Spectral Statistics of Non-Hermitian Matrices and Dissipative Quantum Chaos

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(Received 22 March 2021; accepted 24 September 2021; published 19 October 2021)

We propose a measure, which we call the dissipative spectral form factor (DSFF), to characterize the spectral statistics of non-Hermitian (and nonunitary) matrices. We show that DSFF successfully diagnoses dissipative quantum chaos and reveals correlations between real and imaginary parts of the complex eigenvalues up to arbitrary energy scale (and timescale). Specifically, we provide the exact solution of DSFF for the complex Ginibre ensemble (GinUE) and for a Poissonian random spectrum (Poisson) as minimal models of dissipative quantum chaotic and integrable systems, respectively. For dissipative quantum chaotic systems, we show that the DSFF exhibits an exact rotational symmetry in its complex time argument τ. Analogous to the spectral form factor (SFF) behavior for Gaussian unitary ensemble, the DSFF for GinUE shows a "dip-ramp-plateau" behavior in $|\tau|$: the DSFF initially decreases, increases at intermediate timescales, and saturates after a generalized Heisenberg time, which scales as the inverse mean level spacing. Remarkably, for large matrix size, the "ramp" of the DSFF for GinUE increases *quadratically* in $|\tau|$, in contrast to the *linear* ramp in the SFF for Hermitian ensembles. For dissipative quantum integrable systems, we show that the DSFF takes a constant value, except for a region in complex time whose size and behavior depend on the eigenvalue density. Numerically, we verify the above claims and additionally show that the DSFF for real and quaternion real Ginibre ensembles coincides with the GinUE behavior, except for a region in the complex time plane of measure zero in the limit of large matrix size. As a physical example, we consider the quantum kicked top model with dissipation and show that it falls under the Ginibre universality class and Poisson as the "kick" is switched on or off. Lastly, we study spectral statistics of ensembles of random classical stochastic matrices or Markov chains and show that these models again fall under the Ginibre universality class.

DOI: 10.1103/PhysRevLett.127.170602

Introduction.—The study of spectral statistics is of fundamental importance in theoretical physics due to its universality and utility as a robust diagnosis of quantum chaos [1,2]. While Bohigas, Giannoni and Schmidt conjectured that chaotic quantum systems exhibit spectral correlation as those found in random matrix theory (RMT) in the same symmetry class [2], Berry and Tabor observed that integrable systems follow Poisson statistics of uncorrelated random variables [3]. Both claims have withstood the test of time, and in particular, the signature of level repulsion has been found in a wide range of disciplines, including nuclear resonance spectra [4], mesoscopic physics [5,6], quantum chaos [7], black hole physics [8–10], quantum chromodynamics [11,12], number theory [13], information theory [14], and more.

Non-Hermitian physics has advanced significantly in recent years in the study of optics [15–17], acoustics [18,19], parity-time-symmetric systems [20,21], mesoscopic physics [22–25], cold atoms [26,27], driven dissipative systems [28–32], biological systems [33], and disordered systems [5]. Recent studies on spectral properties have focused on the shape of eigenvalue density, the spectral gap, and the spacing between nearest-neighbor eigenvalues [34–48]. The goal of this Letter is to introduce and analyze a simple indicator that characterizes the level statistics of non-Hermitian matrices up to an arbitrary energy scale (and, equivalently, timescale) and show that it captures universal signatures of dissipative quantum chaos. We treat the complex-valued spectrum as a two-dimensional (2D) gas and introduce the dissipative spectral form factor (DSFF) as the 2D Fourier transform of the density-density correlator of complex eigenvalues, which depends on a complex time parameter τ. We exactly compute the DSFF for the GinUE and for a Poissonian random spectrum (Poisson) as minimal models of dissipative quantum chaotic and integrable systems respectively. In particular, we show that the DSFF for the complex Ginibre ensemble (GinUE) exhibits a dip-ramp-plateau behavior as a function of $|\tau|$, with an asymptotically quadratic ramp, as opposed to the linear ramp in the spectral form factor (SFF) for Gaussian ensembles of Hermitian matrices. We demonstrate the universality of these results by showing that they capture the level statistics of a quantum kicked top model with dissipation and random classical stochastic matrices. We conjecture that, for large enough complex time $|\tau|$, dissipative chaotic systems have DSFF behavior that coincides with GinUE's [49]. As such, the DSFF solution for the GinUE provides an important benchmark of a dissipative quantum chaotic system and can be treated as a complex analog of the SFF solution for the Gaussian unitary ensemble (GUE).

Spectral form factor.—It is instructive to review the behavior of the SFF for closed quantum systems [50], with which the DSFF shares several analogous features. Consider a closed quantum system described by an $N \times N$ Hermitian (or unitary) matrix with the density of states (DOS) $\rho(E) = \sum_n \delta(E - E_n)$, where E_n is the nth eigenlevel (or eigenphase). Note that, in our convention, $\int dE\rho(E) = N$. The correlation between eigenlevels can be quantified by the (so-called "2α point") SFF, which is the (αth power of the) Fourier transform of the two-level correlation function $\langle\rho(E)\rho(E+\omega)\rangle$, and can be directly defined as

$$K_\alpha(t) \coloneqq \left\langle \left[\sum\nolimits_{n,m} e^{-i(E_n - E_m)t}\right]^\alpha \right\rangle, \tag{1}$$

where $\langle\cdot\rangle$ denotes the average over an ensemble of statistically similar systems. Importantly, the SFF captures correlations between eigenlevels at *all* scales, including the level repulsion and spectral rigidity, while at the same time it is one of the simplest nontrivial and analytically tractable diagnostic of quantum chaos [51]. Furthermore, the SFF has recently been shown to capture novel signatures of quantum many-body physics like the deviation from RMT behavior at early time (and, consequently, the onset of chaos) [9,10,53–67]. For integrable systems with Poisson statistics, we have $K_1(t) = N$ for t larger than a timescale set by the inverse width of the DOS. For quantum chaotic systems without symmetries, the generic behavior of the SFF can be understood by studying the GUE. The SFF for the GUE initially decays and then grows linearly until the Heisenberg time t_{Hei}, after which $K(t) = N$ reaches a plateau. Qualitatively, this is referred to as the dip-ramp-plateau behavior. The form of the early time decay is due to the (nonuniversal) form of the DOS, while the linear ramp reflects the phenomenon of spectral rigidity. t_{Hei} is proportional to the inverse of the mean level spacing, and it encodes the largest physically relevant timescale of the system.

Dissipative spectral form factor.—For an $N \times N$ non-Hermitian matrix with complex spectra, the SFF is exponentially growing or decaying in time due to the imaginary parts of the complex eigenvalues. Moreover, traditional methods like Green's function approach fail due to nonanalyticity of Green's function. To circumvent these problems, we consider 2D DOS, $\rho(z) = \sum_n \delta(x - x_n)\delta(y - y_n)$, where $z = x + iy$, $x_n = \mathrm{Re}z_n$, $y_n = \mathrm{Im}z_n$, and $z_n = x_n + iy_n$ is the nth complex eigenvalue. We introduce the (2α-point) DSFF as the ensemble average of the (αth power of the) 2D Fourier transform of the two-level correlation function $\langle\rho(x,y)\rho(x+\omega, y+\omega')\rangle$. We directly define the DSFF as

$$K_\alpha(t,s) \coloneqq \left\langle \left[\sum\nolimits_{m,n} e^{i(x_n - x_m)t + i(y_n - y_m)s}\right]^\alpha \right\rangle, \tag{2}$$

where t and s are two "time" variables conjugate to the $x_n - x_m$ and $y_n - y_m$, respectively. Importantly, the correlation between *both* the real and imaginary parts of two given complex eigenvalues now contributes to the DSFF as phases. Notice that, if the spectrum is real, the DSFF is effectively reduced to the SFF as a function of t for all s. To obtain an intuition of how the DSFF behaves, we write $\vec{z}_{mn} \equiv (x_{mn}, y_{mn}) \equiv (x_m - x_n, y_m - y_n)$ and $\vec{\tau} \equiv (t, s) = (|\tau|\cos\theta, |\tau|\sin\theta)$. The DSFF can now be written as $K_\alpha(t,s) = \langle[\sum_{m,n} e^{i\vec{z}_{mn}\cdot\vec{\tau}}]^\alpha\rangle$, which allows a natural interpretation in the complex plane: At fixed θ and as a function of $|\tau|$ (2α point), the DSFF is the (α point) SFF of the projection of $\{z_m\}$ onto the radial axis specified by angle θ (illustrated in Fig. 1, inset). Reverting back to the notation with complex numbers, we define complex time $\tau = t + is$ and write the DSFF as $K_\alpha(\tau, \tau^*) = \langle|\sum_n e^{i(z_n\tau^* + z_n^*\tau)/2}|^{2\alpha}\rangle$. For ensembles where the two-point correlation function $\langle\rho(z_1)\rho(z_2)\rangle$ is known, the DSFF at $\alpha = 1$ can be written as an integral $K_1 = \int d^2z_1 d^2z_2 \langle\rho(z_1)\rho(z_2)\rangle e^{i(z_1\tau^* + z_1^*\tau - z_2^*\tau - z_2\tau^*)/2}$. For the rest of the Letter, we will drop the subscript and focus on the simplest and most relevant case, $\alpha = 1$.

Dissipative quantum chaotic systems.—We use the GinUE as a minimal model of the dissipative quantum chaotic systems. The joint probability distribution function of eigenvalues of the GinUE is known exactly, and the correlation function of eigenvalues can be expressed in terms of the kernel [68] $\mathcal{K}(z_1, z_2) = (N/\pi)e^{-(N/2)(|z_1|^2 + |z_2|^2)}\sum_{\ell=0}^{N-1}[(Nz_1z_2^*)^\ell/\ell!]$. The one-point correlation function, i.e., the DOS, is given by $\langle\rho(z)\rangle = \mathcal{K}(z,z)$, and the kernel is normalized such that $\int d^2z\langle\rho(z)\rangle = \int d^2z\mathcal{K}(z,z) = N$. Note that the DOS is isotropic and is asymptotically, as $N \to \infty$, flat on a unit disk $|z| < 1$ and vanishing outside. The two-point correlation function is $\langle\rho(z_1)\rho(z_2)\rangle = \mathcal{K}(z_1,z_1)\delta(z_1 - z_2) + \mathcal{K}(z_1,z_1)\mathcal{K}(z_2,z_2) - |\mathcal{K}(z_1,z_2)|^2$. We will refer to the above three terms as the contact, disconnected, and connected term, respectively. Using $\langle\rho(z_1)\rho(z_2)\rangle$, we compute (2) by expanding the exponential factors and using the fact that the integrals over the phases of z_1 and z_2 kill all terms in the sum except the ones depending on $|z_1|$ and $|z_2|$. This gives

$$\begin{aligned} K_{\mathrm{GinUE}}(\tau,\tau^*) = {} & N + N^2\, {}_1F_1\left(N+1; 2; -\frac{|\tau|^2}{4N}\right)^2 \\ & - \sum_{n,m=0}^{N-1} \frac{(\max(m,n)!)^2}{n!m!(|m-n|!)^2} \\ & \times {}_1F_1\left(\max(m,n)+1; |m-n|+1; -\frac{|\tau|^2}{4N}\right)^2, \end{aligned} \tag{3}$$

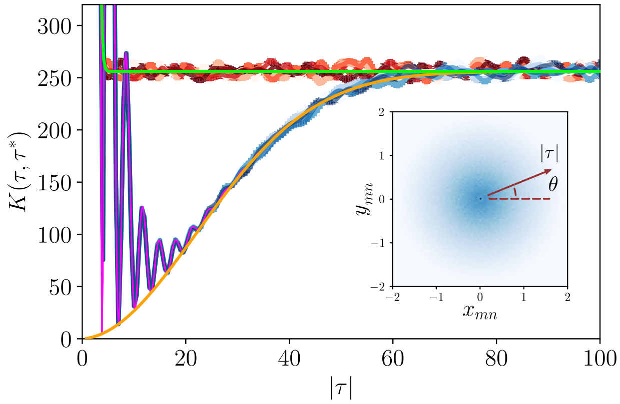

FIG. 1. $K(\tau,\tau^*)$ vs $|\tau|$ for the GinUE and a Poissonian random spectrum [defined above (5)] with matrix size $N = 256$. Writing $\tau = |\tau|e^{i\theta}$, numerical simulations of the GinUE (Poisson) for fixed $\theta = 0$ to $\theta = \pi/2$ in steps of $\pi/4$ are plotted in blue colors (red colors) in multiple shades. The analytical solutions of $K_{\rm GinUE}$ [Eq. (3)] and $K_{\rm Poi}$ [Eq. (5)] are plotted as the purple and green lines on top of the numerical data. The connected part of $K_{\rm GinUE}$ [first and third terms in Eq. (3)] is plotted as the orange line. Inset: two-dimensional histogram of $\{z_{mn} \equiv z_m - z_n\}$ of the GinUE for $N = 1024$, where increasing values are plotted with deeper blue colors. Note that the bin at the origin is occupied by N (diagonal) contributions of $\{z_{nn} = 0\}$, and there is a dip around the origin due to level repulsion. The computation of the DSFF for fixed θ as a function of $|\tau|$ is equivalent to the computation of the SFF as a function of $|\tau|$ of $\{z_m\}$ projected onto the axis defined by θ. The sample sizes are 5000 and 2000 for Poisson and the GinUE, respectively.

where we have listed the three terms in the same ordering as in the two-point correlation function. ${}_1F_1(a,b;z) = \sum_{n=0}^{\infty} a^{(n)} z^n / b^{(n)} n!$ is the Kummer confluent hypergeometric function, where $a^{(n)}$ is the rising factorial. Keeping the leading contributions in N, Eq. (3) becomes

$$K_{\rm GinUE}(\tau,\tau^*) = N + N^2 \frac{4J_1(|\tau|)^2}{|\tau|^2} - N\exp\left(-\frac{|\tau|^2}{4N}\right), \qquad (4)$$

where $J_\mu(x)$ is the Bessel function of the first kind. Along with Eqs. (3) and (4) forms our main result, as they capture the universal spectral correlations of dissipative quantum chaotic systems. First, note that $K_{\rm GinUE}(\tau,\tau^*)$ only depends on the absolute value of τ; i.e., the DSFF is manifestly rotational symmetric in complex time (see Fig. 1). Second, the qualitative behavior of the DSFF as a function of $|\tau|$ for dissipative quantum systems shows a dip-ramp-plateau structure, analogous to the SFF for closed quantum systems: At early time $|\tau| \lesssim \tau_{\rm edge}$, the DSFF dips from $K(0,0) = N^2$ with a form dominated by the disconnected piece (3); at intermediate time $\tau_{\rm edge} \lesssim |\tau| \lesssim \tau_{\rm Hei}$, the DSFF increases quadratically $K_{\rm GinUE} \simeq |\tau|^2/4$ with precise form given by the sum of hypergeometric functions, or the Gaussian function for large N, until it reaches late time $\tau_{\rm Hei} \lesssim |\tau|$ with $\tau_{\rm Hei} \sim \sqrt{N}$, where the DSFF reaches a plateau at N. Note that the DSFF GinUE ramp behavior is drastically different from the corresponding SFF GUE behavior, which is linear in time. Third, in analogy to the SFF, the connected term in the DSFF captures the spectral rigidity in the complex spectral plane. As apparent from the functional form of the connected term, we see that the Heisenberg time scales as the inverse of mean level spacing (in the complex plane), $\tau_{\rm Hei} = O(\Delta^{-1}) = O(\sqrt{N})$ [69]. Again, this is in contrast to the corresponding Heisenberg time scaling for the SFF, which scales as N. Fourth, the nonoscillatory part of the disconnected term asymptotically scales as $N^2|\tau|^{-3}$. Setting the time $\tau_{\rm edge}$, where disconnected and connected contributions are of the same order, we find $\tau_{\rm edge} = O(N^{2/5})$. Note that, as a function of $|\tau|$, the GinUE disconnected term of the DSFF coincides with the GUE disconnected term of the SFF, due to the fact that the projection of the DOS along the axis defined by θ exhibits a semicircle shape, like the DOS of the GUE. Fifth, according to the interpretation described above, the DSFF at fixed θ is equivalent to the SFF of the projected spectrum $\{z_m\}$ along the axis defined by θ. While there is level repulsion in the 2D complex plane, there are accidental degeneracies between distanced pairs of eigenvalues in the set of projected $\{z_m\}$, and remarkably, the lack of level repulsion along the projection axis makes up for the difference between the SFF of the GUE, which has a linear ramp in t with $t_{\rm Hei} = O(N)$, and the DSFF of the GinUE, which has a quadratic ramp in $|\tau|$ with $\tau_{\rm Hei} = O(\sqrt{N})$.

Finally, the GinUE DSFF result can be interpreted via the mapping between the GinUE and 2D log-potential Coulomb gas. the DSFF is equivalent to the 2D static structure factor (SSF), defined as the Fourier transform of density-density fluctuation, where the complex energy and the complex time in (2) take the roles of position and wave vector $\vec{k}$. For the Coulomb gas, with the assumption of "perfect screening," SSF is argued to have an asymptotic behavior of $|\vec{k}|^2$ for small $|\vec{k}|$ [70], which is consistent with the quadratic increase $|\tau|^2$ for large N in (4). Furthermore, a system is "hyperuniform" if its SSF vanishes as $|\vec{k}|$ tends to zero. This implies that density fluctuation is suppressed at very large length scales [71,72]. This leads us to interpret that the spectrum of the GinUE is a 2D gas that displays hyperuniformity with a quadratic power-law form.

The numerical data and analytical solutions are plotted in Fig. 1 for $N = 256$ with excellent agreement. We further computed the DSFF for the real (GinOE) and quaternion real (GinSE) Ginibre ensembles [69]. The rotational symmetry of K in τ is broken, since eigenvalues of quaternion real (real) ensemble (are either real or) come in complex conjugate pairs, which leads to special behavior of z_{mn} near $\theta = 0, \pi/2$ [69]. We define critical angles θ_0^* and $\theta_{\pi/2}^*$ such that the DSFF of the GinOE and GinSE coincide with the one of the GinUE for $\theta \in [\theta_0^*, \pi/2 - \theta_{\pi/2}^*]$. We numerically show and heuristically argue that $\theta_\phi^* \propto N^{-1/2}$ with the only exception of $\theta_{\pi/2}^* \propto N^{-0.56}$ for the GinSE due to the lack of "projected degeneracies" [69]. We therefore conclude that, in large N, the GinOE and GinSE coincide with the GinUE behavior except for the angle $\theta = 0$ and $\pi/2$. This is consistent with the fact that spectral correlations of these ensembles coincide with the GinUE for eigenvalues away from the real axis [73–77].

Dissipative integrable systems.—We model the spectrum of dissipative quantum integrable systems with a set of uncorrelated normally distributed complex eigenvalues. The DOS is $\langle\rho(z)\rangle = N(2\pi)^{-1}e^{-|z|^2/2}$ with $\int d^2z\langle\rho(z)\rangle = N$. The two-point correlation function is $\langle\rho(z_1)\rho(z_2)\rangle = \langle\rho(z_2)\rangle\delta(z_1 - z_2) + [N(N-1)/N^2]\langle\rho(z_1)\rangle\langle\rho(z_2)\rangle$. The DSFF can be evaluated as

$$K_{\mathrm{Poi}}(\tau, \tau^*) = N + N(N-1)e^{-|\tau|^2}. \tag{5}$$

We see that the DSFF for a Poissonian random spectrum takes a constant value of N except for a small region of complex time near the origin. The constant value N is due to the diagonal contribution from the DSFF, and the deviation from N is due to the disconnected part and depends on the details of the DOS. The numerical data and analytical solution are plotted in Fig. 1 for $N = 256$ with excellent agreement.

Dissipative quantum kicked top model.—A simple but rich example of quantum systems that exhibit chaotic and integrable behaviors is the quantum kicked top (QKT) model [78–80], which has been experimentally realized in [81]. The unitary evolution of the QKT is governed by the Hamiltonian [78],

$$H(t) = pJ_z + \frac{k_0}{2j}J_z^2 + \frac{k_1}{2j}J_y^2 \sum_{n=-\infty}^{\infty} \delta(t-n), \tag{6}$$

where $\vec{J} = (J_x, J_y, J_z)$ are angular momentum operators that act on a single spin-j particle and obey $[J_\alpha, J_\beta] = i\epsilon_{\alpha\beta\gamma}J_\gamma$, $\alpha, \beta, \gamma \in \{x, y, z\}$. The first two terms describe the precession of the spin. The third term describes a periodic kick at integer time n. We introduce the dissipation by considering the action of the quantum map in the Kraus form $\Phi(\rho) = \sum_a K_a e^{-iH}\rho e^{iH}K_a^\dagger$, where $K_{1,2} = (J_x \pm iJ_y)/\sqrt{j(j+1)}$ and $K_3 = \sqrt{2}J_z/\sqrt{j(j+1)}$ such that the constraint $\sum_a K_a^\dagger K_a = \mathbb{1}$ is satisfied to ensure trace preservation and complete positivity [82]. The time evolved density matrix is obtained by a successive action of Φ, i.e., $\rho(t) = \Phi^t(\rho)$. We represent Φ as a superoperator; i.e., $\Phi = \sum_a (K_a \otimes K_a^*)(U \otimes U^*)$, where $U = \mathcal{T}\exp[-i\int_0^1 dt H(t)]$ and $\mathcal{T}$ is the time ordering. Note that the total angular momentum $\vec{J}^2$ and the parity are conserved and we will therefore study the restricted Hilbert space of size roughly half of $(2j+1)^2$. We analyze the DSFF of the spectrum of Φ in the symmetric subspace. Note that the spectrum is symmetric across the real axis [69]. As shown

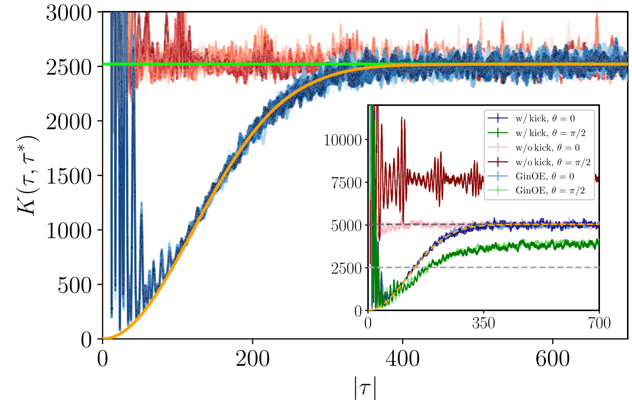

FIG. 2. DSFF of QKT with dissipation for $j = 35$ and Gaussianly distributed $p \in \mathcal{N}(2, 2/3)$ and $k_0 \in \mathcal{N}(10, 3)$. The blue (red) lines in different shades are for the DSFF of QKT with the kick with $k_1 = 8$ (without the kick with $k_1 = 0$) for fixed $\theta \in [\pi/16, 7\pi/16]$ in steps of $\pi/16$. The two cases fit the connected part of GinUE (orange line) and Poisson (green line) predictions as expected. Inset: DSFF of QKT and GinOE for angles $\theta = 0$, $\pi/2$ [69]. The sample sizes for QKT with and without the kick are 2500 and 4300, respectively.

in Fig. 2, as we turn on and off the kick parameter k_1, the DSFF coincides with the GinOE (for all angles [69]) and Poisson behavior with excellent agreement. Note that the DSFF of the corresponding Liouvillian operators of the Lindblad form of the QKT also converges to GinOE DSFF behavior.

Classical stochastic systems.—Another interesting class of non-Hermitian matrices are classical stochastic matrices or Markov chains, which are matrices with real positive entries and with each column summing to unity. A particular way to generate an ensemble of random stochastic matrices is to consider matrix S with entries $S_{ij} = |M_{ij}|^2/\sum_i |M_{ij}|^2$, where M_{ij} is a matrix chosen from

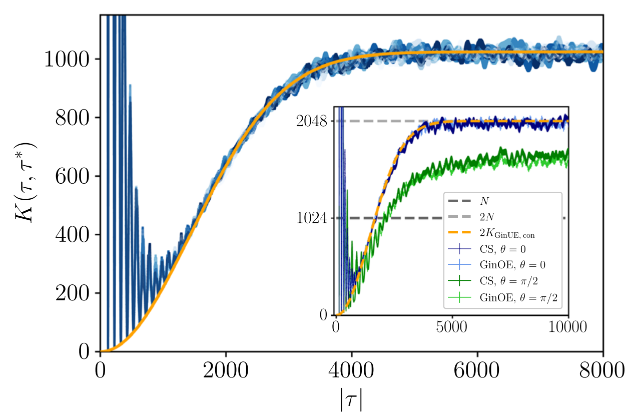

FIG. 3. DSFF of the random classical stochastic ensemble induced by CUE for $N = 1024$ with sample size 5000. The blue colors are for fixed $\theta \in [\pi/16, 7\pi/16]$ in steps of $\pi/16$. The data can be fitted with the connected part of the GinUE (orange line). Inset: DSFF for CS and GinOE at angles $\theta = 0$, $\pi/2$ [69].

a certain matrix ensemble. We consider classical stochastic (CS) matrices induced from two random ensembles: the circular unitary ensemble (CUE) (whose induced stochastic ensemble is called the unistochastic ensemble) and the GinUE. Unistochastic matrices arise in the context of quantum graphs [83–85] and in the theory of majorization and characterization of quantum maps [86–89]. By the Perron-Frobenius theorem, the stochastic matrices have leading eigenvalues of unity, and the spectra are again symmetric across the real axis [69,90]. We plot the DSFF for the unistochastic ensemble in Fig. 3 and the other GinUE-induced ensemble in the Supplemental Material [69]. For both ensembles, the DSFF behavior coincides with the GinOE behavior (for all angles [69]) with excellent agreement.

Discussion.—We have proposed and exactly computed the DSFF for the GinUE and a Poissonian random spectrum as minimal models of dissipative quantum chaotic and integrable systems. In particular, we show that the DSFF for the GinUE has a dip-ramp-plateau behavior with a quadratic ramp and numerically demonstrated the universality of the result with the example of the QKT and random classical stochastic ensembles. This Letter opens up many exciting directions: the DSFF can be used to classify dissipative quantum chaotic systems in different universality and symmetry classes, beyond the nearest-neighbor spacing distribution studied previously [91–94], and to unveil deviation of open quantum many-body systems from RMT behaviors at early time (cf. [53–63]). In particular, it can be used to investigate the spectral properties across the measurement-induced phase transition [95–101]. These directions will be discussed in an upcoming work [49].

Finally, note that the DSFF contrasts with a related observable called the dissipative form factor [35] (DFF) in several ways: the DFF is a one-parameter function defined for the (Lindblad) superoperators, and the correlation between the imaginary parts of eigenvalues contribute to the DFF as an exponential factor (as supposed to a phase in the DSFF). This makes the DFF useful in capturing the scaling of the spectral gap, while the DSFF is beneficial in unraveling the correlation between eigenvalues in the bulk of the spectrum.

We are thankful to Fiona Burnell, David Huse, Abhinev Prem, Shinsei Ryu, Lucas Sá, Shivaji Sondhi, and Salvatore Torquato for helpful discussions. A. C. is particularly grateful to Tankut Can, for pointing out the connections to static structure factor and hyperuniformity, and to John Chalker and Andrea De Luca for collaborations on related projects. A. C. is supported by Croucher foundation and PCTS at Princeton University fellowships. T. P. acknowledges ERC Advanced Grant No. 694544-OMNES and ARRS research program P1-0402.

[1] M. L. Mehta, *Random Matrices* (Academic Press, New York, 2004).
[2] O. Bohigas, M.-J. Giannoni, and C. Schmit, Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws, Phys. Rev. Lett. **52**, 1 (1984).
[3] M. V. Berry and M. Tabor, Level clustering in the regular spectrum, Proc. R. Soc. A **356**, 375 (1977).
[4] T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. M. Wong, Random-matrix physics: Spectrum and strength fluctuations, Rev. Mod. Phys. **53**, 385 (1981).
[5] C. W. J. Beenakker, Random-matrix theory of quantum transport, Rev. Mod. Phys. **69**, 731 (1997).
[6] C. W. J. Beenakker, Random-matrix theory of Majorana fermions and topological superconductors, Rev. Mod. Phys. **87**, 1037 (2015).
[7] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics, Adv. Phys. **65**, 239 (2016).
[8] A. M. García-García and J. J. M. Verbaarschot, Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 126010 (2016).
[9] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, Black holes and random matrices, J. High Energy Phys. 05 (2017) 118.
[10] J. Cotler, N. Hunter-Jones, J. Liu, and B. Yoshida, Chaos, complexity, and random matrices, J. High Energy Phys. 11 (2017) 048.
[11] A. V. Smilga, *Continuous Advances in QCD* (World Scientific, Singapore, 1995), https://www.worldscientific.com/doi/pdf/10.1142/2435.
[12] G. Akemann, *Random matrix theory and quantum chromodynamics*, (Oxford Scholarship Online, 2018), https://doi.org/10.1093/oso/9780198797319.003.0005.
[13] M. Schroeder, *Number Theory in Science and Communication: With Applications in Cryptography, Physics, Digital Information, Computing, and Self-Similarity*, 5th ed. (Springer, New York, 2008).
[14] A. M. Tulino and S. Verdú, Random matrix theory and wireless communications, Found. Trends Commun. Inf. Theory **1**, 1 (2004).
[15] L. Feng, R. El-Ganainy, and L. Ge, Non-Hermitian photonics based on parity-time symmetry, Nat. Photonics **11**, 752 (2017).
[16] H. Carmichael, An open systems approach to quantum optics, Lect. Notes Phys., M: Monogr. **18**, 114 (1993).
[17] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Non-Hermitian physics and PT symmetry, Nat. Phys. **14**, 11 (2018).
[18] G. Ma and P. Sheng, Acoustic metamaterials: From local resonances to broad horizons, Sci. Adv. **2** (2016).
[19] S. A. Cummer, J. Christensen, and A. Alù, Controlling sound with acoustic metamaterials, Nat. Rev. Mater. **1**, 16001 (2016).
[20] C. M. Bender, Making sense of non-Hermitian Hamiltonians, Rep. Prog. Phys. **70**, 947 (2007).
[21] Ş. K. Özdemir, S. Rotter, F. Nori, and L. Yang, Parity-time symmetry and exceptional points in photonics, Nat. Mater. **18**, 783 (2019).

[22] N. Moiseyev, Quantum theory of resonances: Calculating energies, widths and cross-sections by complex scaling, Phys. Rep. **302**, 212 (1998).

[23] J. G. Muga, J. P. Palao, B. Navarro, and I. L. Egusquiza, Complex absorbing potentials, Phys. Rep. **395**, 357 (2004).

[24] H. Cao and J. Wiersig, Dielectric microcavities: Model systems for wave chaos and non-Hermitian physics, Rev. Mod. Phys. **87**, 61 (2015).

[25] S. Rotter and S. Gigan, Light fields in complex media: Mesoscopic scattering meets wave control, Rev. Mod. Phys. **89**, 015005 (2017).

[26] A. J. Daley, Quantum trajectories and open many-body quantum systems, Adv. Phys. **63**, 77 (2014).

[27] S. Kuhr, Quantum-gas microscopes: A new tool for cold-atom quantum simulators, Natl. Sci. Rev. **3**, 170 (2016).

[28] H. Deng, H. Haug, and Y. Yamamoto, Exciton-polariton Bose-Einstein condensation, Rev. Mod. Phys. **82**, 1489 (2010).

[29] M. Müller, S. Diehl, G. Pupillo, and P. Zoller, Engineered open systems and quantum simulations with atoms and ions, Opt. Phys. **61**, 1 (2012).

[30] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Cold atoms in cavity-generated dynamical optical potentials, Rev. Mod. Phys. **85**, 553 (2013).

[31] L. M. Sieberer, M. Buchhold, and S. Diehl, Keldysh field theory for driven open quantum systems, Rep. Prog. Phys. **79**, 096001 (2016).

[32] T. Chou, K. Mallick, and R. K. P. Zia, Non-equilibrium statistical mechanics: From a paradigmatic model to biological transport, Rep. Prog. Phys. **74**, 116601 (2011).

[33] M. C. Marchetti, J. F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, Hydrodynamics of soft active matter, Rev. Mod. Phys. **85**, 1143 (2013).

[34] G. Akemann, M. Kieburg, A. Mielke, and T. Prosen, Universal Signature from Integrability to Chaos in Dissipative Open Quantum Systems, Phys. Rev. Lett. **123**, 254101 (2019).

[35] T. Can, Random Lindblad dynamics, J. Phys. A **52**, 485302 (2019).

[36] T. Can, V. Oganesyan, D. Orgad, and S. Gopalakrishnan, Spectral Gaps and Midgap States in Random Quantum Master Equations, Phys. Rev. Lett. **123**, 234103 (2019).

[37] L. Sá, P. Ribeiro, and T. Prosen, Complex Spacing Ratios: A Signature of Dissipative Quantum Chaos, Phys. Rev. X **10**, 021019 (2020).

[38] L. Sá, P. Ribeiro, and T. Prosen, Spectral and Steady-State Properties of Random Liouvillians, J. Phys. A **53**, 305303 (2020).

[39] L. Sá, P. Ribeiro, T. Can, and T. Prosen, Spectral transitions and universal steady states in random Kraus maps and circuits, Phys. Rev. B **102**, 134310 (2020).

[40] L. Sá, P. Ribeiro, and T. Prosen, Integrable non-unitary open quantum circuits, Phys. Rev. B **103**, 115132 (2021).

[41] Á. Rubio-García, R. A. Molina, and J. Dukelsky, From integrability to chaos in quantum Liouvillians, arXiv: 2102.13452.

[42] K. Wang, F. Piazza, and D. J. Luitz, Hierarchy of Relaxation Timescales in Local Random Liouvillians, Phys. Rev. Lett. **124**, 100604 (2020).

[43] S. Denisov, T. Laptyeva, W. Tarnowski, D. Chruściński, and K. Życzkowski, Universal spectra of random Lindblad operators, Phys. Rev. Lett. **123** (2019).

[44] Y. Huang and B. I. Shklovskii, Anderson transition in three-dimensional systems with non-Hermitian disorder, Phys. Rev. B **101**, 014204 (2020).

[45] R. Hamazaki, K. Kawabata, and M. Ueda, Non-Hermitian Many-Body Localization, Phys. Rev. Lett. **123**, 090603 (2019).

[46] A. F. Tzortzakakis, K. G. Makris, and E. N. Economou, Non-Hermitian disorder in two-dimensional optical lattices, Phys. Rev. B **101**, 014202 (2020).

[47] T. Peron, B. M. F. de Resende, F. A. Rodrigues, L. F. Costa, and J. A. Méndez-Bermúdez, Spacing ratio characterization of the spectra of directed random networks, Phys. Rev. E **102**, 062305 (2020).

[48] F. Borgonovi, I. Guarneri, and D. L. Shepelyansky, Statistics of quantum lifetimes in a classically chaotic system, Phys. Rev. A **43**, 4517 (1991).

[49] J. Li, T. Prosen, and A. Chan (to be published).

[50] F. Haake, *Quantum Signatures of Chaos* (Springer, New York, 2010).

[51] The SFF involves only two sums over eigenlevels as supposed to four sums in observables like the out-of-time-order correlator or the quantum purity. Note also that the finer correlations between eigenlevels can be captured by the so-called higher-point SFF [10,52].

[52] J. Liu, Spectral form factors and late time quantumchaos, Phys. Rev. D **98**, 086026 (2018).

[53] P. Kos, M. Ljubotina, and T. Prosen, Many-Body Quantum Chaos: Analytic Connection to Random Matrix Theory, Phys. Rev. X **8**, 021062 (2018).

[54] B. Bertini, P. Kos, and T. Prosen, Exact Spectral form Factor in a Minimal Model of Many-Body Quantum Chaos, Phys. Rev. Lett. **121**, 264101 (2018).

[55] D. Roy and T. Prosen, Random matrix spectral form factor in kicked interacting fermionic chains, Phys. Rev. E **102**, 060202 (2020).

[56] A. Flack, B. Bertini, and T. Prosen, Statistics of the spectral form factor in the self-dual kicked Ising model, Phys. Rev. Res. **2**, 043403 (2020).

[57] A. Chan, A. De Luca, and J. T. Chalker, Solution of a Minimal Model for Many-Body Quantum Chaos, Phys. Rev. X **8**, 041019 (2018).

[58] A. Chan, A. De Luca, and J. T. Chalker, Spectral Statistics in Spatially Extended Chaotic Quantum Many-Body Systems, Phys. Rev. Lett. **121**, 060601 (2018).

[59] A. J. Friedman, A. Chan, A. De Luca, and J. T. Chalker, Spectral Statistics and Many-Body Quantum Chaos with Conserved Charge, Phys. Rev. Lett. **123**, 210603 (2019).

[60] S. Moudgalya, A. Prem, D. A. Huse, and A. Chan, Spectral statistics in constrained many-body quantum chaotic systems, Phys. Rev. Research **3**, 023176 (2021).

[61] S. J. Garratt and J. T. Chalker, Many-Body Quantum Chaos and the Local Pairing of Feynman Histories, Phys. Rev. X **11**, 021051 (2021).

[62] H. Gharibyan, M. Hanada, S. H. Shenker, and M. Tezuka, Onset of random matrix behavior in scrambling systems, J. High Energy Phys. 07 (2018) 124.

[63] A. Chan, A. De Luca, and J. T. Chalker, Spectral Lyapunov exponents in chaotic and localized many-body quantum systems, Phys. Rev. Research **3**, 023118 (2021).

[64] P. Saad, S. H. Shenker, and D. Stanford, A semiclassical ramp in SYK and in gravity, arXiv:1806.06840.

[65] B. Bertini, P. Kos, and T. Prosen, Random matrix spectral form factor of dual-unitary quantum circuits, Commun. Math. Phys. **387**, 597 (2021).

[66] M. Winer and B. Swingle, Hydrodynamic theory of the connected spectral form factor, arXiv:2012.01436.

[67] J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, Quantum chaos challenges many-body localization, Phys. Rev. E **102**, 062144 (2020).

[68] J. Ginibre, Statistical ensembles of complex, quaternion, and real matrices, J. Math. Phys. (N.Y.) **6**, 440 (1965).

[69] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.170602 for the DSFF of additional ensembles (with modified spectra), discussion of the DSFF near $\theta = 0$ and $\pi/2$, analysis on critical angles θ^*_ϕ, details about DOS, and the scaling of τ_{Hei}.

[70] M. Baus and J.-P. Hansen, Statistical mechanics of simple coulomb systems, Phys. Rep. **59**, 1 (1980).

[71] S. Torquato and F. H. Stillinger, Local density fluctuations, hyperuniformity, and order metrics, Phys. Rev. E **68**, 041113 (2003).

[72] S. Torquato, Hyperuniform states of matter, Phys. Rep. **745**, 1 (2018).

[73] N. Lehmann and H.-J. Sommers, Eigenvalue Statistics of Random Real Matrices, Phys. Rev. Lett. **67**, 2403(E) (1991).

[74] P. J. Forrester and T. Nagao, Eigenvalue Statistics of the Real Ginibre Ensemble, Phys. Rev. Lett. **99**, 050603 (2007).

[75] A. Borodin and C. D. Sinclair, The Ginibre ensemble of real random matrices and its scaling limits, Commun. Math. Phys. **291**, 177 (2009).

[76] G. Akemann and E. Kanzieper, Integrable structure of Ginibre's ensemble of real random matrices and a Pfaffian integration theorem, J. Stat. Phys. **129**, 1159 (2007).

[77] H.-J. Sommers and W. Wieczorek, General eigenvalue correlations for the real Ginibre ensemble, J. Phys. A **41**, 405003 (2008).

[78] R. Grobe, F. Haake, and H.-J. Sommers, Quantum Distinction of Regular and Chaotic Dissipative Motion, Phys. Rev. Lett. **61**, 1899 (1988).

[79] F. Haake, M. Kuś, and R. Scharf, Classical and quantum chaos for a kicked top, Z. Phys. B Condens. Matter **65**, 381 (1987).

[80] T. Gorin, T. Prosen, T. H. Seligman, and M. Žnidarič, Dynamics of Loschmidt echoes and fidelity decay, Phys. Rep. **435**, 33 (2006).

[81] G. A. Smith, S. Chaudhury, A. Silberfarb, I. H. Deutsch, and P. S. Jessen, Continuous Weak Measurement and Nonlinear Dynamics in a Cold Spin Ensemble, Phys. Rev. Lett. **93**, 163602 (2004).

[82] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, Cambridge, England, 2010).

[83] G. Tanner, Unitary-stochastic matrix ensembles and spectral statistics, J. Phys. A **34**, 8485 (2001).

[84] G. Tanner, Spectral statistics for unitary transfer matrices of binary graphs, J. Phys. A **33**, 3567 (2000).

[85] S. Gnutzmann and U. Smilansky, Quantum graphs: Applications to quantum chaos and universal spectral statistics, Adv. Phys. **55**, 527 (2006).

[86] A. W. Marshall, I. Olkin, and B. C. Arnold, *Inequalities: Theory of Majorization and its Applications*, 2nd ed. (Springer, New York, 2011), Vol. 143.

[87] M. A. Nielsen, Conditions for a Class of Entanglement Transformations, Phys. Rev. Lett. **83**, 436 (1999).

[88] M. A. Nielsen, Probability distributions consistent with a mixed state, Phys. Rev. A **62**, 052308 (2000).

[89] K. Hammerer, G. Vidal, and J. I. Cirac, Characterization of nonlocal gates, Phys. Rev. A **66**, 062321 (2002).

[90] K. Zyczkowski, M. Kus, W. Słomczyński, and H.-J. Sommers, Random unistochastic matrices, J. Phys. A **36**, 3425 (2003).

[91] U. Magnea, Random matrices beyond the Cartan classification, J. Phys. A **41**, 045203 (2008).

[92] K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Symmetry and Topology in Non-Hermitian Physics, Phys. Rev. X **9**, 041015 (2019).

[93] D. Bernard and A. LeClair, A classification of non-Hermitian random matrices, in *Statistical Field Theories*, edited by A. Cappelli and G. Mussardo (Springer Netherlands, Dordrecht, 2002), pp. 207–214.

[94] R. Hamazaki, K. Kawabata, N. Kura, and M. Ueda, Universality classes of non-Hermitian random matrices, Phys. Rev. Res. **2**, 023286 (2020).

[95] B. Skinner, J. Ruhman, and A. Nahum, Measurement-Induced Phase Transitions in the Dynamics of Entanglement, Phys. Rev. X **9**, 031009 (2019).

[96] Y. Li, X. Chen, and M. P. A. Fisher, Quantum Zeno effect and the many-body entanglement transition, Phys. Rev. B **98**, 205136 (2018).

[97] Y. Li, X. Chen, and M. P. A. Fisher, Measurement-driven entanglement transition in hybrid quantum circuits, Phys. Rev. B **100**, 134306 (2019).

[98] A. Chan, R. M. Nandkishore, M. Pretko, and G. Smith, Unitary-projective entanglement dynamics, Phys. Rev. B **99**, 224307 (2019).

[99] Y. Bao, S. Choi, and E. Altman, Theory of the phase transition in random unitary circuits with measurements, Phys. Rev. B **101**, 104301 (2020).

[100] C.-M. Jian, Y.-Z. You, R. Vasseur, and A. W. W. Ludwig, Measurement-induced criticality in random quantum circuits, Phys. Rev. B **101**, 104302 (2020).

[101] A. Zabalo, M. J. Gullans, J. H. Wilson, S. Gopalakrishnan, D. A. Huse, and J. H. Pixley, Critical properties of the measurement-induced transition in random quantum circuits, Phys. Rev. B **101**, 060301(R) (2020).