMP type DM having mass below MeV. It is also interesting to note that the final DM relic density is independent to the mass of N_1 . This observation stems from the fact that (a) the crucial parameter responsible for generating the dark matter abundance is $\epsilon_i \propto \sqrt{m_1 M_1}$ and (b) the dominant production of N_1 is from W and Z decays. The corresponding decay width (and hence Y_{N_1} also) turns out to be proportional to m_1/M_1 [see Eq. (5)]. Then the final relic density $\Omega_{N_1} h^2$ being related to $M_1 Y_{N_1}$, the M_1 dependence is canceled out and m_1 is uniquely fixed to satisfy the relic. This leads to an interesting prediction for lightest active neutrino mass $\sim (1.07-1.12) \times 10^{-12}$ eV (considering the 3σ range of DM relic density) so that the model remains falsifiable in nature. Off course, if one incorporates the

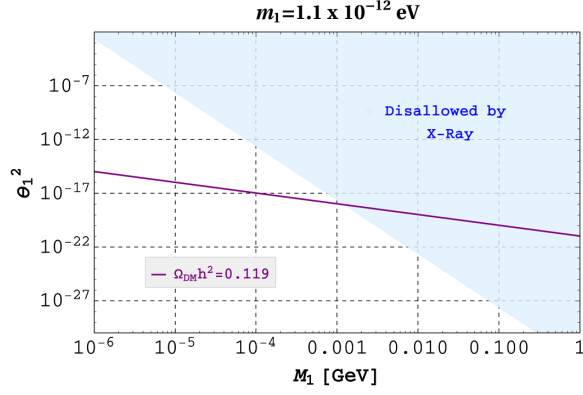


FIG. 2. Relic satisfaction contour (solid purple line) in $\theta_1^2 - M_1$ plane. Constraint on θ_1^2 from x-ray observation due to $N_1 \rightarrow \gamma\nu$ decay excludes the blue shaded region.

effect of both m_1 and the extra rotation φ , this value of m_1 serves as the upper limit of lightest neutrino mass.

We have verified that the nonthermal condition $\Gamma/H < 1$ at $T \sim m$ is satisfied where Γ corresponds to the relevant decay width for a particular production channel of N_1 and m is the mass of the decaying particle. Hence, the DM particles produced (having a mass range of 1 keV–1 MeV) cannot have sufficient energy to be associated with large free streaming length, thereby treated as cold dark matter, contrary to the DW mechanism associated with warm dark matter (~ 2 –10 keV). The lower limit on M_1 is considered as 1 keV to be consistent with Tremaine-Gunn bound [48] on sterile neutrino mass. A detailed study on the nature of DM in this range is beyond the scope of the Letter. Finally, considering all these constraints, the range of DM mass turns out to be restricted within 1 keV–1 MeV.

We now proceed to discuss the role of two other heavier RHNs, $N_{2,3}$ and their cosmological evolution. While they help in realizing the correct order of light neutrino mass and mixing, we find their contribution to DM production is almost negligible. That being said, their masses can be anywhere between a few hundred GeV to a very large scale. However, considering the fact that their decay can explain baryon asymmetry of the Universe via leptogenesis, we can now have a complete picture including neutrino mass, dark matter, and lepton asymmetry which will also tell us about these otherwise unspecified mass scales.

Being heavier than the Higgs mass, $N_{2,3}$ are expected to decay into lepton doublet and Higgs via the Yukawa interaction of Eq. (1). This out of equilibrium decay along with the CP violation present in Y_ν will be crucial in leptogenesis. Note that $\text{Im}[\theta_R]$ serving as the source of CP violation via Eq. (3), apart for a subdominant contribution from Dirac CP phase in U , is the same one which mostly restricts the production of DM from the decay of $N_{2,3}$ while in tension with the EW vacuum stability.

It is preferable to keep the heavy neutrino masses as low as possible in view of naturalness of hierarchy within RHNs, and hence we opt for flavor leptogenesis here. As $M_2 < M_3$, the CP asymmetry $\epsilon_{2\alpha}^{CP}$ is effectively generated from the decay of N_2 to a specific flavor l_α . Using the standard expression [30], we evaluate $\epsilon_{2\alpha}^{CP}$ first and then proceed for estimating the final lepton asymmetry employing the set of Boltzmann equations

$$sH z' \frac{dY_{N_2}}{dz'} = - \left\{ \left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) (\gamma_D + 2\gamma_{H_s} + 4\gamma_{H_t}) \right\}, \quad (11)$$

$$sH z' \frac{dY_{\Delta_\alpha}}{dz'} = - \left\{ \left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) \epsilon_{2\alpha}^{CP} \gamma_D + K_\alpha^0 \sum_\beta \left[\frac{1}{2} (C_{\alpha\beta}^\ell + C_\beta^H) \gamma_D + \left(\frac{Y_{N_2}}{Y_{N_2}^{\text{eq}}} - 1 \right) \left(C_{\alpha\beta}^\ell \gamma_{H_s} + \frac{C_\beta^H}{2} \gamma_{H_t} \right) + (2C_{\alpha\beta}^\ell + C_\beta^H) \left(\gamma_{H_t} + \frac{1}{2} \gamma_{H_s} \right) \right] \frac{Y_{\Delta_\beta}}{Y^{\text{eq}}} \right\}, \quad (12)$$

where $K_\alpha^0 = \{[(Y_\nu^*)_{\alpha 2} (Y_\nu)_{\alpha 2}] / [(Y_\nu^\dagger Y_\nu)_{22}]\}$ is known as flavor projector and C^ℓ, C^H matrices connect the asymmetries in lepton and Higgs sectors to asymmetries in $\Delta_\alpha = B/3 - L_\alpha$ expressed in terms of $Y_{\Delta_\alpha = e, \mu, \tau}$. Here, γ_X is the corresponding reaction rate density [49]. The final baryon asymmetry is obtained as $Y_B = (28/79) \sum_\alpha Y_{\Delta_\alpha}$.

Figure 3 depicts the variation of individual components of lepton asymmetry Y_{Δ_α} as well as the total baryon asymmetry Y_B with respect to $z' = M_2/T$. It turns out that the observed baryon asymmetry results for the lowest possible value of $M_2 = 3.5(5) \times 10^9$ GeV with $\text{Re}[\theta_R] = 0.2(0)$ and $\text{Im}[\theta_R] = -0.45(-0.4)$. The corresponding

value of M_3 is found to be $7.5(15) \times 10^{10}$ GeV. At this temperature range $\sim M_2$ value, muon and tau Yukawa interactions come to equilibrium and hence lepton asymmetries along all the flavor directions become relevant (see Fig. 3). We also infer from Fig. 3 that the abundance of Y_{N_2} with such a large M_2 is falling sharply as temperature decreases and hence is expected to be vanishingly small in the EW broken phase where N_1 production is mainly taking place from the decay of the SM gauge bosons. This along with the fact that production of N_1 from N_2 decay is also suppressed (via $V^\dagger V$ as stated earlier) eventually indicates that N_2 contributes effectively nothing to N_1 production as

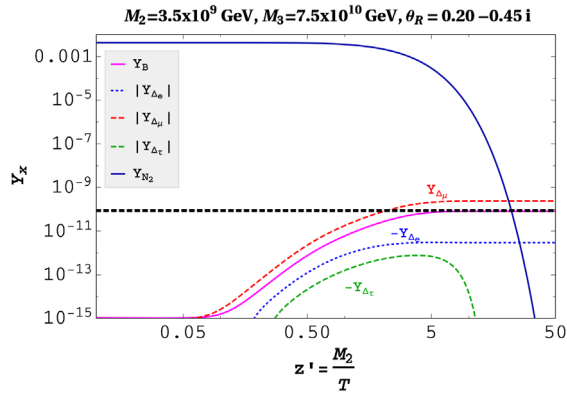


FIG. 3. Evolution of individual flavor asymmetries as well as baryon asymmetry with respect to $z' = M_2/T$. Black dashed lines: range of observed baryon asymmetry of the Universe.

seen from the right hand side of Eq. (7) (first term). The same conclusion holds for N_3 as well.

In summary, we have shown that the conventional type-I seesaw scenario itself has the potential to offer a FIMP type of dark matter in the form of lightest RHN, the relic density of which is mainly governed by the decay of the SM gauge bosons in the electroweak symmetry broken phase. The hypothesis that in the limit of zero lightest active neutrino mass the dark matter is absolutely stable, implies that production and stability of the dark matter both are effectively controlled by the tiny active neutrino mass. The proposal predicts an upper bound on this lightest neutrino mass as $m_1 \lesssim \mathcal{O}(10^{-12})$ eV which makes it falsifiable if ongoing (or future) experiments such as the KATRIN [50] and PROJECT-8 Collaborations [51] succeed to probe it. In this way, the smallness of couplings involved in a generic FIMP type model, related to dark matter production, can now be connected with the lightness of active neutrino mass m_1 . While we find the DM mass ~ 1 keV–1 MeV satisfies the correct relic density as well as the stringent limits from x-ray observation, the dark matter phenomenology does not restrict the mass scales of two other heavy RHNs. Then we incorporate the flavor leptogenesis scenario to show that they can be $\sim 10^9$ – 10^{10} GeV to explain the baryon asymmetry of the Universe. So the minimal setup of type-I seesaw can simultaneously address the origin of neutrino mass, non-thermal production of dark matter, and matter-antimatter asymmetry without any additional fields. The presence of active-sterile neutrino mixing in the setup is suggestive of the rare lepton flavor violating decays. The most relevant branching ratio in this context is related to $\mu \rightarrow e\gamma$ which turns out to be a function of active-sterile neutrino mixing V as well as RHN masses M_i . Employing values of M_i (used in producing Figs. 1 or 3) and corresponding V elements, the branching ratio of $\mu \rightarrow e\gamma$ is found to be

negligibly small compared to the present experimental limit [52]. We also evaluate the effective neutrino mass parameter involved in the half-life of neutrinoless double beta decay (function of m_i , lepton mixing angles, and phases) and find it to be insignificant.

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