

## Efficient Narrow-Band Terahertz Radiation from Electrostatic Wakefields in Nonuniform Plasmas

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It is shown that electrostatic plasma wakefields can efficiently radiate at harmonics of the plasma frequency when the plasma has a positive density gradient along the propagation direction of a driver. The driver propagating at a subluminal group velocity excites the plasma wakefield with the same phase velocity. However, due to the positive density gradient, the wake phase velocity steadily increases behind the driver. As soon as the phase velocity becomes superluminal, the electrostatic wakefield couples efficiently to radiative electromagnetic modes. The period of time when the phase velocity stays above the speed of light depends on the density gradient scale length. The wake radiates at well-defined harmonics of the plasma frequency in the terahertz band. The angle of emission depends on the gradient scale and the time passed behind the driver. For appropriate plasma and driver parameters, the wake can radiate away nearly all its energy, which potentially results in an efficient, narrow-band, and tunable source of terahertz radiation.

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The terahertz band of electromagnetic radiation spans the frequency range  $3 \times 10^{11}$  Hz to  $3 \times 10^{13}$  Hz [1]. From the engineering point of view, there are currently few practical radiation sources in this terahertz gap [2]. The standard vacuum devices (gyrotrons, magnetrons, synchrotrons, free electron lasers, etc.) could in principle be modified to work in this range. However, these devices are still in prototype form, are not compact, or may require cryogenic temperatures to operate. The laser technology operates at the higher frequencies, with the wavelengths below  $10 \mu\text{m}$ . At the same time, radiation sources in the terahertz band are required to study rotational energy levels in complex molecules, oscillations in solid crystals, etc. [3]. If the terahertz field is strong enough (above  $100 \text{ MV/m}$ ), nonlinear interaction with solid state materials becomes possible including excitation of a diverse zoo of oscillatory degrees of freedom (spin waves, phonons, magnons, excitons, etc.) [4–6]. New physics related to control of nonequilibrium processes in solid state, initiation of surface chemical reactions, security, location, etc. require powerful sources of narrow-band terahertz radiation with a tunable central frequency [7]. The terahertz radiation also has a significant potential in medical diagnosis and treatment because its frequency range corresponds to the characteristic energy of biomolecular motion. Advantageously, terahertz-specific low photon energy does not cause the ionization of biomolecules [8].

After the advent of intense short laser pulses, the pioneering experiments by Hamster *et al.* [9] revealed strong terahertz emission both from solid and gas targets.

Recently, there was a lot of activity on laser- and accelerator-based sources of powerful terahertz sources [10]. Most of these generate just a single period of high amplitude terahertz emission. It is still a challenge to create a narrow-band terahertz source with the pulse energy beyond  $1 \mu\text{J}$ . Presently, the most powerful sources of terahertz radiation (1–10 MW,  $10 \mu\text{J}$  energy per pulse) are free electron lasers [11] which are expensive and not compact.

One of the possibilities to generate a narrow-band terahertz radiation is to exploit plasma oscillations—or wakefield—excited by a relativistic driver. The driver can be a short pulse laser or short bunch of charged particles. When the driver propagates through plasma, it displaces the plasma electrons from their equilibrium positions. This is accomplished either by the laser ponderomotive force, or by the transverse fields of the charged particles bunch. The plasma electrons continue to oscillate behind the driver at the local plasma frequency. This plasma wave is called the wakefield. The wakefield phase velocity  $v_{\text{ph}}$  simply equals the group velocity  $v_g$  of the driver. The wakefield is an electrostatic plasma oscillation that normally does not couple to electromagnetic waves.

Yet, it is possible to cause the plasma waves to radiate. One of the options is to collide two plasma wakefields by using superpropagating drivers [12,13]. Ginzburg and Zheleznyakov [14] first guessed that the radioemission from colliding electrostatic plasma waves is responsible for solar radio bursts. Tsytovich developed a kinetic theory of nonlinear waves coupling in plasmas [15]. Later, this mechanism was widely accepted in astrophysics [16–18].

It is also known that wakes generated by a laser pulse at an angle to plasma density gradients can emit broadband radiation via mode conversion [19–21]. This mechanism corresponds to the inverse resonant absorption. Another known way to cause the wakefields radiating is to apply an external magnetic field [22].

In this Letter, we suggest a different coupling mechanism between the electrostatic wakefield and electromagnetic radiation. The nonlinear wakefield current contains a nonvanishing curl. When the plasma has a positive density gradient along the driver propagation direction, the wake phase velocity is continuously increasing with time after the driver has passed over. At some point, the wake phase velocity becomes superluminal and the nonlinear curl of the plasma wave current can efficiently couple to electromagnetic modes. The wake phase velocity can remain above the vacuum speed of light for many plasma periods. This time is defined by the plasma density gradient scale length and can be sufficiently long to radiate nearly all energy stored in the wake field. Thus, an efficient regime of energy conversion from the driver to the plasma wakefield and then to terahertz radiation can be found.

In the following, we assume the plasma wakefield is generated by a short pulse laser, although a similar formalism can be applied to charged particles bunches. We assume the plasma is tenuous,  $\omega_p \ll \omega_0$ , where  $\omega_0 = 2\pi c/\lambda_0$  is the laser frequency with  $\lambda_0$  being the laser wavelength,  $\omega_p = \sqrt{4\pi e^2 n/m}$  the plasma frequency,  $n$  the background electron density,  $m$  the electron mass, and  $e$  the elementary charge. The laser pulse with the dimensionless amplitude  $a = \sqrt{0.74I/I_{18}}\lambda_0/\mu\text{m}$  excites a (quasi)linear wakefield with the potential  $\Phi$  [23]

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2(z)\right)\Phi(t, z) = \omega_p^2(z)\frac{mc^2}{e}a^2(t, z). \quad (1)$$

Here,  $I$  is the laser intensity,  $I_{18} = 10^{18} \text{ W/cm}^2$ ,  $a(t, z) = a[t - \int^z v_g^{-1}(\xi)d\xi]$ , and  $v_g(z) \approx c[1 - \omega_p^2(z)/2\omega_0^2]$  is the laser group velocity. We assume the laser pulse is weakly relativistic,  $a \ll 1$  and the density scale  $L = \omega_p(d\omega_p/dz)^{-1}$  is very long,  $\omega_p L/c \gg 1$ . The solution of this linear equation is a harmonic oscillation

$$\Phi(t, z, \mathbf{r}_\perp) = \Phi_0(\mathbf{r}_\perp) \exp[\mathbf{i}\varphi(t, z)] + \text{c.c.} \quad (2)$$

with the wake phase  $\varphi(t, z) = -\omega_p(z)[t - \int^z v_g^{-1}(\xi)d\xi]$ . The wake phase velocity simply equals the driver group velocity  $v_g(z) \approx c$  in the tenuous plasma. The full formula gives only a small correction, while it complicates the analytics. For this reason, we omit the small difference and assume  $v_g = c$  in the following formulas.

At larger amplitudes, the wakefield becomes anharmonic. In the 1D case, its potential satisfies the nonlinear oscillator equation [24]

$$\begin{aligned} \frac{d^2\Phi}{d\varphi^2} &= -\frac{mc^2}{e} \frac{1}{2} \left[ 1 - \frac{1}{(1 + \frac{e\Phi}{mc^2})^2} \right] \\ &= -\frac{mc^2}{e} \left[ \frac{e\Phi}{mc^2} - \frac{3}{2} \left( \frac{e\Phi}{mc^2} \right)^2 + 2 \left( \frac{e\Phi}{mc^2} \right)^3 + \dots \right]. \end{aligned} \quad (3)$$

The quasilinear solution contains higher harmonics of the plasma frequency  $\omega_l = l\omega_p$ , where  $l$  is the harmonic number. The wake potential scales linearly with the driver laser intensity  $I_0$ , so that  $e\Phi/mc^2 \sim a_0^2$ .

Our ansatz for the wake phase is  $\varphi(t, z) = -\omega_p(z)\tau$ , where  $\tau = t - z/c$  is the fast time. The wake phase frequency is then  $\omega = -\partial\varphi/\partial t = \omega_p(z)$  and the longitudinal wave number is  $k_z = \partial\varphi/\partial z = \omega_p(z)/c - \tau\partial\omega_p(z)/\partial z$ . The wake phase velocity along the driver propagation direction is not constant anymore, but changes with time after the driver passage

$$v_{\text{ph}z} = \frac{\omega}{k_z} = \frac{c}{1 - c\tau/L}. \quad (4)$$

The phase velocity (4) stays superluminal for times

$$0 < \tau < 2\frac{L}{c}. \quad (5)$$

Within the period of time (5), the wakefield is resonant with electromagnetic waves and radiates. Mention, the phase velocity passes through a singularity at  $\tau = L/c$ , then reverses and becomes negative afterward. It has been recently shown that this reversal can be used to generate a high-power broadband Cherenkov signal [25,26]. Here, instead we discuss a narrow-band emission at plasma frequency harmonics.

The radiation source is obtained from the wave equation, which we write on the magnetic field  $\mathbf{B}$ :

$$-\frac{\partial^2}{\partial t^2}\mathbf{B} + c^2\nabla^2\mathbf{B} - \omega_p^2\mathbf{B} = -4\pi c\nabla \times \mathbf{j}_{\text{wake}}, \quad (6)$$

where  $\mathbf{j}_{\text{wake}} = -e(n + \delta n)\mathbf{v}_{\text{wake}}$  is the current density generated by the wakefield,  $n$  is the background electron density,  $\delta n$  is the density perturbation due to the plasma wave, and  $\mathbf{v}_{\text{wake}}$  is the corresponding electron velocity. The nonvanishing curl of the wake current appears due to the nonlinear term  $-e\delta n\mathbf{v}_{\text{wake}}$  that causes the mode mixing. We Fourier transform the wakefield potential in plasma frequency harmonics  $\Phi = \int \sum_l \Phi_{l\mathbf{k}_\perp} \exp(-\mathbf{i}l\omega_p t + \mathbf{i}lk_z z + \mathbf{i}\mathbf{k}_\perp \mathbf{r}_\perp) d\mathbf{k}_\perp$ , with  $|l|$  being the harmonic number, and obtain the radiation source [compare Eq. (12.11) in [15] and Eq. (1) in [17]]:

$$\begin{aligned} \mathbf{R}_{l,\mathbf{k}_\perp} &= \frac{\mathbf{i}ec}{2m\omega_p} \sum_{l_1, l_2 = -\infty}^{\infty} \iint \delta_{l_1+l_2, l} \delta(\mathbf{k}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \\ &\quad \times \left( \frac{k_1^2}{l_2} - \frac{k_2^2}{l_1} \right) \mathbf{k}_1 \times \mathbf{k}_2 \Phi_{l_1\mathbf{k}_{1\perp}} \Phi_{l_2\mathbf{k}_{2\perp}} d\mathbf{k}_{1\perp} d\mathbf{k}_{2\perp}. \end{aligned} \quad (7)$$

The radiated frequency is  $\omega = (l_1 + l_2)\omega_p$  and the wave vector  $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$  have to satisfy the electromagnetic dispersion relation  $\omega^2 = \omega_p^2 + c^2k^2$ . The source (7) is proportional to the wake field potential squared. The wake-field amplitude in turn scales linearly with the laser intensity  $I_L$ . Assuming the wave numbers scale with  $k \sim \omega_p/c$ , we obtain that for weakly relativistic wakefields, the power of emitted terahertz radiation at the second harmonic scales as  $P_{\text{THz}} \sim |\mathbf{R}_{\omega, \mathbf{k}}|^2 \sim n^2 I_L^4$ . For higher harmonics with  $|l| \geq 4$ , the simple formula (7) requires relativistic corrections, which will be published elsewhere.

The formula (7) tells us further that plane plasma waves do not radiate. The radiation requires the presence of noncollinear wave vectors  $k_1$  and  $k_2$  with  $k_1^2 \neq k_2^2$  in the wave. Thus, a transversely symmetric wakefield never radiates exactly forward.

The emission direction can be obtained from the resonance condition. The angle of emission  $\theta_p$  inside the plasma column is  $\cos \theta_p = (1 - c\tau/L)/\sqrt{1 - \omega_p^2/\omega^2}$ . This angle depends not only on the delay  $\tau$  after the driver has passed, but also on the radiated frequency  $\omega$ . However, when the radiation leaves the plasma column and passes the lateral plasma-vacuum boundary, it undergoes an additional refraction, so that the observed angle of emission  $\theta_{\text{vacuum}}$  does not depend anymore on the frequency and all harmonics exit plasma at the same angle

$$\cos \theta_{\text{vacuum}} = 1 - \frac{c\tau}{L}. \quad (8)$$

The expression (8) defines the angle of emission as a function of the delay  $\tau$  after the driver passage.

We simulate the terahertz emission from a plasma wakefield by the fully relativistic three-dimensional (3D) particle-in-cell code VLPL (Virtual Laser-Plasma Laboratory) [27]. In the simulation, the background gas is assumed to be hydrogen ( $\text{H}_2$ ) with a linear density gradient so that the molecular density rises from  $0.85 \times 10^{18} \text{ cm}^{-3}$  to  $1.7 \times 10^{18} \text{ cm}^{-3}$  over the distance of  $L = 0.5 \text{ mm}$ . The hydrogen is dynamical ionized by the linearly polarized (in  $Y$  direction) short pulse laser with the intensity  $I_L = 5 \times 10^{17} \text{ W/cm}^2$  and FWHM pulse duration  $T_L = 30 \text{ fs}$ . This pulse duration is close to the ‘‘resonant’’ one [28]. The laser pulse has the wavelength  $\lambda_0 = 800 \text{ nm}$  and is focused to a focal spot with the FWHM diameter  $14 \mu\text{m}$ . The focal plane position is at  $z = 100 \mu\text{m}$ . This laser pulse has 40 mJ energy. The laser pulse group velocity here is  $v_g \approx (1 - \omega_p^2/2\omega_0^2)c \approx 0.9995c$ . The three-dimensional simulation box was  $L_x \times L_y \times L_z = 240 \mu\text{m} \times 240 \mu\text{m} \times 160 \mu\text{m}$  sampled by grid steps  $h_z = 0.14 \mu\text{m}$  and  $h_x = h_y = 0.4 \mu\text{m}$  the time step was  $\Delta = ch_z$  [29]. The boundary conditions were open for both fields and particles on all sides of the simulation box.

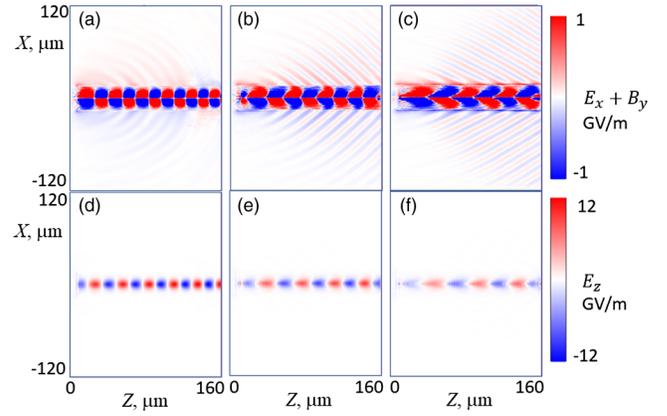


FIG. 1. (a)–(c) Snapshots of terahertz emission from a laser-plasma wakefield at the times  $t_{1,2,3} = 650, 1450, 2000 \text{ fs}$ . (d)–(f) Evolution of the longitudinal field  $E_z$ . The effective wavelength of the wake increases with time. The angle of terahertz emission changes correspondingly.

For simplicity and to save computational resources, we simulate only the first  $150 \mu\text{m}$  of the density ramp. As the laser pulse propagates, it ionizes the background gas within the focal spot and produces a plasma column. Outside of the plasma column, the gas remains unionized. Figure 1 shows 2D ( $z, x$ ) snapshots of the terahertz emission (the right-bound field  $E_x + B_y$ ) and the longitudinal wake field  $E_z$  at the times  $t_{1,2,3} = 650, 1450, 2000 \text{ fs}$ . The excited wakefield has the amplitude  $e\Phi/mc^2 \approx 0.1$  immediately behind the laser pulse [23]. This corresponds to the longitudinal field of  $E_z \approx 12 \text{ GV/m}$ .

Evolution of the on-axis longitudinal plasma wakefield is shown in Fig. 2. One sees as the wakefield period increases with time, thus increases the effective phase velocity. The continuous emission of electromagnetic waves leads to the steady drop of the wakefield amplitude.

Simultaneously, one sees as the plasma wakefield emits terahertz radiation at an angle to the laser propagation direction. The angle of emission  $\theta$  can be estimated by the inclination of wave phase fronts outside of the plasma column. This angle increases with time as predicted by Eq. (8). Figure 3 shows the angle of emission observed in the 3D PIC simulation (circles) as a function of time. The solid line gives the formula (8).

The radiated wave is radially polarized. We record its field at the right boundary at the point located  $100 \mu\text{m}$  away from the laser optical axis. The field itself and the spectrum are shown in Figs. 4(a) and 4(b). We see a nearly monochromatic signal at the second plasma frequency  $2\omega_p$ , which lasts for couple picoseconds. The wakefield in this case is only weakly nonlinear, so the emission at the harmonics of plasma frequency is low. Yet, we see a weaker emission at  $3\omega_p$  and even at the plasma frequency itself. The electromagnetic wave at  $\omega_p$  has an implicit transverse wave number  $k_{\perp}$ , so it is an evanescent wave inside plasma.

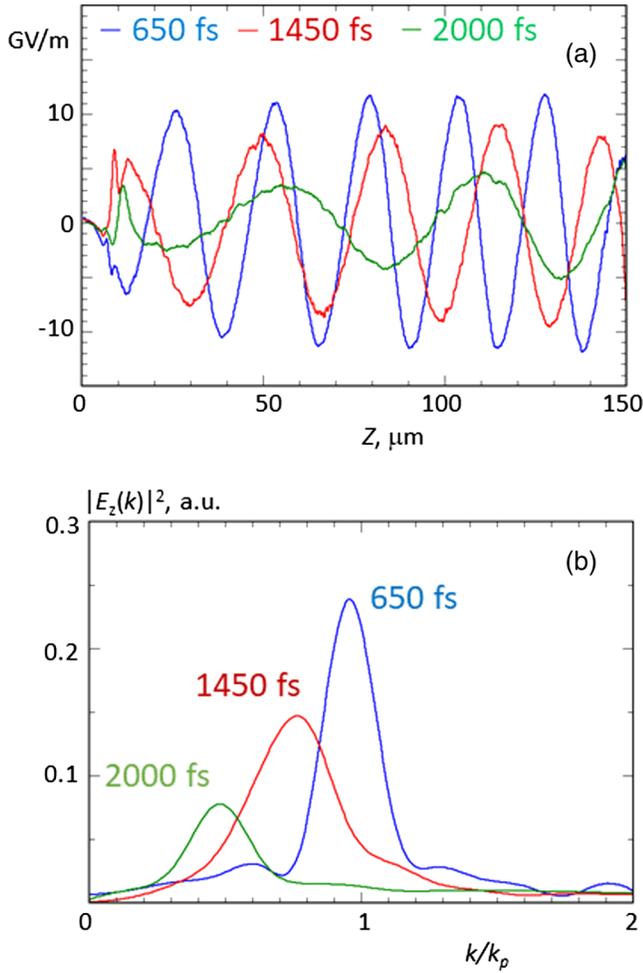


FIG. 2. (a) The on-axis wakefield  $E_z$  for the three times  $t_{1,2,3} = 650, 1450, 2000$  fs. It is seen that the effective wavelength increases. At the same time the wakefield amplitude drops continuously due the radiation losses. (b) Fourier transformation of the wakefields. At the earlier time, the wake wave number is centered at  $k_p = \omega_p/c$ . At the later times, the peak is shifting to smaller wave numbers.

However, as the plasma column radius is comparable with the plasma skin depth  $c/\omega_p$ , the evanescent wave can reach the plasma-vacuum boundary and escape. In addition, we see in Fig. 4(b) a radiation at a very low frequency. This is the Sommerfeld wave [30,31] attached to the plasma filament. The Sommerfeld wave is a slow wave with a subluminal phase velocity. Because it has a very long wavelength, it can be quasiresonantly excited even without any density gradient. The terahertz pulse energy was  $\approx 4 \mu\text{J}$  with the peak power  $P_{\text{THz}} \approx 2 \text{ MW}$ . This is a significant part of  $\approx 8 \mu\text{J}$  energy stored in the wakefield.

To highlight the terahertz emission at harmonics of plasma frequency, we did another simulation with the same gas target, but focused the laser pulse to a twice smaller spot of  $7 \mu\text{m}$  and peak intensity  $I_L = 2 \times 10^{18} \text{ W/cm}^2$ . The plasma wave produced by the higher intensity laser pulse has a

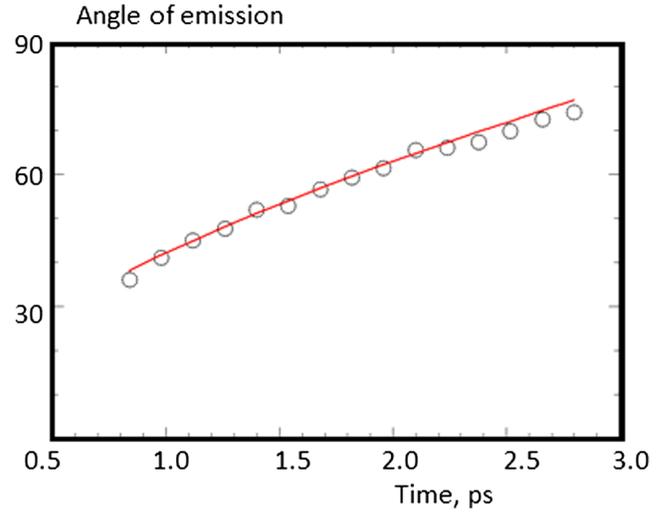


FIG. 3. Mean angle of terahertz emission as a function of time. Circles mark the simulation results. The solid line is given by the formula (8).

larger amplitude and radiates more at the higher harmonics. The terahertz signal recorded at the same position is shown in Fig. 4(c). The field amplitude exceeds  $0.5 \text{ GV/m}$  and lasts for about half a picosecond. During this time, the wakefield radiates away nearly all its energy. The spectrum of radiation is shown in Fig. 4(d). Although the second plasma harmonic still dominates, we see at least five harmonics of the plasma frequency. Mention also the emission at the very low frequencies, around  $f \approx 0$ . This is again the Sommerfeld mode (cylindrical plasma surface wave). This mode causes the significant asymmetry of the

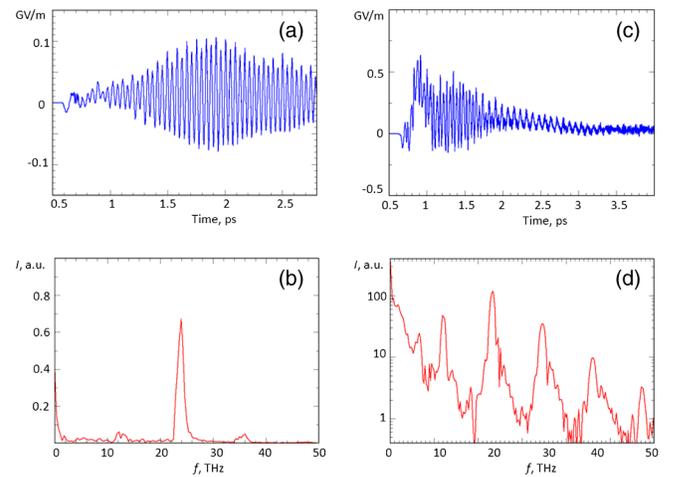


FIG. 4. Terahertz signal and its spectra recorded at the right boundary the radial position  $r = 100 \mu\text{m}$  outside of the plasma column (a),(b) for the laser intensity  $I_L = 5 \times 10^{17} \text{ W/cm}^2$  and focal spot diameter  $14 \mu\text{m}$ ; (c),(d) for the laser intensity  $I_L = 2 \times 10^{18} \text{ W/cm}^2$  and focal spot diameter  $7 \mu\text{m}$ . Radiation emission at the harmonics of plasma frequency  $f_p \approx 12 \text{ THz}$  are observed.

terahertz field seen in Fig. 4(c). The radiated terahertz energy is  $\approx 50 \mu\text{J}$  with the peak power  $P_{\text{THz}} \approx 50 \text{ MW}$ . A guided laser propagation over longer distances is required to increase the terahertz pulse power and energy.

In conclusion, we have shown that a plasma wakefield can efficiently radiate electromagnetic modes, when there is a positive density gradient along the driver propagation direction. The density scale length  $L$  defines how long the wake phase velocity remains superluminal and the coupling to the electromagnetic waves is possible. The low amplitude linear wake emits a narrow-band terahertz radiation at the second plasma harmonic,  $\omega = 2\omega_p$ . At higher amplitudes, the nonlinear plasma wave emits at the higher harmonics as well. The angle of emission is defined by the delay  $\tau$  behind the driver and the density scale length  $L$ . The intensity of emission scales as the wake amplitude to the fourth power. Because there is no other dissipation mechanism in tenuous plasmas, one can adjust the density gradient so that most of the wake energy is depleted by the THz radiation. In this work, we explain the physics of interaction, provide simple estimates and show results of full 3D PIC simulations. Accurate analytical formulas for typical Gaussian laser pulses will be presented elsewhere.

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