Squeezed Lasing

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We introduce the concept of a squeezed laser, in which a squeezed cavity mode develops a macroscopic photonic occupation due to stimulated emission. Above the lasing threshold, the emitted light retains both the spectral purity of a laser and the photon correlations characteristic of quadrature squeezing. Our proposal, implementable in optical setups, relies on a combination of the parametric driving of the cavity and the excitation by a broadband squeezed vacuum to achieve lasing behavior in a squeezed cavity mode. The squeezed laser can find applications that go beyond those of standard lasers thanks to the squeezed character, such as the direct application in Michelson interferometry beyond the standard quantum limit, or its use in atomic metrology.

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Squeezed states of light are one of the most important resources in current quantum-optical technologies. They have particularly important applications in quantum metrology, with remarkable examples such as the enhanced sensitivities reported in gravitational-wave interferometers [1]. A particularly interesting possibility is the application of squeezed states in atomic quantum metrology, where they have been proposed as a way to generate spin squeezing [2–9] and enhance the sensitivity of atomic interferometers [10–12] and magnetometers [13–15]. Efficient coherent coupling between squeezed light and long-lived atomic transitions—which would have important applications for atomic metrology or the design of quantum memories [3,6,10,16]—requires the generation of narrow band squeezed states [16]. For this purpose, an ideal source of squeezed light would have the narrow linewidth characteristic of the lasers usually employed in atom optics, which is a consequence of stimulated emission above the lasing threshold. While the standard source of squeezed light, the optical parametric oscillator (OPO), has a similar threshold due to stimulated emission, it is incompatible with the generation of squeezing, since above threshold the squeezed vacuum turns into a mixture of two coherent states [17]. In this work, we propose the implementation of a squeezed laser, introducing the mechanism of stimulated emission into a squeezed cavity mode. We show that this generates coherent squeezed states retaining both the linewidth and coherence time characteristic of a laser, and the photon correlation properties of squeezed states. This novel source of light can open new regimes of exploration of atomic physics with quantum states of light.

The mechanism that we propose requires driving the cavity that contains the gain medium by (i) a detuned parametric drive, which sets a basis of squeezed states as the natural photonic eigenstates of the system, and (ii) a resonant, broadband squeezed vacuum, which eliminates the squeezedlike noise that emerges naturally in such a basis from spontaneous photon emission from the cavity Our model consists of a single cavity mode of frequency ω_c with bosonic annihilation operator $\hat{a}$, interacting with N two-level atoms with ground and excited energy levels $\{|g\rangle_i, |e\rangle_i\}$, lowering operators $\hat{\sigma}_i \equiv |g\rangle_i\langle e|_i$, and transition frequency $\omega_{\sigma,i}$. The cavity is parametrically driven by a detuned drive of amplitude Ω_p, achieved through the down-conversion of a coherent drive of frequency $2\omega_p$ into photon pairs at frequency ω_p (slightly detuned from the cavity frequency ω_c) by means of a nonlinear $\chi^{(2)}$ crystal inside the cavity [see Fig. 1(a)]. A resonant version of this type of parametric drive is the typical mechanism to generate a squeezed vacuum just below the OPO threshold. Here, the coupling to the atoms will provide a gain mechanism, amplifying the vacuum of this squeezed mode into a coherent squeezed state by stimulated emission, and yielding laserlike coherence times. Contrary to the case of stimulated emission in the OPO phase transition, this macroscopic population buildup occurs into a squeezed mode and preserves the squeezing properties rather than degrading them. As we prove in the Supplemental Material [18], the pump intensities required to turn normal lasing action into lasing into a squeezed mode are well within reach in current experimental platforms.

In a frame rotating at a frequency ω_p, the total Hamiltonian reads, as setting $\hbar = 1$, $\hat{H}=\Delta_c\hat{a}^\dagger\hat{a}+\Omega_p(e^{-i\theta}\hat{a}^2+\mathrm{H.c.})/2+\sum_{i=1}^N\Delta_{\sigma,i}\hat{\sigma}_i^\dagger\hat{\sigma}_i+g(\hat{a}^\dagger\hat{\sigma}_i+\hat{a}\hat{\sigma}_i^\dagger)$, where $\Delta_c \equiv \omega_c - \omega_p$, $\Delta_{\sigma,i} \equiv \omega_{\sigma,i} - \omega_p$, and θ is the phase of the

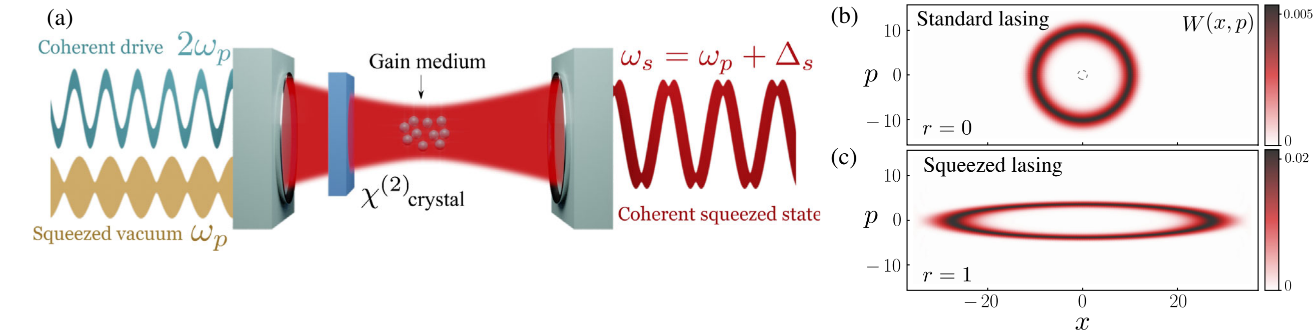

FIG. 1. (a) Sketch of the proposed setup. A single cavity mode of frequency $\omega_c = \omega_p + \Delta_c$ (with $\Delta_c \ll \omega_p$) is parametrically driven through the down-conversion of pump photons of frequency $2\omega_p$ by a nonlinear crystal $\chi^{(2)}$. The cavity includes a gain medium (e.g., an ensemble of two-level atoms) and is driven by a broadband squeezed vacuum centered at the frequency ω_p, which eliminates squeezedlike noise in the squeezed basis originating from cavity spontaneous emission. If the laser is imposed with a well-defined phase, the output emission corresponds to a coherent squeezed state of frequency $\omega_s = \omega_p + \Delta_s$. (b)–(c) Exact calculation of the Wigner function for the steady state in the lasing regime of (b) a standard laser ($r = 0$) and (c) a squeezed laser ($r = 1$), for $N = 1$, $\gamma = 0$, $C_s = 2$, $n_q = 50$, and $\theta = \pi$. For comparison, dashed lines mark the uncertainty contour of a vacuum state (b).

coherent drive. We have assumed a Jaynes-Cummings type of light-matter coupling, requiring the rotating-wave approximation (RWA) $g \ll \omega_c, \omega_\sigma$. The purely photonic part of the Hamiltonian can be diagonalized by a Bogoliouvov transformation corresponding to a unitary squeezing operator $\hat{S}(re^{-i\theta}) = \exp[r(e^{i\theta}\hat{a}^2 - e^{-i\theta}\hat{a}^{\dagger 2})/2]$, with $r \equiv \ln[(1+\alpha)/(1-\alpha)]/4$, and $\alpha \equiv \Omega_p/\Delta_c$, so that $\hat{a} \to \hat{a}_s \cosh r - \hat{a}_s^\dagger e^{-i\theta} \sinh r$, where $\hat{a}_s$ denotes the annihilation operator in the new, squeezed basis. The Hamiltonian in the squeezed basis then approximately becomes $\hat{H}_s \approx \Delta_s \hat{a}_s^\dagger \hat{a}_s + \sum_{i=1}^N \Delta_{\sigma,i} \hat{\sigma}_i^\dagger \hat{\sigma}_i + \tilde{g}(\hat{a}_s^\dagger \hat{\sigma}_i + \hat{a}_s \hat{\sigma}_i^\dagger)$ where $\Delta_s \equiv \Delta_c \sqrt{1-\alpha^2}$ and $\tilde{g} \equiv g \cosh r$. In the last step we have performed a new RWA under the requirement that the collective coupling remains small compared with the effective free frequencies, $\sqrt{N} g \sinh r \ll \Delta_s, \Delta_\sigma$. Although this requirement can in principle always be satisfied for any r by increasing both Δ_c and Ω_p so that the ratio α remains constant, it will ultimately be limited by the realistic impositions on the driving amplitude Ω_p and by the condition that Δ_c remains smaller than the free spectral range of the cavity. The resulting light-matter coupling rate $\tilde{g}$ is exponentially enhanced with respect to the bare coupling by the factor $\cosh r$: this enhancement and its implications in different setups have been proposed and discussed in several recent works [38–42]. Here, we explore another consequence of this type of coupling: the possibility of developing a macroscopic photonic phase in the squeezed cavity mode a_s through a lasing mechanism.

Lasing is described with a driven-dissipative model in which the previous Hamiltonian is supplemented by Lindblad operators that describe incoherent driving of the atoms and dissipative decay of cavity photons and atomic excitations. As discussed in previous works [38,41], a cavity coupled to a thermal reservoir behaves as a cavity coupled to a squeezed reservoir when moving to the squeezed basis. This effective squeezed noise is detrimental for lasing action in the squeezed cavity mode, and needs to be removed. One can achieve this by driving the system with a broadband squeezed vacuum with an opposite squeezing angle and a properly tuned squeezing parameter [41], which can be obtained from the output of an OPO of frequency ω_p and with a linewidth much larger than the cavity decay rate [43]. We define κ as the cavity photon loss rate through the first mirror, which couples to the squeezed photonic reservoir (i.e., the output of the driving OPO). We will also consider other sources of photon loss—e.g., intracavity losses or losses through the second cavity mirror—with a total associated decay rate $\eta\kappa$, where η is a dimensionless factor, and assume $\eta \lesssim 1$. The total decay rate in the cavity is then $\kappa(1+\eta)$. As we elaborate in Ref. [18], considering driving by a squeezed vacuum with squeezing parameter $r_e e^{i\theta_e}$, a lasing master equation can be obtained by setting $r_e \approx r + \frac{1}{2}\eta \sinh(2r)$, (the last equality being valid for $\eta \ll \sinh 2r$) and $\theta_e = \pi - \theta$. This yields the following master equation for the atoms-cavity system:

$$\partial_t \hat{\rho} = -i[\hat{H}_s, \hat{\rho}] + \frac{\kappa}{2}(1 + \eta + N_s) D_{\hat{a}_s}[\hat{\rho}] + \frac{\kappa}{2} N_s D_{\hat{a}_s^\dagger}[\hat{\rho}] + \sum_{i=1}^N \left[\frac{P}{2} D_{\hat{\sigma}_i^\dagger}[\hat{\rho}] + \frac{\gamma}{2} D_{\hat{\sigma}_i}[\hat{\rho}]\right], \quad (1)$$

where P and γ are, respectively, the incoherent pumping and spontaneous emission rates of the atoms, and N_s is an effective thermal photon number given by $N_s = \eta \sinh^2 r + \sqrt{1 + \eta^2 \sinh^2(2r)}/2 - 1/2$. Thanks to the broadband squeezed drive, Eq. (1) contains only standard incoherent single-photon decay and pumping terms, instead of squeezedlike noise that would be detrimental for lasing action [18].

Equation (1) describes a extensively studied model of a laser in contact with a thermal bath [43], and therefore describes lasing in the squeezed cavity mode. Intracavity losses $\eta > 0$ lower the degree of squeezing with respect to the squeezed drive, i.e., $r < r_e$. In exchange, the output of the squeezed laser will have the spectral profile and correlation times characteristic of a laser, which can, e.g., be used for the resonant excitation of long-lived atomic transitions with quantum light. The thermal photon number can be kept at $N_s \sim 1$ or lower provided $\eta \lesssim 1$, as shown in Ref. [18]. In that case, the detrimental effect of the thermal fluctuations for the generation of a macroscopic, coherent population of photons is negligible. Therefore, for simplicity, we will set $N_s = 0$ for the rest of the work. In general, Eq. (1) or very similar ones have been used to study lasing in a wide variety of situations, e.g., including the effect of inhomogeneously broadened emitters [44,45]. Hence, all the results and knowledge from laser theory apply directly to the squeezed laser, with the difference that the macroscopic, coherent state develops in a squeezed cavity mode.

We will now discuss a generic lasing scenario, and fix $\omega_{\sigma,i} = \omega_\sigma$. A mean-field description predicts a photon population $n_s \equiv \langle \hat{a}_s^\dagger \hat{a}_s \rangle = n_0(p_s - 1)H(p_s - 1)$, with $H(x)$ the Heaviside step function, featuring a dissipative phase transition at $p_s = 1$, where $\tilde{\gamma} \equiv (P + \gamma)/2$, $C_s \equiv 2g_s^2/(\tilde{\gamma}\kappa)$, $p_s \equiv NC_s(P - \gamma)/(P + \gamma)$ and $n_0 = \tilde{\gamma}^2/2g_s^2$. The photonic population in the bare cavity mode $n \equiv \langle \hat{a}^\dagger \hat{a} \rangle$ can then be expressed in terms of n_s by undoing the squeezing transformation, $n = n_s \cosh(2r) + \sinh^2 r$. Here, we explore the quantum properties of the squeezed lasing phase through exact, numerical calculations in the limit $N = 1$, $\gamma = 0$, so that $n_s = n_q(1 - 1/p_s)H(p_s - 1)$, with $n_q = P/2\kappa$ being the photon saturation number at high pump. This limit is taken for numerical convenience and without loss of generality, since it keeps all the general characteristics of lasing. Henceforth, quadrature operators are defined as $\hat{X}_\phi \equiv \hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi}$ (with $\hat{x} = \hat{X}_0$, $\hat{p} = \hat{X}_{\pi/2}$), so that in vacuum, $\langle (\Delta \hat{X}_\phi)^2 \rangle = 1$.

Stationary state.—Well within the lasing phase, a lasing state is well approximated by a mixture of coherent states $|\sqrt{n_s} e^{i\varphi}\rangle$ with a fixed amplitude $\sqrt{n_s}$, over all possible phases, $\hat{\rho}_s = (1/2\pi) \int_0^{2\pi} d\varphi |\sqrt{n_s} e^{i\varphi}\rangle\langle\sqrt{n_s} e^{i\varphi}|$. This mixed stationary state reflects the effect of phase diffusion in the limit of infinite time, and it is a common property of all quantum models of a laser. In phase space, this state has the annular shape shown in Fig. 1(b). However, these models also predict that the timescale of phase diffusion is extremely slow and related to the inverse of the spectral linewidth, yielding the characteristically long coherence times of the laser. Since the lasing state is developed in the squeezed basis, in order to recover the stationary photonic state of cavity mode in the original basis $\hat{\rho}_a$, we must apply a squeezing transformation, giving

$$\hat{\rho}_a = \frac{1}{2\pi} \int_0^{2\pi} d\varphi S(re^{i\theta}) |\sqrt{n_s} e^{i\varphi}\rangle\langle\sqrt{n_s} e^{i\varphi}| S(re^{i\theta})^\dagger. \quad (2)$$

This mixture of displaced-squeezed states gives the squeezed annular shape in phase space displayed in Fig. 1(c). To the best of our knowledge, a similar squeezed-lasing state has only been considered in a previous proposal in the microwave domain [46], which required a fast modulation of qubit energies in superconducting circuits. Our proposal allows one to implement this novel type of lasing in the optical domain, or any other system where a broadband squeezed vacuum and efficient down-conversion mechanisms are available.

Symmetry broken solutions.—Just as in a conventional laser, despite having a phase-diffused stationary state, a state with a well-defined phase will remain stable for an extremely long time. This phase can be spontaneously selected under an homodyne measurement [47] or, as in any other physical system with a symmetry, can be imposed by a small external perturbation that breaks the symmetry [48]. To elaborate further on this point, Fig. 2 summarizes the effect of seeding the laser with a small coherent drive with a well-defined phase, described by the Hamiltonian term $\hat{H}_{\text{drive}} = \Omega(\hat{a}_s e^{-i\phi_d} + \hat{a}_s^\dagger e^{i\phi_d})$. The stationary population of photons that would be established in the squeezed basis solely by the action of this drive is $n_d = 4\Omega^2/\kappa^2$. As shown

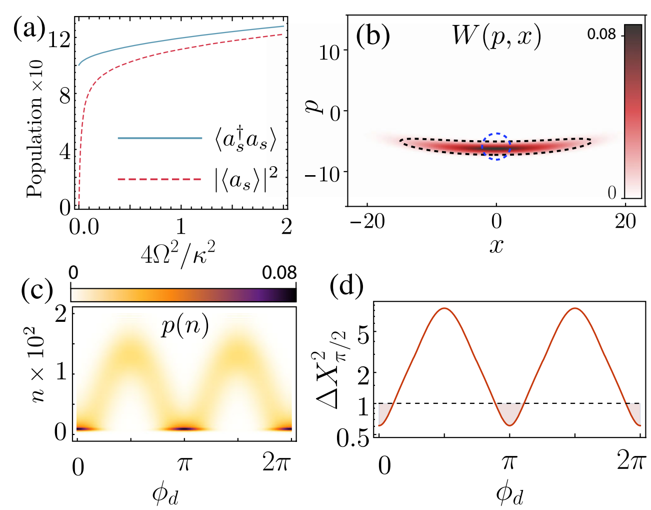

FIG. 2. Enforcing a well-defined phase. (a) Stationary photon population in the squeezed basis, compared with the square of the mean field, versus the amplitude of the coherent drive. Parameters, $n_q = 200$; $C = 2$. (b) Wigner function of the steady state of the squeezed laser with a small seeding field. Dashed black lines are contour lines for 0.1 $\max[W(x,p)]$, blue dashed lines are the equivalent for a coherent field with the same coherent amplitude. Same parameters as in (a), and $r = 0.7$. Seeding laser parameters, $n_d = 4\Omega^2/\kappa^2 = 2$; $\phi_d = 0$. (c) Photon number distribution and (d) quadrature fluctuations versus the phase of the driving field, with the same parameters as in (b). Dashed line marks vacuum-level fluctuations. Squeezing is achieved with $\phi_d = 0, \pi$.

in Fig. 2(a), a small drive giving $n_d \approx 1$ is enough to force a phase in a lasing state of ~ 100 photons in the squeezed basis. This is evidenced by $|\langle a_s \rangle|^2$ going from zero to $\sim \langle \hat{a}_s^\dagger \hat{a}_s \rangle$, indicating that the annular distribution in phase space is symmetry broken toward a coherent state. Consequently, this coherent state turns into a squeezed coherent state in the standard cavity basis, as shown in Fig. 2(b) (note that, in this finite-size exact calculation, some degree of phase diffusion is still present). Symmetry-broken states are of the form $\hat{\rho}_\varphi = |\xi, \alpha\rangle\langle\xi, \alpha|$, with $|\xi, \alpha\rangle \equiv \hat{S}(\xi)\hat{D}(\alpha)|0\rangle$ and $\xi = re^{i\theta}$, $\alpha = \sqrt{n_s}e^{i\varphi}$. They display an oscillatory dependence of photon number on the relative phase between the coherent component and squeezing angle, as depicted in Figs. 2(c)–2(d), showing how the degree of squeezing, photon statistics, and mean photon number can be controlled through the seed phase. Note that the squeezed-coherent states created are in fact coherent-squeezed states, but with a coherent component γ different from α and dependent on r and θ, $\hat{S}(\xi)\hat{D}(\alpha) = \hat{D}(\gamma)\hat{S}(\xi)$, where $\gamma = \alpha \cosh r - e^{i\theta}\alpha^* \sinh r$. In Ref. [18], we describe how these states can be directly applied in Michelson interferometers to obtain a metrological quantum advantage, as an alternative to a mixture of coherent states and squeezed vacua.

Spectrum of emission.—We now consider the spectrum of the light emitted through the second cavity mirror, $S(\omega) = \lim_{t\to\infty}(1/\pi n)\mathrm{Re}\int_0^\infty d\tau e^{i\omega\tau}\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle$ [49]. As we explain in detail in Ref. [18], this spectrum takes exactly the same form predicted for a standard laser described by Eq. (1), i.e., the change of basis from the squeezed to the standard basis does not change the form of the spectrum, and therefore $S(\omega)$ corresponds to the spectrum of a laser with photon number n_s and cooperativity C_s, sharply peaked around the frequency of the squeezed mode $\omega_s \equiv \omega_p + \Delta_s$. We can therefore conclude that the squeezed laser will inherit the spectral properties of a standard laser, described extensively in the literature [43,50,51], including the extreme line narrowing developed in the lasing phase, with a linewidth that in the thermodynamic limit will be given by $\Gamma = \kappa C_s / 4n_s$ [43]. The timescale of phase diffusion is given by Γ^{-1}. Well within the lasing regime, this linewidth is inversely proportional to the saturation photon number n_q, and therefore will vanish in the thermodynamic limit $n_q \to \infty$. This trend is confirmed by numerical calculations of the spectrum of the squeezed laser (see Fig. 3), where we show the values of Γ extracted from the spectrum versus the effective cooperativity of the squeezed mode C_s (with the lasing transition at $C_s = 1$) and increasing values of the saturation photon number n_q.

Quadrature squeezing.—Quadrature fluctuations are usually assessed by difference photocurrents in homodyne measurements, proportional to the quadrature $\hat{X}_\phi^{\mathrm{out}}$ of the output field selected by the local oscillator. The power spectrum of this signal contains contributions from quantum fluctuations that are encoded in the spectrum of squeezing [52], $S_\phi(\omega) = 1 + :S_\phi(\omega):$, where using standard input-output theory one can write : $S_\phi(\omega) \coloneqq 2\kappa\int_0^\infty[\langle \hat{a}^\dagger(t+\tau), \hat{a}(t)\rangle + e^{-2i\phi}\langle \hat{a}(t+\tau), \hat{a}(t)\rangle +$ c.c.$]\cos(\omega\tau)d\tau$. Negative (positive) values of $:S_\phi(\omega):$ indicate fluctuations below (above) the shot noise limit. The dependence on t is kept in order to study transient, long-lived symmetry-broken states. Figure 4 summarizes the time evolution of the squeezing properties of an initial, phase-locked state $\hat{\rho}_\varphi$, with $\varphi = (\theta + \pi)/2$. As shown in Figs. 4(a) –4(b), the profile of the spectrum of squeezing of the antisqueezed quadrature $:S(\omega)_{[(\theta+\pi)/2]}:$ reproduces the profile of the emission spectrum, i.e., a narrow linewidth Γ set by the vanishing Liouvillian gap. On the other hand, for the squeezed quadrature, $:S(\omega)_{(\theta/2)}:$ shows a similarly narrow feature that evolves from negative to positive values in a timescale $\sim\kappa^{-1}$, on top of a broader, negative profile with a linewidth $\sim\kappa$. When integrated in frequencies, this broader negative profile is the main contribution to the intracavity quadrature fluctuation, which fulfills $\langle :\Delta X_\phi^2: \rangle = 1/(2\pi\kappa)\int_{-\infty}^\infty :S(\omega)_\phi: d\omega$. Thus, the spectrum of the squeezed quadrature has a bandwidth $\sim\kappa$, while the spectrum of the antisqueezed quadrature replicates the laserlike linewidth of the emission spectrum, narrowing as one goes deep into the lasing phase. The long-lived character of squeezed fluctuations is evidenced by the time evolution of $\langle :\Delta\hat{X}_{\theta/2}^2: \rangle$ [see Fig. 4(e)], which shows that the quadrature remains squeezed for a time much longer than the cavity lifetime κ^{-1}.

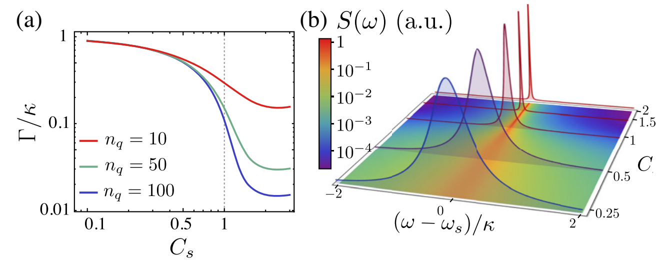

FIG. 3. Line narrowing of the emission spectrum in a squeezed laser. (a) Emission linewidth (related to phase diffusion rate) versus cooperativity $C_s = p_s$ for different values of saturation photon number n_q. The lasing phase transition occurs at $C_s = 1$. This result depends on r only through C_s. (b) Spectrum versus cooperativity C_s for saturation photon number $n_q = 100$. Frequency defined with respect to the squeezed mode frequency ω_s.

Second-order correlation function.—The lasing system that we present here extends the definition of a laser in terms of Glauber's criterion of coherence. Contrary to a standard laser, photons emitted by the squeezed laser exhibit positive correlations between each other, characterized by a value of the zero-delay, stationary second order correlation function $g^{(2)}(0)$ greater than one. Considering the steady state given

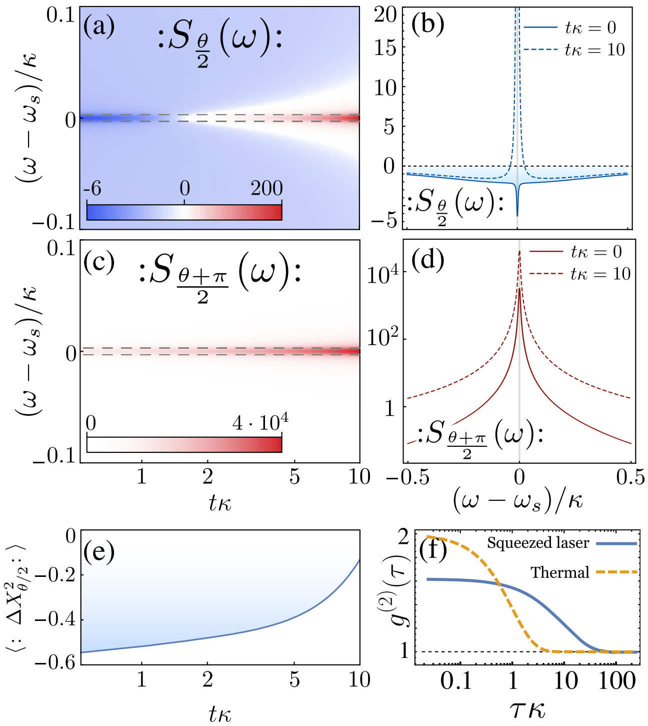

FIG. 4. Photon correlations of a squeezed laser. (a),(c) Squeezing spectrum versus time (negative values indicate fluctuations below the shot noise). (b),(c) Cuts of (a),(b) at initial and final time. We observe survival of squeezing for times much longer than cavity lifetime. Parameters, $n_q = 250$; $C_s = 2$; $\varphi = (\theta+\pi)/2$; $r = 0.45$. (e) Evolution of normalized quadrature variance. (f) $g^{(2)}(\tau)$ of the squeezed laser displaying positive correlations surviving for extremely long correlation times $\propto 1/\Gamma$, with $\Gamma \sim 0.1\kappa$ in this particular case. A thermal state, shown for comparison, displays a much shorter positive correlation time of the order $1/\kappa$. Parameters, $C_s = 1.5$; $r = 1$; $n_q = 50$.

by Eq. (2), in the deep-lasing regime $n_s \gg 1$, $g^{(2)}(0)$ reads as $g^{(2)}(0) \approx [3 - \mathrm{sech}(2r)^2]/2$, which saturates with increasing r to a value of $3/2$. For the symmetry-broken states $\hat{\rho}_\varphi$, the $g^{(2)}(0)$ will approach that of a displaced-squeezed state [53], which can be larger than $3/2$, or even antibunched. We emphasize that, contrary to the case of, e.g., a thermal state, these correlations exhibit an extraordinarily long coherence time $\tau_c = 1/\Gamma$, that can be made orders of magnitude longer than the natural lifetime of the cavity photons $1/\kappa$, as shown in Fig. 4(f). In conclusion, the squeezed laser inherits both the long coherence times typical of any laser and the strong photon-photon correlations and quadrature fluctuations of squeezed states, which can find important applications in the field of atomic metrology.

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