Destabilization of Black Holes and Stars by Generalized Proca Fields

Sebastian Garcia-Saenz,[1,3,*] Aaron Held,[1,†] and Jun Zhang[1,2,‡]

[1]*Theoretical Physics, Blackett Laboratory, Imperial College London, SW7 2AZ London, United Kingdom*

[2]*Illinois Center for Advanced Studies of the Universe and Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

[3]*Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China*

(Received 6 May 2021; accepted 3 September 2021; published 23 September 2021)

We demonstrate that black holes and stars in general relativity can be destabilized by perturbations of nonminimally coupled vector fields. Focusing on static and spherically symmetric backgrounds, our analysis shows that black holes with sufficiently small mass and stars with sufficiently high densities are subject to ghost- or gradient-type instabilities. This holds for a large class of Einstein-Proca theories with nonminimal couplings, including generalized Proca models that have sparked attention for their potential role in cosmology and astrophysics. The stability criteria translate into bounds of relevance for low-scale theories of dark energy and for ultralight dark matter scenarios.

DOI: 10.1103/PhysRevLett.127.131104

Introduction.—Compact astrophysical objects afford a unique opportunity to probe the existence of new light particles whose couplings to ordinary matter are too weak for direct detection. Observational signatures, including potentially dramatic effects [1–8], bear the potential to reveal otherwise hidden gravitationally induced bosonic condensates. Black holes and compact stars, thus, constitute precious targets for testing particle physics and cosmology via strong-field gravity and multimessenger astronomy [9,10].

Theory needs to inform experiment by mapping out the "phase diagram" of black holes and compact stars. In a first step, an ambitious research program drives the development of theoretically consistent bosonic field theories and their interaction with gravity [11–13]. In a second step, astrophysically relevant solutions are analyzed, asking (i) whether they can account for the observed compact objects and (ii) whether potential instabilities can occur.

Here, we advance the second step and focus on vector bosons [14,15]. These are predicted by a number of scenarios beyond the standard model [16,17] and serve as viable candidates for dark matter [18–20] and dark energy [21–26]. If such a light vector particle were to arise from a hidden sector and its dominant interaction with visible matter is mediated by gravity, one is led to consider Einstein-Proca theory of a massive vector field coupled to gravity. While astrophysical solutions in the minimal version of the theory are constrained by the no-hair theorem [27,28], this does not necessarily apply when including nonminimal couplings, i.e., couplings beyond a covariantization of the kinetic term.

The general class of Einstein-Proca theories, thus, accommodates exotic solutions, including hairy black holes [29–35], boson stars [36], and vectorized stars [37–39]. These may be constrained by current and future observations through the beyond-general-relativity effects of Einstein-Proca theories studied, e.g., in Refs. [40–51].

Nevertheless, all Einstein-Proca theories which admit the same solutions as general relativity (GR) (supplemented with a vanishing vector-field background) remain unconstrained. In this situation, the crucial phenomenological question, thus, concerns the stability of GR solutions.

In this Letter, we show that Schwarzschild black holes (and stars) destabilize if their mass (inverse density) drops below a threshold value, related to the nonminimal Einstein-Proca couplings. This allows us to either directly constrain the theory or conclude that GR solutions evolve into non-GR solutions with a nonvanishing vector field.

Although, in the linearized approximation, it is only the vector field and not the metric which suffers from an instability, the backreaction of the vector beyond linear order is expected to render the whole system unstable. Crucially, the uncovered destabilization differs from what is known as "vectorization" [52–55]. In analogy to scalarization [56], vectorization occurs due to tachyonic modes, i.e., wrong-sign mass terms. Here, we find that destabilization is always driven by a ghost or gradient mode, i.e., a wrong-sign kinetic or gradient operator. The latter instability is expected to be far more dramatic than the tachyonic one, with potentially unique astrophysical observables. This calls for numerical-relativity investigations (cf. Refs. [57,58] for linear Proca fields and Refs. [59–67] for related numerical studies in other beyond-GR theories), as one may, in principle, expect a significant signal in gravitational waves sourced by the exponentially growing vector modes beyond linear order.

This novel destabilization channel and the related astrophysical bounds apply to all Einstein-Proca theories with nonminimal couplings that contribute to the linearized dynamics. This includes generalized Proca (GP) theory [68–70], which has received much attention recently in studies of dark matter and dark energy as well as on the potential role of new light particles in astrophysical phenomena. We find that destabilization of stellar-mass Schwarzschild black holes constrains cosmological models in which the associated nonminimal coupling is set by the energy scale $\Lambda \sim (M_{\rm Pl} H_0^2)^{1/3}$, where $M_{\rm Pl}$ is the Planck scale and H_0 is the Hubble constant. Moreover, if stellar mass black holes acquire transient charges [71,72], destabilization could also constrain fuzzy dark-matter models [73].

General quadratic Lagrangian.—We consider a metric tensor $g_{\mu\nu}$ and vector field A_μ with an action

$$S[g,A] = \int d^4x\sqrt{-g}\Bigg[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\mu^2}{2}A^\mu A_\mu + G_{4,X}A^\mu A^\nu G_{\mu\nu} - \frac{G_6}{4}(F^{\mu\nu}F_{\mu\nu}R - 4F^{\mu\rho}F^\nu_{\ \rho}R_{\mu\nu} + F^{\mu\nu}F^{\rho\sigma}R_{\mu\nu\rho\sigma})\Bigg]. \tag{1}$$

Here, $G_{4,X}$ and G_6 are the two constant parameters that define the model, $M_{\rm Pl} = (1/\sqrt{8\pi G})$ is the Planck mass, and μ is the mass of the vector field (we assume $\mu^2 > 0$). For instance, Eq. (1) follows from expanding the complete GP theory to quadratic order in the vector field about $\langle A_\mu \rangle = 0$ on an arbitrary curved background (see Supplemental Material, Sec. A [74]).

GP is the complete generalization of the standard Proca theory; i.e., its interactions preserve the existence of a (local) frame in which the component A_0 is nondynamical. Although sufficient, this is not necessary for consistency with respect to the number of degrees of freedom [75,76]; see [77–79] for alternative extensions. Nevertheless, Eq. (1) is the most general vector-tensor model that is (i) quadratic in the vector field, (ii) a function of the vector field and its first derivative only, and (iii) at most linear in the undifferentiated curvature.

Condition (i) follows because we are investigating the linear stability of GR solutions without a vector condensate. Condition (ii) is a sufficient condition to avoid extra degrees of freedom as in GP theory. Condition (iii) is motivated by our focus on astrophysical GR backgrounds with subleading higher-derivative terms.

Stability and quasinormal modes of a minimally coupled Proca field on black-hole spacetimes are studied in Refs. [80–83]. The coupling proportional to the Einstein tensor was considered previously, e.g., in Refs. [29–31], although restricted to unperturbed backgrounds. The interaction terms involving the field strength, which are reminiscent of the Drummond-Hathrell effective action [84], were studied in Ref. [85]. We confirm their results on the stability of GR black holes as a special case of our more general setup.

Stability conditions.—We focus on static and spherically symmetric backgrounds, for which the metric can be chosen as

$$g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2}$$

Perturbations of the metric about GR backgrounds with a vanishing vector field decouple and can be ignored. The vector field can be decomposed in vector spherical harmonics (see, e.g., [86]):

$$A_\mu = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{I=1}^{4} C^{(I)}_{l,m}(t,r)(Z^{(I)}_{l,m})_\mu(\theta,\phi). \tag{3}$$

Explicit expressions for $Z^{(I)}_{l,m}$ are given in Supplemental Material, Sec. B [74]. The mode functions $C^{(I)}_{l,m}$ with $I = 1$, 2, 3 correspond to perturbations with polar parity, while $C^{(4)}_{l,m}$ corresponds to an axial-parity mode. Parity is a "good quantum number." Hence, polar and axial perturbations decouple at linear order.

The stability of localized perturbations—with physical size much smaller than all the length scales of the background—is dictated by the structure of the causal cones (see [87–90] for related discussions). In other words, to address the question of *local* stability, one may neglect background variations and evaluate all metric functions at fixed radius r_0. The propagator matrix for the mode functions $C^{(I)}_{l,m}$ is defined in Fourier space. Gradient and tachyonic instabilities can be determined by the dispersion relations, defined by the poles of the inverse propagator matrix. The presence of ghosts follows from the matrix of residues. See Supplemental Material, Sec. C [74], for details. Henceforth, we drop the subscript on the fixed radius r_0.

The axial sector has a single degree of freedom. Its dispersion relation follows from the decomposed action

$$\frac{\mathcal{H}_1}{f}\omega^2 - g\mathcal{H}_2 k^2 - \left(\mathcal{N}_m + \frac{l(l+1)}{r^2}\mathcal{N}_j\right) = 0, \tag{4}$$

where ω and k are the comoving (as opposed to proper) frequency and radial wave number, respectively, and

$$\mathcal{H}_1 = 1 - G_6\frac{g'}{r}, \qquad \mathcal{H}_2 = 1 - G_6\frac{f'g}{fr},$$
$$\mathcal{N}_m = \mu^2 + G_{4,X}(R - 2r^2R^{\theta\theta}),$$
$$\mathcal{N}_j = 1 + G_6\left(R - 4r^2R^{\theta\theta} + \frac{2(1-g)}{r^2}\right). \tag{5}$$

Here, a prime denotes differentiation with respect to r. The curvature terms R and $R^{\theta\theta}$ are known in terms of f and g.

For $l \geq 1$, only two combinations of the three polar mode functions $C^{(1,2,3)}_{l,m}$ are dynamical. Integrating out the nondynamical mode (see Supplemental Material, Sec. B [74]), we can infer the two-by-two (inverse) propagator matrix $\mathcal{P}$. Its components read

$$\mathcal{P}_{11} = \frac{a_0^2}{g(\mathcal{M}_2 + \mathcal{H}_2 \frac{l(l+1)}{r^2})}\omega^2 - \frac{f a_0^2}{(\mathcal{M}_1 + \mathcal{H}_1 \frac{l(l+1)}{r^2})}k^2 - \sigma_0,$$
$$\mathcal{P}_{22} = \frac{\mathcal{M}_1\mathcal{H}_1}{f r^2(\mathcal{M}_1 + \mathcal{H}_1 \frac{l(l+1)}{r^2})}\omega^2 - \frac{g\mathcal{M}_2\mathcal{H}_2}{r^2(\mathcal{M}_2 + \mathcal{H}_2 \frac{l(l+1)}{r^2})}k^2 - \frac{\mathcal{N}_m}{r^2},$$
$$\mathcal{P}_{12} = \frac{\sigma_0 a_0 \sqrt{l(l+1)}(\mathcal{M}_1\mathcal{H}_2 - \mathcal{M}_2\mathcal{H}_1)}{r^2(\mathcal{M}_1 + \mathcal{H}_1 \frac{l(l+1)}{r^2})(\mathcal{M}_2 + \mathcal{H}_2 \frac{l(l+1)}{r^2})}\omega k. \tag{6}$$

Here, $a_0 = \sqrt{(g|\mathcal{G}_1|/f)}$, $\sigma_0 = \mathrm{sgn}(\mathcal{G}_1)$, and

$$\mathcal{G}_1 = 1 + 2G_6 \frac{1-g}{r^2},$$
$$\mathcal{M}_1 = \mu^2 - 2G_{4,X}\left(\frac{g'}{r} - \frac{1-g}{r^2}\right),$$
$$\mathcal{M}_2 = \mu^2 - 2G_{4,X}\left(\frac{f'g}{fr} - \frac{1-g}{r^2}\right). \tag{7}$$

The dispersion relations are defined by the roots $\omega_\pm^2$ of the equation $\det \mathcal{P} = 0$.

Monopole perturbations with $l = 0$ are special in that only $C_{0,0}^{(1)}$ is present in the polar sector. Its dispersion relation reads

$$\frac{|\mathcal{G}_1|}{f\mathcal{M}_2}\omega^2 - \frac{g|\mathcal{G}_1|}{\mathcal{M}_1}k^2 - \sigma_0 = 0. \tag{8}$$

Notice that the dispersion relations of the polar and axial sector involve the same functions $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{N}_m$. *A priori*, these functions need not be in any way related—in fact, they are not for the monopole modes. This coincidence has important consequences for the stability conditions.

The stability of axial perturbations under ghosts and radial gradients dictates that $\mathcal{H}_1 > 0$ and $\mathcal{H}_2 > 0$ for all physical radii. Stability of these modes under angular gradients, which means that ω^2 must be positive in the limit $l \to \infty$, requires $\mathcal{N}_j > 0$. Similarly, stability of the polar monopole mode implies that $\mathcal{M}_1 > 0$ and $\mathcal{M}_2 > 0$. Given these conditions, it then follows that all polar modes with $l \geq 1$ are stable under ghosts and radial gradients; see Supplemental Material, Sec. C [74]. Furthermore, the stability of these modes under angular gradients gives independent constraints, namely, $\mathcal{N}_m > 0$ and $\mathcal{G}_1 > 0$. In turn, these last two conditions imply the absence of tachyonic instabilities for all the perturbations.

An important outcome is that tachyonlike instabilities are absent—for, if such modes are excited, they are necessarily accompanied by ghosts and/or gradient-unstable modes with a much faster growth rate. Hence, vector condensates cannot form as a result of a standard vectorization mechanism—which by definition follows from a tachyon- or Jeans-type destabilization—starting from *any* static spherically symmetric GR state and for *any* Einstein-Proca theory that reduces to Eq. (1) at linear order.

Black holes.—For the Schwarzschild black hole (BH) of mass M, i.e., for $f = g = 1 - (r_s/r)$ with $r_s = 2GM$, the stability conditions simplify. Whenever $g = f$ holds, $\mathcal{H}_1 = \mathcal{H}_2$, $\mathcal{M}_1 = \mathcal{M}_2$, and the propagator matrix in Eq. (6) is diagonal. Moreover, for vacuum GR solutions, the dependence on $G_{4,X}$ drops out; cf. Eq. (1). For the Schwarzschild spacetime, one finds that $\mathcal{N}_m = \mathcal{M}_1 = \mathcal{M}_2 = \mu^2$ are automatically positive, while $\mathcal{H}_1 = \mathcal{H}_2 = 1 - (G_6 r_s/r^3)$ and $\mathcal{N}_j = \mathcal{G}_1 = 1 + (2G_6 r_s/r^3)$. Positivity of these functions for all $r \geq r_s$ requires

$$-\frac{1}{2} < \frac{G_6}{r_s^2} < 1, \tag{9}$$

in order for Schwarzschild BHs to be stable. This bound is in agreement with the results of Ref. [85]. It implies that small enough BHs are always unstable whenever G_6 is nonzero.

Order-of-magnitude estimates (assuming validity of the vector theory on all involved scales) reveal that the stability bound in Eq. (9) could be of relevance both in late-time cosmology as well as for primordial BHs. In the cosmological setting, nonlinear operators are typically controlled by an energy scale $\Lambda \sim (M_{\mathrm{Pl}} H_0^2)^{1/3}$, where H_0 is the Hubble constant [91,92]. (This estimate is based on scalar-tensor theories and implicitly assumes the existence of a decoupling limit in which the vector-tensor models we consider can be approximated by scalar-tensor interactions.) Taking, for example, $G_6 \sim \Lambda^{-2}$, this yields $G_6 \sim (10^3\ \mathrm{km})^2$, implying the destabilization of stellar-mass BHs while supermassive BHs remain stable. For smaller values of G_6, stellar-mass BHs remain stable while primordial BHs in the experimentally preferred range $r_s \sim 10^{-10}$ m [93] would still be subject to the instability.

Constraining the parameter $G_{4,X}$ requires to look at non-Ricci-flat GR solutions. We consider the Reissner-Nordström (RN) metric as a first example. Although astrophysical BHs are unlikely to exhibit significant electric (or magnetic) charge, small and transient charges remain viable. For instance, stellar-mass BHs could accrete charges up to the order of 10^{-7}, in units of the BH mass [72], through the Wald mechanism [71] in a merger with a strongly magnetized neutron star. (In Refs. [71,72], the charging effect requires a spinning BH; however, a more recent study [94] has shown that rotation is not needed and that the relative motion between the coalescing BH and neutron star can generate charges of comparable magnitude.)

The RN metric is defined by $f = g = 1 - (r_s/r) + (r_Q^2/4r^2)$. Here, $r_Q = 2\sqrt{G}Q$ in terms of the hole's electric charge, and we recall the extremality bound $r_Q \leq r_s$. The stability conditions now depend on the scale r_Q; cf. Fig. 1 and Supplemental Material, Sec. D [74], for the analytic expressions. For G_6, we observe a nontrivial dependence of

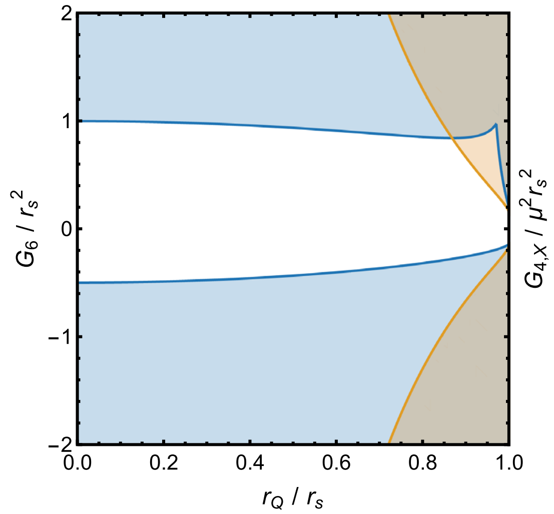

FIG. 1. Region plot of the GP parameters for which a destabilization of the RN BH occurs, blue for G_6 and orange for $G_{4,X}/\mu^2$ (normalized by the Schwarzschild radius).

the stability bounds on the charge (cf. Ref. [85]); in particular, they are most restrictive for an extremal BH, for which $|G_6|/r_s^2 < 1/8$.

More interestingly, we find a novel bound on $G_{4,X}$:

$$\frac{|G_{4,X}|}{\mu^2 r_s^2} < \frac{\left[1+\sqrt{1-(r_Q/r_s)^2}\right]^4}{8(r_Q/r_s)^2}. \tag{10}$$

Remarkably, for any fixed $G_{4,X}$, r_s, and r_Q, this bound implies a lower limit on the vector-boson mass μ. As a concrete example, for $G_{4,X} = \mathcal{O}(1)$, as typically considered in the literature, stability of a stellar-mass BH with $r_s \sim 10$ km that acquires the aforementioned estimate $r_Q \sim 10^{-7} r_s$ implies $\mu \sim 10^{-17}$ eV as the critical vector-boson mass. Comparison with the typical mass range $10^{-22}-10^{-20}$ eV for fuzzy dark matter [73] exemplifies the significance of Eq. (10) for the study of ultralight particles. We note that $G_{4,X}$ may not be independent of μ: If the operators that break gauge invariance were to arise from a Higgs-type mechanism, we would expect $G_{4,X} \propto \mu^2$ [95] and our stability criteria would not directly constrain the mass μ but rather the scale of symmetry breaking.

Stars.—We have analyzed the stability conditions for static perfect fluid stars governed by the Tolman-Oppenheimer-Volkoff equations. Although for generic equations of state (EOS)—relating the pressure p to the density ρ—the metric cannot be determined in analytic form, critical values for the parameters G_6 and $G_{4,X}$ can still be obtained if one assumes that the functions that determine the stability are minimized at the center of the star. We have checked analytically that this assumption is correct for a uniform-density star and also numerically for a polytropic star with EOS $p = K\rho^{5/3}$; see Supplemental Material, Sec. D [74]. It is plausible that the assumption is true for all realistic EOS, including ones for imperfect fluids, and we plan to come back to this question in a dedicated work.

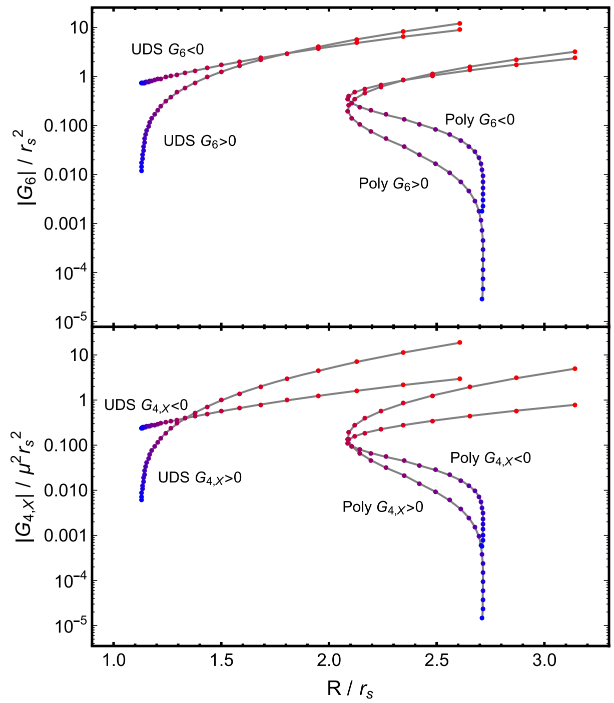

FIG. 2. Critical values of the GP parameters for which an instability is triggered in stars modeled by uniform density ("UDS") and $\gamma = 5/3$ polytropic index ("Poly") as inferred from Eq. (11). Colored points label different values of the central pressure, ranging from 10^{-2} (red; upper-right end) to 10^4 (blue; lower end) in arbitrary units such that $K = 1$ (the constant appearing in the polytropic EOS). Despite this arbitrariness, the comparison between different pressures and between the two stellar models is meaningful.

Under this premise, we can infer the following bounds on the GP coupling constants:

$$\begin{aligned} -\frac{3}{2\rho_c} < \frac{G_6}{M_{\rm Pl}^2} < \frac{3}{\rho_c + 3p_c}, \\ -\frac{1}{2\rho_c} < \frac{G_{4,X}}{\mu^2 M_{\rm Pl}^2} < \frac{1}{2p_c}, \end{aligned} \tag{11}$$

where p_c and ρ_c are the pressure and density at the center, respectively. Figure 2 shows the critical values of G_6 and $G_{4,X}/\mu^2$ for stellar models with uniform density and $\gamma = 5/3$ polytropic EOS, plotted as functions of the normalized star's radius. We observe an interesting dependence on the EOS, with the bounds for a polytrope being up to 3 orders of magnitude stronger than for a uniform-density star with the same central pressure and density.

The stability window for both coupling parameters shrinks to zero as the star's central pressure and density increase. For a neutron star with $\rho_c \sim 10^{18}\,\mathrm{kg\,m^{-3}} \sim 10^{-76} M_{\rm Pl}^4$, one has $\Lambda/M_{\rm Pl} \gtrsim 10^{-38}$ if we take $|G_6| \sim |G_{4,X}|/\mu^2 \sim \Lambda^{-2}$. This bound on Λ may seem mild but again could be violated in very low-scale models like the ones envisioned in cosmology and in the context of ultralight dark matter.

Discussion.—We identify a novel destabilization channel for static and spherically symmetric GR backgrounds triggered by nonminimally coupled vector perturbations. Any nonvanishing nonminimal coupling destabilizes small enough BHs and dense enough stars. The implied astrophysical constraints ultimately depend on the scales at hand. We find relevant constraints for theories of dark energy and ultralight dark matter. The nonminimal couplings that source destabilization [cf. Eq. (1)] naturally appear in all of these scenarios unless the model is fine-tuned to avoid them.

More specifically, avoiding instabilities of stellar-mass BHs and/or neutron stars constrains the respective nonminimal coupling at cosmologically relevant scales set by $\Lambda \sim (M_{\rm Pl} H_0^2)^{1/3}$, with $M_{\rm Pl}$ the Planck scale and H_0 the Hubble constant. In turn, transient charges, potentially induced by nearby strongly magnetized neutron stars, imply further destabilization constraints involving the Proca mass and are of relevance for ultralight vector dark-matter models.

Notably, destabilization differs from vectorization. The latter describes a transition between GR and non-GR solutions via a tachyonic growth mode. Here, we find that a potential tachyonic instability is always accompanied by a dominant ghost or gradient instability. Hence, destabilization is controlled by the highest growth rates in the problem. The timescale and fate of the instability, thus, remain uncertain.

We emphasize that the destabilization channel concerns only GR solutions and does not constrain solutions with nontrivial vector hair. Formally, we have shown only a linear instability in the vector field and not in the metric. However, as one may expect for ghost or gradient modes, interactions beyond linear order will generically destabilize the full system. Future nonlinear studies are necessary to strictly discard the possibility that the vector field may settle into a condensed state with the GR metric being kept intact, as occurs with so-called "stealth" solutions [96–99].

From the perspective of radiative corrections, the inclusion of higher-derivative operators may quench the instability, similarly to the phenomenon of ghost condensation [100]. This possibility calls for a detailed study to determine the role of higher-order operators. Assuming the transition can be made sense of in a controlled theoretical framework, our analysis makes a strong case for simulating Einstein-Proca theories in numerical relativity.

There are also several avenues for future work within the present setup of linear perturbations about GR backgrounds. This includes (i) studying a broader set of stellar models in order to verify the robustness of our bounds in Eq. (11); (ii) effects of a cosmological constant, potentially related to extended vector fields in holographic models (see, e.g., [101–103]); and, of course, (iii) an extension to nonstatic systems, in particular, rotating BHs and stars.

We thank Claudia de Rham, Lavinia Heisenberg, Shinji Mukohyama, and an anonymous referee for useful comments. S. G.-S. and J. Z. are supported by the European Union's Horizon 2020 Research Council Grant No. 724659 MassiveCosmo ERC-2016-COG. A. H. is supported by a Royal Society Newton International fellowship [NIF\R1 \191008].

[*]s.garcia-saenz@imperial.ac.uk
[†]a.held@imperial.ac.uk
[‡]jun.zhang@imperial.ac.uk

[1] W. H. Press, Astrophys. J. Lett. **170**, L105 (1971).
[2] Y. B. Zel'Dovich, JETP Lett. **14**, 180 (1971).
[3] W. H. Press and S. A. Teukolsky, Nature (London) **238**, 211 (1972).
[4] T. Damour, N. Deruelle, and R. Ruffini, Lett. Nuovo Cimento **15**, 257 (1976).
[5] T. J. M. Zouros and D. M. Eardley, Ann. Phys. (N.Y.) **118**, 139 (1979).
[6] S. L. Detweiler, Phys. Rev. D **22**, 2323 (1980).
[7] H. Yoshino and H. Kodama, Prog. Theor. Phys. **128**, 153 (2012).
[8] R. Brito, V. Cardoso, and P. Pani, Lect. Notes Phys. **906**, 1 (2015).
[9] E. Berti *et al.*, Classical Quantum Gravity **32**, 243001 (2015).
[10] V. Cardoso and P. Pani, Living Rev. Relativity **22**, 4 (2019).
[11] C. Deffayet and D. A. Steer, Classical Quantum Gravity **30**, 214006 (2013).
[12] D. Langlois, Int. J. Mod. Phys. D **28**, 1942006 (2019).
[13] T. Kobayashi, Rep. Prog. Phys. **82**, 086901 (2019).
[14] P. Galison and A. Manohar, Phys. Lett. **136B**, 279 (1984).
[15] B. Holdom, Phys. Lett. **166B**, 196 (1986).
[16] S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze, and A. Ringwald, J. High Energy Phys. 07 (2008) 124.
[17] M. Goodsell, J. Jaeckel, J. Redondo, and A. Ringwald, J. High Energy Phys. 11 (2009) 027.
[18] N. Arkani-Hamed and N. Weiner, J. High Energy Phys. 12 (2008) 104.
[19] A. E. Nelson and J. Scholtz, Phys. Rev. D **84**, 103501 (2011).
[20] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo, and A. Ringwald, J. Cosmol. Astropart. Phys. 06 (2012) 013.
[21] A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, S. Tsujikawa, and Y.-l. Zhang, J. Cosmol. Astropart. Phys. 06 (2016) 048.
[22] A. De Felice, L. Heisenberg, R. Kase, S. Mukohyama, S. Tsujikawa, and Y.-l. Zhang, Phys. Rev. D **94**, 044024 (2016).
[23] J. B. Jimenez, L. Heisenberg, R. Kase, R. Namba, and S. Tsujikawa, Phys. Rev. D **95**, 063533 (2017).
[24] A. de Felice, L. Heisenberg, and S. Tsujikawa, Phys. Rev. D **95**, 123540 (2017).

[25] L. Heisenberg and H. Villarrubia-Rojo, J. Cosmol. Astropart. Phys. 03 (2021) 032.
[26] G.-P. Nicosia, J. Levi Said, and V. Gakis, Eur. Phys. J. Plus **136**, 191 (2021).
[27] J. D. Bekenstein, Phys. Rev. D **5**, 1239 (1972).
[28] J. D. Bekenstein, Phys. Rev. Lett. **28**, 452 (1972).
[29] J. Chagoya, G. Niz, and G. Tasinato, Classical Quantum Gravity **33**, 175007 (2016).
[30] M. Minamitsuji, Phys. Rev. D **94**, 084039 (2016).
[31] E. Babichev, C. Charmousis, and M. Hassaine, J. High Energy Phys. 05 (2017) 114.
[32] L. Heisenberg, R. Kase, M. Minamitsuji, and S. Tsujikawa, Phys. Rev. D **96**, 084049 (2017).
[33] L. Heisenberg, R. Kase, M. Minamitsuji, and S. Tsujikawa, J. Cosmol. Astropart. Phys. 08 (2017) 024.
[34] R. Kase, M. Minamitsuji, S. Tsujikawa, and Y.-L. Zhang, J. Cosmol. Astropart. Phys. 02 (2018) 048.
[35] R. Kase, M. Minamitsuji, and S. Tsujikawa, Phys. Lett. B **782**, 541 (2018).
[36] M. Minamitsuji, Phys. Rev. D **96**, 044017 (2017).
[37] S. Nakamura, R. Kase, and S. Tsujikawa, Phys. Rev. D **96**, 084005 (2017).
[38] R. Kase, M. Minamitsuji, and S. Tsujikawa, Phys. Rev. D **97**, 084009 (2018).
[39] R. Kase, M. Minamitsuji, and S. Tsujikawa, Phys. Rev. D **102**, 024067 (2020).
[40] P. Pani, V. Cardoso, L. Gualtieri, E. Berti, and A. Ishibashi, Phys. Rev. Lett. **109**, 131102 (2012).
[41] H. Witek, V. Cardoso, A. Ishibashi, and U. Sperhake, Phys. Rev. D **87**, 043513 (2013).
[42] R. Brito, V. Cardoso, C. A. R. Herdeiro, and E. Radu, Phys. Lett. B **752**, 291 (2016).
[43] C. Herdeiro, E. Radu, and H. Rúnarsson, Classical Quantum Gravity **33**, 154001 (2016).
[44] N. Sanchis-Gual, C. Herdeiro, E. Radu, J. C. Degollado, and J. A. Font, Phys. Rev. D **95**, 104028 (2017).
[45] V. Cardoso, O. J. C. Dias, G. S. Hartnett, M. Middleton, P. Pani, and J. E. Santos, J. Cosmol. Astropart. Phys. 03 (2018) 043.
[46] M. Minamitsuji, Phys. Rev. D **97**, 104023 (2018).
[47] N. Sanchis-Gual, C. Herdeiro, J. A. Font, E. Radu, and F. Di Giovanni, Phys. Rev. D **99**, 024017 (2019).
[48] N. M. Santos, C. L. Benone, L. C. B. Crispino, C. A. R. Herdeiro, and E. Radu, J. High Energy Phys. 07 (2020) 010.
[49] J. C. Bustillo, N. Sanchis-Gual, A. Torres-Forné, J. A. Font, A. Vajpeyi, R. Smith, C. Herdeiro, E. Radu, and S. H. W. Leong, Phys. Rev. Lett. **126**, 081101 (2021).
[50] A. Caputo, S. J. Witte, D. Blas, and P. Pani, Phys. Rev. D **104**, 043006 (2021).
[51] S. Barton, B. Hartmann, B. Kleinhaus, and J. Kunz, Phys. Lett. B **817**, 136336 (2021).
[52] F. M. Ramazanoğlu, Phys. Rev. D **96**, 064009 (2017).
[53] F. M. Ramazanoğlu, Phys. Rev. D **98**, 044013 (2018).
[54] L. Annulli, V. Cardoso, and L. Gualtieri, Phys. Rev. D **99**, 044038 (2019).
[55] M. Minamitsuji, Phys. Rev. D **101**, 104044 (2020).
[56] T. Damour and G. Esposito-Farese, Phys. Rev. Lett. **70**, 2220 (1993).
[57] W. E. East and F. Pretorius, Phys. Rev. Lett. **119**, 041101 (2017).
[58] W. E. East, Phys. Rev. Lett. **121**, 131104 (2018).
[59] N. Sanchis-Gual, J. C. Degollado, P. J. Montero, J. A. Font, and C. Herdeiro, Phys. Rev. Lett. **116**, 141101 (2016).
[60] P. Bosch, S. R. Green, and L. Lehner, Phys. Rev. Lett. **116**, 141102 (2016).
[61] C. A. R. Herdeiro, E. Radu, N. Sanchis-Gual, and J. A. Font, Phys. Rev. Lett. **121**, 101102 (2018).
[62] J. L. Ripley and F. Pretorius, Phys. Rev. D **101**, 044015 (2020).
[63] J. L. Ripley and F. Pretorius, Classical Quantum Gravity **37**, 155003 (2020).
[64] M. Okounkova, Phys. Rev. D **102**, 084046 (2020).
[65] W. E. East and J. L. Ripley, Phys. Rev. D **103**, 044040 (2021).
[66] H. O. Silva, H. Witek, M. Elley, and N. Yunes, Phys. Rev. Lett. **127**, 031101 (2021).
[67] A. Held and H. Lim, arXiv:2104.04010.
[68] G. Tasinato, J. High Energy Phys. 04 (2014) 067.
[69] L. Heisenberg, J. Cosmol. Astropart. Phys. 05 (2014) 015.
[70] M. Hull, K. Koyama, and G. Tasinato, Phys. Rev. D **93**, 064012 (2016).
[71] R. M. Wald, Phys. Rev. D **10**, 1680 (1974).
[72] J. Levin, D. J. D'Orazio, and S. Garcia-Saenz, Phys. Rev. D **98**, 123002 (2018).
[73] W. Hu, R. Barkana, and A. Gruzinov, Phys. Rev. Lett. **85**, 1158 (2000).
[74] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.131104 for technical definitions and derivations used in the Letter.
[75] V. Errasti Díez, B. Gording, J. A. Méndez-Zavaleta, and A. Schmidt-May, Phys. Rev. D **101**, 045009 (2020).
[76] V. Errasti Díez, B. Gording, J. A. Méndez-Zavaleta, and A. Schmidt-May, Phys. Rev. D **101**, 045008 (2020).
[77] L. Heisenberg, R. Kase, and S. Tsujikawa, Phys. Lett. B **760**, 617 (2016).
[78] R. Kimura, A. Naruko, and D. Yoshida, J. Cosmol. Astropart. Phys. 01 (2017) 002.
[79] C. de Rham and V. Pozsgay, Phys. Rev. D **102**, 083508 (2020).
[80] R. A. Konoplya, Phys. Rev. D **73**, 024009 (2006).
[81] J. G. Rosa and S. R. Dolan, Phys. Rev. D **85**, 044043 (2012).
[82] D. Baumann, H. S. Chia, J. Stout, and L. ter Haar, J. Cosmol. Astropart. Phys. 12 (2019) 006.
[83] J. Percival and S. R. Dolan, Phys. Rev. D **102**, 104055 (2020).
[84] I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22**, 343 (1980).
[85] J. Beltran Jimenez, R. Durrer, L. Heisenberg, and M. Thorsrud, J. Cosmol. Astropart. Phys. 10 (2013) 064.
[86] H.-P. Nollert, Classical Quantum Gravity **16**, R159 (1999).
[87] C. de Rham and S. Melville, Phys. Rev. D **95**, 123523 (2017).
[88] E. Babichev, C. Charmousis, G. Esposito-Farèse, and A. Lehébel, Phys. Rev. D **98**, 104050 (2018).
[89] C. de Rham and J. Zhang, Phys. Rev. D **100**, 124023 (2019).

[90] C. de Rham, J. Francfort, and J. Zhang, Phys. Rev. D **102**, 024079 (2020).
[91] A. Nicolis and R. Rattazzi, J. High Energy Phys. 06 (2004) 059.
[92] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).
[93] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, arXiv: 2002.12778.
[94] K. Chen and Z. G. Dai, Astrophys. J. **909**, 4 (2021).
[95] C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, J. High Energy Phys. 03 (2019) 182.
[96] E. Babichev and C. Charmousis, J. High Energy Phys. 08 (2014) 106.
[97] J. Chagoya and G. Tasinato, J. Cosmol. Astropart. Phys. 01 (2018) 046.
[98] M. Minamitsuji and H. Motohashi, Phys. Rev. D **98**, 084027 (2018).
[99] M. Minamitsuji, Phys. Rev. D **102**, 044048 (2020).
[100] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, and S. Mukohyama, J. High Energy Phys. 05 (2004) 074.
[101] J. Jing and S. Chen, Phys. Lett. B **686**, 68 (2010).
[102] J. Jing, L. Wang, Q. Pan, and S. Chen, Phys. Rev. D **83**, 066010 (2011).
[103] Z. Zhao, Q. Pan, S. Chen, and J. Jing, Nucl. Phys. **B871**, 98 (2013).