# Enhancement of Spin-Charge Conversion in Dilute Magnetic Alloys by Kondo Screening

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We derive a kinetic theory capable of dealing both with large spin-orbit coupling and Kondo screening in dilute magnetic alloys. We obtain the collision integral nonperturbatively and uncover a contribution proportional to the momentum derivative of the impurity scattering $S$ matrix. The latter yields an important correction to the spin diffusion and spin-charge conversion coefficients, and fully captures the so-called side-jump process without resorting to the Born approximation (which fails for resonant scattering), or to otherwise heuristic derivations. We apply our kinetic theory to a quantum impurity model with strong spin-orbit, which captures the most important features of Kondo-screened Cerium impurities in alloys such as $Ce_xLa_{1-x}Cu_6$. We find (1) a large zero-temperature spin-Hall conductivity that depends solely on the Fermi wave number and (2) a transverse spin diffusion mechanism that modifies the standard Fick's diffusion law. Our predictions can be readily verified by standard spin-transport measurements in metal alloys with Kondo impurities.

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*Introduction.*—Topological materials with strong electronic correlation and large spin-orbit coupling (SOC) [1–7] are promising platforms for the realization of exotic phases of matter, with potential applications in spintronics [8–13]. One recent example that is being intensively researched are Weyl-Kondo semimetals in heavy fermion compounds [14–16]. Below the (Kondo) coherence temperature, the local magnetic moments in these materials form a topologically nontrivial band with Weyl points pinned at the Fermi level. The existence of the latter is believed to lead to the giant Hall effect in $Ce_3Bi_4Pd_3$ [17]. In the opposite limit of a periodic arrangement that yields a coherent band structure, a giant spin-Hall effect has been observed in disordered alloys of FePt-Au [18]. A theoretical explanation has been put forward for the latter in terms of an orbital-dependent Kondo effect [19,20].

Driven by these exciting developments, in this Letter we report a kinetic theory capable of describing the coupled spin and charge transport in dilute magnetic alloys with Kondo screened impurities as well as other types of impurities. Note that, unlike ordinary potential scattering, Kondo screening is a strong correlation phenomenon that arises from the antiferromagnetic exchange interaction between a local magnetic moment and the conduction electrons. The screening of an impurity magnetic moment results in a strong (often resonant) scattering of the conduction electrons at the Fermi energy when the temperature is lower than the Kondo temperature. Under such conditions and in the presence of large SOC, we have found that the spin-Hall effect is substantially enhanced and the spin diffusion coefficients become spin anisotropic. The abundance of dilute magnetic alloys allows our predictions to be readily tested by existing experimental techniques (e.g., [21]). Below, we develop a model that can be applied to alloys containing rare earth impurities, such as Cerium in $Ce_x$ $La_{1-x}Cu_6$ for which a robust Kondo effect has been observed in electrical resistivity measurements [22], but no spin-transport measurements have been carried out so far to the best of our knowledge.

The most direct manifestations of SOC in transport experiments are the anomalous Hall effect [23] and the spin-Hall effect [24]. Depending on the origin of SOC, one usually distinguishes between intrinsic and extrinsic contributions to the transverse conductivity. The former is related to SOC generated by the periodic crystal potential of the lattice and encoded in the electronic band structure, while the latter originates from the SOC of randomly distributed impurities. In turn, the extrinsic contribution is further divided into two distinct mechanisms: skew scattering and side jump. Skew scattering arises due to the angular asymmetry of the scattering cross section and therefore it can be readily incorporated in the collision integral of the kinetic (Boltzmann-like) equation. Among all mechanisms, the side jump [25–32] appears to be the least understood.

 

Physically it can be attributed to a spin-dependent transverse shift (jump) of a wave packet scattered off the impurity. Since this effect does not show up in the scattering cross section, its inclusion in the kinetic theory is by no means straightforward. It is typically done heuristically by defining a coordinate "jump" $\delta\boldsymbol{r}$ of a wave packet, introducing the related anomalous velocity and a modified carrier energy dispersion, and incorporating these ingredients into the kinetic equation using reasonable, but still nonrigorous arguments [28,33,34]. On the other hand, a formal justification of the above procedure and/or derivations of the side-jump contribution from the rigorous quantum kinetic theory practically always rely on the lowest order Born approximation [32]. Such an approach fails for magnetic impurities of heavy elements in the Kondo regime when neither scattering nor SOC can be considered weak. This motivates us to construct a kinetic theory to properly describe all extrinsic mechanisms (including side jump) self-consistently without resorting to any finite order Born approximation. We achieved this by computing the lesser impurity self-energy to first order in spatial derivative (but all orders in disorder potential strength). This gives rise to an additional contribution to the collision integral. When the kinetic theory is solved in the presence of the additional contribution, the side-jump correction to spin-Hall conductivity and diffusion constants follows automatically without any heuristic arguments.

Our theory predicts that the standard Fick's law of spin diffusion is modified by SOC when we go beyond the Born approximation: in addition to the standard Laplacian operator $\nabla^2\mathbf{s}$, the diffusion operator acquires a new term $\sim\nabla(\nabla\cdot\mathbf{s})$ because SOC breaks the spin-rotation symmetry. This correction occurs at second order in SOC magnetic field.

*Kinetic theory.*—We start from the Kadanoff-Baym equation for the nonequilibrium Green functions. Keeping only leading order terms in the impurity density $n_{\rm im}$ we sum up exactly the entire Born series and perform gradient expansion, which allows us to obtain the following kinetic equations [35] for the spin-density matrix $\hat{n}_{\boldsymbol{p}} \equiv \hat{n}_{\boldsymbol{p}}(\boldsymbol{r},t)$:

$$\partial_t\hat{n}_{\boldsymbol{p}} + \boldsymbol{v}_{\boldsymbol{p}}\cdot\nabla_{\boldsymbol{r}}\hat{n}_{\boldsymbol{p}} + i[\Sigma^H_{\boldsymbol{p}},\hat{n}_{\boldsymbol{p}}] = \hat{I}_0[\hat{n}_{\boldsymbol{p}}] + \hat{I}_1[\hat{n}_{\boldsymbol{p}}]. \quad (1)$$

Here $\epsilon_p = p^2/(2m^*)$ is the single-particle energy dispersion, $\boldsymbol{v}_{\boldsymbol{p}} = \nabla_{\boldsymbol{p}}\epsilon_p$, and $\Sigma^H_{\boldsymbol{p}} = n_{\rm im}(T^R_{\boldsymbol{pp}} + T^A_{\boldsymbol{pp}})/2$ is the mean field generated by impurities, where $T^{R(A)}_{\boldsymbol{pk}}$ is the exact single-impurity retarded (advanced) scattering $T$ matrix. The $T$ matrix also determines the collision integrals in the right-hand side of Eq. (1), which describes, amongst other effects, the momentum and spin relaxation caused by impurity scattering:

$$\begin{aligned}\hat{I}_0[\hat{n}_{\boldsymbol{p}}]_{\alpha,\beta} &= 2\pi n_{\rm im}\sum_{\boldsymbol{k}}\delta(\epsilon_{\boldsymbol{p}}-\epsilon_{\boldsymbol{k}}) \\ &\quad\times\left(T^R_{\boldsymbol{pk}}\hat{n}_{\boldsymbol{k}}T^A_{\boldsymbol{kp}} - \frac{1}{2}\{T^R_{\boldsymbol{pk}}T^A_{\boldsymbol{kp}},\hat{n}_{\boldsymbol{p}}\}\right)_{\alpha\beta} \\ &\quad\to -\frac{n_{\rm im}}{2\pi}\sum_{\boldsymbol{k}}\Lambda_{\alpha\beta,\gamma\delta}(\boldsymbol{p},\boldsymbol{k})\delta\hat{n}_{\boldsymbol{k},\gamma\delta}, \end{aligned} \quad (2)$$

$$\begin{aligned}\hat{I}_1[n_{\boldsymbol{p}}]_{\alpha,\beta} &= \pi n_{\rm im}\sum_{\boldsymbol{k}}\delta(\epsilon_{\boldsymbol{p}}-\epsilon_{\boldsymbol{k}}) \\ &\quad\times i(T^R_{\boldsymbol{pk}}(\nabla_{\boldsymbol{r}}\hat{n}_{\boldsymbol{k}})\cdot(\boldsymbol{D}_{\boldsymbol{pk}}T^A_{\boldsymbol{kp}}) - \text{H.c.})_{\alpha\beta} \\ &\quad\to \pi n_{\rm im}\sum_{\boldsymbol{k}}\boldsymbol{V}_{\alpha\beta,\gamma\delta}(\boldsymbol{p},\boldsymbol{k})\cdot\nabla_{\boldsymbol{r}}\delta\hat{n}_{\boldsymbol{k},\gamma\delta}, \end{aligned} \quad (3)$$

where $\boldsymbol{D}_{\boldsymbol{pk}} = \nabla_{\boldsymbol{p}} + \nabla_{\boldsymbol{k}}$ is a momentum shift generator.

Eqs. (2) and (3) are the main results of this work and provide the basis for our combined treatment of strong scattering resulting from Kondo screening and large SOC. Equation (2) is the matrix generalization [44,45] of the golden-rule collision integral derived by Luttinger and Kohn [46], which has a Lindbladian structure often encountered in open quantum systems [47]. As we explain in what follows, the leading gradient correction to the collision integral, $\hat{I}_1[n_{\boldsymbol{p}}]$ in Eq. (3), accounts for the side-jump mechanism. Indeed, the role of $\hat{I}_1$ is twofold. First, because $\hat{I}_1 \sim \nabla_{\boldsymbol{r}}\hat{n}_{\boldsymbol{k}}$, it renormalizes the velocity entering the drift term of Eq. (1), thus generating an anomalous contribution to the current as $\boldsymbol{D}_{\boldsymbol{pk}}T^A_{\boldsymbol{pk}} = i\langle\boldsymbol{p}|[T^A,\boldsymbol{r}]|\boldsymbol{k}\rangle$ which has its origin in the impurity potential. Second, in the presence of an external field that can be introduced by trading the density for the electrochemical potential (i.e., $\boldsymbol{\nabla}\rho = \sum_{\boldsymbol{k}}\mathrm{Tr}\nabla_{\boldsymbol{r}}\hat{n}_{\boldsymbol{k}} \to N_F\nabla_{\boldsymbol{r}}\mu = eN_F\boldsymbol{E}$, where $\boldsymbol{E}$ is the electric field and $N_F$ is the density of states at the Fermi energy), it generates a coupling to the electric field, proportional to $n_{\rm im}$. The latter leads to the very special scaling with the impurity concentration of the side-jump contribution to the transport coefficients. In particular, the corresponding contribution to the spin-Hall conductivity is independent on $n_{\rm im}$—the well known signature of the side-jump mechanism [23,24]. When the $T$ matrix is replaced with the *bare* impurity potential, $\pi n_{\rm im}\boldsymbol{V}$ in Eq. (3) becomes the anomalous velocity derived in Ref. [25] within the Born approximation.

In the most practically important linear regime, the deviation of $\hat{n}_{\boldsymbol{k}}$ from the Fermi function $n_F(\epsilon_{\boldsymbol{k}})$ is bound to the Fermi surface (FS), $\hat{n}_{\boldsymbol{k}} - n_F = \delta(\epsilon_{\boldsymbol{k}}-\epsilon_F)\delta\hat{n}_{\boldsymbol{k}}$, where $\epsilon_F$ is the Fermi energy. In this regime the collision integrals $\hat{I}_0$ and $\hat{I}_1$ simplify as shown by arrows in Eqs. (2) and (3), respectively. The fourth rank tensors $\check{\Lambda}(\boldsymbol{p},\boldsymbol{k})$ and $\check{\boldsymbol{V}}(\boldsymbol{p},\boldsymbol{k})$ depend only on directions of momenta on the FS and act as superoperators on the FS density matrix $\delta\hat{n}_{\boldsymbol{k}}$. They are conveniently expressed in terms of the scattering $S$ matrix $S_{\alpha\beta}(\boldsymbol{p},\boldsymbol{k})$ and the on shell $T$ matrix $t_{\alpha\beta}(\boldsymbol{p},\boldsymbol{k}) = [1/(2\pi i)][\delta_{\boldsymbol{pk}}\delta_{\alpha\beta} - S_{\alpha\beta}(\boldsymbol{p},\boldsymbol{k})] \equiv \delta(\epsilon_{\boldsymbol{p}}-\epsilon_{\boldsymbol{k}})T_{\alpha\beta}(\boldsymbol{p},\boldsymbol{k})$:

$$\Lambda_{\alpha\beta,\gamma\delta}(\boldsymbol{p},\boldsymbol{k}) = \delta_{\boldsymbol{pk}}\delta_{\alpha\gamma}\delta_{\beta\delta} - S_{\alpha\gamma}(\boldsymbol{p},\boldsymbol{k})S^*_{\beta\delta}(\boldsymbol{p},\boldsymbol{k}), \quad (4)$$

$$\boldsymbol{V}_{\alpha\beta,\gamma\delta}(\boldsymbol{p},\boldsymbol{k}) = t_{\alpha\gamma}(\boldsymbol{p},\boldsymbol{k})i(\vec{D}_{\boldsymbol{pk}} - \overleftarrow{D}_{\boldsymbol{pk}})t^*_{\beta\delta}(\boldsymbol{p},\boldsymbol{k}). \quad (5)$$

$\Lambda_{\alpha\beta,\gamma\delta}$ has a typical form of a relaxation superoperator commonly used to describe spin decoherence in atoms and molecules [48,49]. The vector-valued "velocity superoperator" $\boldsymbol{V}_{\alpha\beta,\gamma\delta}$ is related to the momentum gradient of the scattering phase and thus to the coordinate shift of the scattered wave packet. In fact, Eqs. (5) and (3) provide a precise nonperturbative definition of the side-jump process and clarify the way it enters a consistent quantum kinetic theory.

*Diffusive limit.*—In a typical transport situation the momentum relaxation length (mean free path) is much shorter than characteristic scales of space inhomogeneities. In this so-called diffusive regime the distribution function $\delta\hat{n}_{\boldsymbol{k}}$ becomes almost isotropic and is fully determined by its 0th $\sum_{\boldsymbol{k}}\delta\hat{n}_{\boldsymbol{k}}$ and first $\sum_{\boldsymbol{k}}\boldsymbol{k}\delta\hat{n}_{\boldsymbol{k}}$ moments:

$$N_F\delta\hat{n}_{\boldsymbol{k}} \approx \rho\mathbb{1} + s_a\sigma_a + 3k_i(g_{i0}\mathbb{1} + g_{ia}\sigma_a)v_F^{-1}, \quad (6)$$

where $\sigma_a$ are Pauli matrices, $\mathbb{1}$ is a $2\times 2$ unit matrix, $N_F$ ($v_F$) is the density of states (Fermi velocity) at the FS, $\rho$ and $\boldsymbol{s}$ are the charge and spin densities, and $g_{i0}$ and $g_{ia}$ are charge and spin parts of the first moment. By substituting Eq. (6) into the kinetic equation and taking its 0th and first moments we arrive at a system of equations coupled by the moments of superoperators $\check{\Lambda}$ and $\check{\boldsymbol{V}}$. Then, elimination of $g_{i0}$ and $g_{ia}$ yields a closed set of equations of motion for $\rho$ and $\boldsymbol{s}$—the charge-spin diffusion equations which we now derive explicitly.

To be specific, we assume *isotropic* disorder potential which leads to a $T$ matrix that is invariant under time-reversal, parity and the full spin-orbit rotations [50]. With these assumptions, we diagonalized the kinetic equation by taking suitable linear combinations of the *ansatz* [rhs of Eq. (6)], and a solution can be obtained without assuming the collision integral is small (see the Supplemental Material Ref. [35] for full details). The 0th moment of the kinetic equation yields the charge and spin continuity equations

$$\partial_t\rho + \partial_j\mathbb{J}_j = 0, \qquad \partial_t s_b + \partial_j\mathbb{J}_{jb} = -s_b/\tau_s, \quad (7)$$

where the charge $\mathbb{J}_j$ and spin $\mathbb{J}_{jb}$ currents are linear combinations of the charge and spin first moments of $\delta\hat{n}_{\boldsymbol{k}}$ [35]. The spin relaxation time $\tau_s$ in Eq. (7) is determined by the angular average of the relaxation superoperator $\check{\Lambda}$, $\tau_s^{-1} \sim n_{\rm im}\mathrm{tr}\langle\sigma_a\check{\Lambda}\sigma_a\rangle$.

By taking the first moment of the kinetic equation, and solving it for the first moments of $\delta\hat{n}_{\boldsymbol{k}}$, we relate the currents to charge and spin-density gradients [35]:

$$\mathbb{J}_j = -D_c\partial_j\rho - D\theta_{\rm SH}\epsilon_{jka}\partial_k s_a \quad (8)$$

$$\mathbb{J}_{jb} = -\sum_{m=0}^{2} D_m P^m_{jb} - D\theta_{\rm SH}\epsilon_{jkb}\partial_k\rho, \quad (9)$$

where $P^m_{jb}$ are irreducible tensors of spin gradients:

$$P^{m=0}_{ja} = \frac{1}{3}\delta_{aj}\partial_i s_i, \qquad P^{m=1}_{ja} = \frac{1}{2}(\partial_j s_a - \partial_a s_j),$$
$$P^{m=2}_{ja} = \frac{1}{2}(\partial_j s_a + \partial_a s_j) - \frac{1}{3}\delta_{aj}\partial_i s_i \quad (10)$$

The diffusion currents are parameterized by the spin-Hall angle $\theta_{\rm SH}$, the charge diffusion constant $D_c$ and three spin diffusion constants $D_m$, which are related to different angular averages of the superoperators $\check{\Lambda}$ and $\check{\boldsymbol{V}}$ [35],

$$\theta_{\rm SH} = \frac{(1-\Omega_c-\Omega_1)\theta_{\rm sk} - \Omega_{\rm cs} - \Omega_{\rm sc}\gamma_1}{\gamma_1 + 2\theta_{\rm sk}^2} \quad (11)$$

$$D_c = D\frac{\gamma_1(1-2\Omega_c) + 4\theta_{\rm sk}\Omega_{\rm cs}}{\gamma_1 + 2\theta_{\rm sk}^2} \quad (12)$$

$$D_1 = D\frac{(1-2\Omega_1) + 4\theta_{\rm sk}\Omega_{\rm sc}}{\gamma_1 + 2\theta_{\rm sk}^2} \quad (13)$$

$$D_m = D(1-2\Omega_m)/\gamma_m, \qquad m = 0, 2 \quad (14)$$

Here the coefficients $\Omega_c$, $\Omega_m$, $\Omega_{\rm cs}$ and $\Omega_{\rm sc}$ are generated by the velocity superoperator $\check{\boldsymbol{V}}$, e.g., $\Omega_{\rm sc} \sim n_{\rm im}\mathrm{tr}\langle\boldsymbol{\sigma}\cdot(\boldsymbol{k}\times\check{\boldsymbol{V}})\mathbb{1}\rangle$. Physically, $\Omega_c$ and $\Omega_m$ renormalize the effective charge and spin velocities, while $\Omega_{\rm cs}$ and $\Omega_{\rm sc}$ account for the side-jump mechanism of the charge-to-spin conversion. Finally, $D = \frac{1}{3}v_F^2\tau_{\rm tr}$, $\gamma_m$, and $\theta_{\rm sk}$, together with $\tau_s$ in Eq. (7) parametrize the superoperator $\check{\Lambda}$. The explicit formula for all these coefficients are provided in the Supplemental Material [35].

The above expressions provide the complete solution to kinetic theory in the diffusive limit. Equations (7), (8), and (9) describe the diffusion of spin and charge for any value of single-impurity potential strength in the dilute limit. The linear response to an external field can be read off from the diffusion equations using the Einstein relation, i.e., by introducing an electric field as described under Eq. (3), both the charge and the transverse spin-Hall conductivity can be obtained from Eqs. (8) and (9), which yields $\sigma_c = e^2D_cN_F$ and $\sigma_{\rm SH} = eD\theta_{\rm SH}N_F$.

Instead of writing the spin current in terms of the coefficients $D_m$, it is also instructive to separate explicitly the divergenceless part of $\mathbb{J}_{jb}$ and rewrite Eq. (9) as follows:

$$\mathbb{J}_{jb} = -D_s^T\partial_j s_b - (D_s^L - D_s^T)\partial_b s_j$$
$$-\kappa(\partial_b s_j - \delta_{jb}\partial_k s_k) - D\theta_{\rm SH}\epsilon_{jkb}\partial_k\rho, \quad (15)$$

where $D_s^T = (D_1 + D_2)/2$, $D_s^L = (D_0 + 2D_2)/3$, and $\kappa = (D_2 - D_0)/3$. The third term entering this equation with the coefficient $\kappa$ is the "swapping current" predicted in [44]. Since the swapping current and the spin-Hall current have zero divergence, only the first line in Eq. (15) contributes to the bulk spin diffusion equation,

$$\partial_t \boldsymbol{s} - D_s^T \nabla^2 \boldsymbol{s} - (D_s^L - D_s^T)\nabla(\nabla \cdot \boldsymbol{s}) = -\boldsymbol{s}/\tau_s. \quad (16)$$

Besides the usual Fick's term $\sim\nabla^2 \boldsymbol{s}$ [51,52], the diffusion operator above contains an additional term $\sim\nabla(\nabla \cdot \boldsymbol{s})$ that breaks the spin-rotation symmetry while preserving the full space+spin-rotation invariance respected by SOC. Physically, the new term leads to different diffusion laws for the transverse $\boldsymbol{s}^T$ (with $\nabla \cdot \boldsymbol{s}^T = 0$) and longitudinal $\boldsymbol{s}^L$ (with $\nabla \times \boldsymbol{s}^L = 0$) components of the spin density. In fact, $D_s^T$ and $D_s^L$ are the diffusion constant for $\boldsymbol{s}^T$ and $\boldsymbol{s}^L$, respectively. To the leading order in SOC, we find $D_s^L \approx D_s^T$, so a sufficiently large SOC is needed to make the effect observable as we discuss next.

*Quantum impurity model.*—We now use a simple quantum impurity model [35] to demonstrate the effect of Kondo screening on spin-Hall conductivity $\sigma_{\rm SH}$ and the anisotropic spin diffusion parameter $D_s^L/D_s^T$. This model is intended to capture some of the basic features of the Ce impurities in the Kondo-screened regime in dilute alloys such as $Ce_xLa_{1-x}Cu_6$ with $x < 0.7$ [36,53]. Since Cu has negligible SOC, we can use Eq. (1) to describe (extrinsic) spin transport in this alloy. The ground state of a single $f$ electron in the Ce atom is a doublet which is separated by $\sim$100 K from a quartet [36,53] due to the crystal environment. We model this low-lying multiplet structure using a $l = 1$ orbital that is split by an effective SOC into a doublet with $j = 1/2$ and a quartet with $j = 3/2$, as shown in Fig. 1. Thus, we find that, contrary to conventional wisdom [23], the spin-Hall conductivity arises entirely from the side-jump mechanism when the T matrix is dominated by a single non $s$-wave scattering channel.

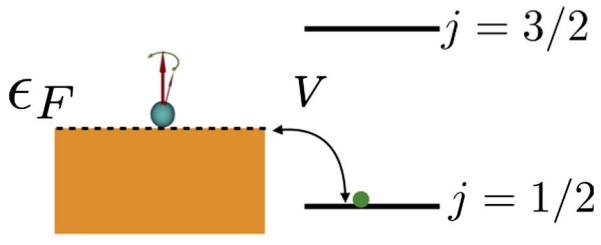


FIG. 1. Sketch of the minimal quantum impurity model to which we have applied our kinetic theory. The impurity contains a single electron in an $l = 1$ orbital that, by virtue of strong spin-orbit coupling, splits into a $j = 1/2$ doublet and a $j = 3/2$ quartet. Strong electron correlation leads to the formation of a local moment. Kondo screening of the latter by the conduction electrons induces a scattering phase shift $\eta_1 = \pi/2$ at the Fermi energy. See the Supplemental Material [35] for a detailed explanation of how this model captures some essential features of Ce impurities in alloys like $Ce_xLa_{1-x}Cu_6$ for $x < 0.7$.

In the Kondo-screened regime (i.e., $T \ll T_K \sim 1$ K), the on shell $T$ matrix at the FS of this quantum impurity model can be derived using standard many-body technique described in the Supplemental Material Ref. [35], and the result is

$$\hat{T}^R_{\boldsymbol{kp}} = -\frac{e^{i\eta_0}\sin\eta_0 + (e^{i\eta_1}\sin\eta_1 + 2e^{i\eta_2}\sin\eta_2)\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{k}}}{\pi N_F}\mathbb{1} - \left[\frac{e^{i\eta_1}\sin\eta_1 - e^{i\eta_2}\sin\eta_2}{\pi N_F}\right] i(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{p}})\cdot\boldsymbol{\sigma}. \quad (17)$$

Here $\eta_1$ ($\eta_2$) represents the scattering phase shifts of the $l = 1$, $j = 1/2$, and ($l = 1$, $j = 3/2$) channel shown in Fig. 1; $\eta_0$ is the phase shift of the usual $s$ wave $l = 0$, $j = 1/2$ channel. The doublet ground state of Ce is Kondo-screened, and therefore Ce behaves as a nonmagnetic scatterer that induces a resonant scattering phase shift. Thus, in our model conduction electrons undergo the strongest scattering in the $\eta_1$ channel for which $\eta_1 = \pi/2$. If we set $\eta_0 = \eta_2 = 0$, then $\theta_{\rm SH} = -\Omega_{\rm cs} - \Omega_{\rm sc}$, which yields a spin-Hall conductivity that arises entirely from the side-jump mechanism:

$$\sigma_{\rm SH} = -\frac{eD(\Omega_{\rm cs} + \Omega_{\rm sc})N_F}{\hbar} = \frac{4}{9\pi^2}\frac{ek_F}{\hbar}. \quad (18)$$

We have reintroduced $\hbar$ above. It is interesting to point out in the single scattering channel limit, $\sigma_{\rm SH}$ does not depend on the impurity density and the specific value of $\eta_1$. This is because the $n_{\rm im}$ and $\eta_1$ dependence of $D = \frac{1}{3}v_F^2\tau_{\rm tr} \propto (n_{\rm im}\sin^2\eta_1)^{-1}$ is exactly canceled by $\theta_{\rm SH} \propto \Omega_{\rm cs} \propto n_{\rm im}\sin^2\eta_1$. Importantly, unlike the case of ordinary impurities with $d$ orbitals [33,54,55] where the relationship between $\eta_1$ and $\eta_2$ is determined by SOC, in our model $\eta_1 = \pi/2$ is determined by the Kondo screening. For three scattering channels, $\sigma_{\rm SH}$ is a complicated function of the phase shifts $\eta_0$, $\eta_1$, and $\eta_2$. For $\eta_0, \eta_2 \ll 1$ channels weakly coupled to the Kondo-screened $\eta_1 = \pi/2$ channel, we find that Eq. (18) receives a skew scattering contribution:

$$\sigma_{\rm SH}^{\rm sk} \simeq -\frac{\eta_0}{12\pi}\left(\frac{n_c}{n_{\rm im}}\right)\left(\frac{ek_F}{\hbar}\right), \quad (19)$$

where $n_c = k_F^3/3\pi^2$ is the carrier density. The ratio of Eq. (18) to Eq. (19) is $\approx 2\eta_0^{-1}(n_{\rm im}/n_c)$. Numerically, $e\sigma_{\rm SH}^{sJ} \simeq 2.72 \times 10^{-6} k_F\,\Omega^{-1}$. If we use the standard estimate for $\eta_0 \simeq 0.1$ [19,20,33], then Eq. (18) becomes comparable in magnitude to $\sigma_{\rm SH}^{\rm sk}$ for $n_{\rm im}/n_c \simeq 5\%$, for which the resistivity still shows the low temperature saturation characteristic of isolated Kondo-screened impurities. [22].

Finally, let us compute the correction to the naïve Fick's law by calculating the deviation of $D_s^L/D_s^T$ from unity. In the limit where the doublet is Kondo screened $\eta_1 = \pi/2$, and the other two orbitals are weakly coupled (i.e. $|\eta_0|$, $|\eta_2| \ll 1$), we obtain

$$\frac{D_s^L}{D_s^T} \simeq 1 - \frac{8n_i}{3\pi n_c}\left(\frac{\eta_0}{3} + \eta_2\right) + \frac{4\eta_0^2}{9}. \tag{20}$$

It is interesting to point out that $(D_s^L/D_s^T) - 1 \propto B^2$ where $B$ is the spin-orbit magnetic field defined by the square bracket in Eq. (17). When we assume all the phase shifts are small, i.e., $|\eta_{0,1,2}| \ll 1$, the leading corrections to $D_s^L/D_s^T$ are third order in the phase shifts so the spin anisotropy cannot be captured by the first Born approximation.

Equations (16), (18), and (20) demonstrate that spin-charge conversion mechanisms can be both quantitatively and qualitatively modified as a consequence of the strong scattering induced in one of the scattering channels by Kondo screening [56].

*Summary and discussion.*—We have developed a kinetic theory that provides a general framework to study spin transport in alloys containing dilute random ensembles of impurities with $d$ and $f$ orbitals. Scattering with such impurities is treated nonperturbatively, allowing us to deal with the strong scattering of conduction electrons on the Fermi surface coupled with strong local spin orbit (SOC). We have reported an analytical solution of the kinetic equations for a rotationally invariant system and applied it to simple quantum impurity model designed to capture the essential features of (Kondo-screened) Cerium impurities in alloys such as $Ce_x La_{1-x}Cu_6$ with $x < 0.7$. We find the combination of strong scattering and local SOC lead to a large contribution to the spin-Hall conductivity $\sigma_{SH}$ that stems entirely from the side jump and in the limit where interference with other channels can be neglected takes a value that depends only on the Fermi wave number. In addition, our nonperturbative treatment of impurity scattering allows us to show that the spin diffusion coefficients is spin anisotropic.

The above predictions can be readily tested in spin-valve devices where the spin current is injected from a ferromagnetic contact along different directions, thus allowing one to measure the different spin diffusion lengths associated with to $D_s^{L,T}$ as well as the spin-Hall conductivity $\sigma_{SH}$. When the injected spin is polarized in the direction parallel (perpendicular) to the direction of the current, it measures the longitudinal $D_s^L$ (transverse $D_s^T$) spin diffusion constant. Because of SOC, $D_s^L \neq D_s^L$, and this will be the most direct test of our theoretical predictions.

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