

Breaking of Gauge Symmetry in Lattice Gauge Theories

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We study perturbations that break gauge symmetries in lattice gauge theories. As a paradigmatic model, we consider the three-dimensional Abelian-Higgs (AH) model with an N -component scalar field and a noncompact gauge field, which is invariant under $U(1)$ gauge and $SU(N)$ transformations. We consider gauge-symmetry breaking perturbations that are quadratic in the gauge field, such as a photon mass term and determine their effect on the critical behavior of the gauge-invariant model, focusing mainly on the continuous transitions associated with the charged fixed point of the AH field theory. We discuss their relevance and compute the (gauge-dependent) exponents that parametrize the departure from the critical behavior (continuum limit) of the gauge-invariant model. We also address the critical behavior of lattice AH models with broken gauge symmetry, showing an effective enlargement of the global symmetry, from $U(N)$ to $O(2N)$, which reflects a peculiar cyclic renormalization-group flow in the space of the lattice AH parameters and of the photon mass.

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Introduction.—Gauge symmetries play a key role in the construction of the theoretical models of fundamental interactions [1,2] and in the description of emergent phenomena in condensed-matter and statistical physics [3–8]. They may be exact, as in the standard model of fundamental interactions, or effectively emerge at low energies, as in some many-body systems. Effectively emergent gauge symmetries have also been discussed in the context of fundamental interactions; see, e.g., Refs. [3,9–11]. In this case, they may arise from microscopic interactions of a different nature, such as string models [12].

To correctly interpret experimental results in terms of models with an emergent gauge symmetry, a solid understanding of the effects of gauge-symmetry violations is essential. This issue is crucial in the context of analog quantum simulations, for example, when controllable atomic systems are engineered to effectively reproduce the dynamics of gauge-symmetric theoretical models, with the purpose of obtaining physical information from the experimental study of their quantum dynamics in laboratory. Several proposals of artificial gauge-symmetry realizations have been reported; see, e.g., Refs. [13,14] and references therein (see also Refs. [15–20] for some experimental realizations), in which the gauge symmetry is expected to effectively emerge in the low-energy dynamics.

A possible strategy is that of adding a *penalty* term to the Hamiltonian, which suppresses the interactions violating the gauge symmetry. This strategy assumes that gauge-symmetry breaking (GSB) terms become negligible at low energies, thereby making the dynamics effectively gauge invariant in this limit [13,21,22]. In spite of the relevance of these issues, there is at present little understanding of the effects of GSB perturbations on the continuum limit of quantum or statistical systems with gauge symmetries or, equivalently, on the critical behavior close to continuous transitions, where long-range correlations develop, realizing the corresponding quantum field theory.

In this Letter, we address this problem by considering three-dimensional (3D) lattice gauge theories, obtained by discretizing the action of corresponding quantum field theories. We study the role of GSB perturbations at the critical transitions of gauge-invariant models, to understand whether and when they are relevant, i.e., they break gauge invariance in the low-energy or large-distance behavior (continuum limit). If this is the case, GSB terms may lead to different continuum limits, as we shall see.

The model.—As a paradigmatic model, we consider the 3D scalar electrodynamics or Abelian-Higgs (AH) field theory, with an N -component complex scalar field $\Phi(\mathbf{x})$ coupled to the electromagnetic field $A_\mu(\mathbf{x})$. Its Lagrangian density reads [2]

$$\mathcal{L} = |D_\mu \Phi|^2 + w \Phi^* \Phi + \frac{u}{4} (\Phi^* \Phi)^2 + \frac{1}{4g^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2, \quad (1)$$

where $D_\mu \equiv \partial_\mu + iA_\mu$. The AH theory is invariant under $U(1)$ gauge and $SU(N)$ global transformations. Its 3D

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renormalization-group (RG) flow has a stable charged (with nonzero gauge coupling) fixed point (FP) for $N \geq N_c$ [23,24], with $N_c = 7(2)$ [25,26]. According to the RG theory [27–30], the charged FP is expected to describe the critical behavior, and therefore the continuum limit, of U(1) gauge models with SU(N) global symmetry.

Lattice representations of the continuum theory (1) differ for the topological nature of the lattice gauge field. One can either use the real field $A_{x,\mu}$ as in the continuum theory (noncompact model) or the link variables $\lambda_{x,\mu} \in \text{U}(1)$ (compact model, corresponding to $e^{iA_{x,\mu}}$). In this Letter, we mostly consider the 3D noncompact AH (NCAH) model defined on cubic lattices of size L^3 , which has a continuous transition line for $N > N_c$, along which the continuum limit is described by the 3D AH field theory (1) [25,31,32]. The fundamental fields are unit-length N -component complex vectors \mathbf{z}_x ($\bar{\mathbf{z}}_x \cdot \mathbf{z}_x = 1$) defined on the lattice sites \mathbf{x} and real fields $A_{x,\mu}$ defined on the lattice links. The lattice action is

$$S_{\text{AH}}(\mathbf{z}, \mathbf{A}) = -JN \sum_{\mathbf{x}, \mu} 2\text{Re}(\bar{\mathbf{z}}_x \cdot \lambda_{x,\mu} \mathbf{z}_{x+\hat{\mu}}) + \frac{1}{4g_0^2} \sum_{\mathbf{x}, \mu\nu} (\Delta_\mu A_{x,\nu} - \Delta_\nu A_{x,\mu})^2, \quad (2)$$

where $\lambda_{x,\mu} \equiv e^{iA_{x,\mu}}$, g_0 is the lattice gauge coupling, $\hat{\mu}$ are unit vectors along the lattice directions, and $\Delta_\mu A_{x,\nu} = A_{x+\hat{\mu},\nu} - A_{x,\nu}$. The action S_{AH} has a global SU(N) symmetry, $\mathbf{z}_x \rightarrow V\mathbf{z}_x$ with $V \in \text{SU}(N)$, and a local U(1) gauge symmetry, $\mathbf{z}_x \rightarrow e^{i\theta_x} \mathbf{z}_x$ and $A_{x,\mu} \rightarrow A_{x,\mu} + \theta_x - \theta_{x+\hat{\mu}}$. We consider C^* boundary conditions [25,34,35] (see also the Supplemental Material [36]) to remove the degeneracy under $A_{x,\mu} \rightarrow A_{x,\mu} + 2\pi n_\mu$ with $n_\mu \in \mathbb{Z}$, obtaining well-defined expectation values for gauge-invariant operators $O(\mathbf{z}, \mathbf{A})$,

$$\langle O(\mathbf{z}, \mathbf{A}) \rangle = \frac{\sum_{\{\mathbf{z}, \mathbf{A}\}} O(\mathbf{z}, \mathbf{A}) e^{-S_{\text{AH}}(\mathbf{z}, \mathbf{A})}}{\sum_{\{\mathbf{z}, \mathbf{A}\}} e^{-S_{\text{AH}}(\mathbf{z}, \mathbf{A})}}. \quad (3)$$

The phase diagram of the NCAH model (2) with $N \geq 2$ is characterized by a Coulomb phase for small J (short-ranged scalar and long-ranged gauge correlations), a Higgs phase for large J and small g_0 (condensed scalar-field and gapped gauge correlations), and a molecular phase for large J and g_0 (condensed scalar-field and long-ranged gauge correlations) [25]. They are separated by three transition lines, which are continuous or of first order depending on N . In particular, for $N > N_c = 7(2)$, the NCAH model undergoes continuous transitions between the Coulomb and Higgs (CH) phases, for $0 < g_0^2 \lesssim 4$. The corresponding critical behavior is described by the charged FP of the 3D AH field theory [25]. For $g_0 \rightarrow 0$, one has $A_{x,\mu} \rightarrow 1$ modulo gauge transformations, so that one recovers the O($2N$)

vector model. We consider the gauge-invariant bilinear operator

$$Q_x^{ab} = \frac{\bar{z}_x^a z_x^b}{N} - \frac{1}{N} \delta^{ab}, \quad (4)$$

which transforms as $Q_x \rightarrow V^\dagger Q_x V$ under global SU(N) transformations. It provides an effective order parameter for the spontaneous breaking of the global SU(N) symmetry.

GSB perturbations.—We study how perturbations breaking the U(1) gauge symmetry affect the CH critical behavior. In this exploratory study we consider the quadratic perturbation

$$P_M = \frac{r}{2} \sum_{\mathbf{x}, \mu} A_{x,\mu}^2, \quad (5)$$

which can be interpreted as a photon mass term. Such a mass term is generally introduced as an infrared regulator in perturbative computations in quantum electrodynamics [2]. We also consider the local quadratic operators

$$P_L = \frac{a}{2} \sum_{\mathbf{x}} \left(\sum_{\mu} \Delta_\mu A_{x,\mu} \right)^2, \\ P_A = \frac{b}{2} \sum_{\mathbf{x}} \left(\sum_{\mu} n_\mu A_{x,\mu} \right)^2, \quad (6)$$

where n_μ is an arbitrary unit vector. When added to the NCAH action, all quadratic terms defined in Eqs. (5) and (6) break gauge invariance, leaving a global U(N) symmetry $\mathbf{z}_x \rightarrow U\mathbf{z}_x$, $U \in \text{U}(N)$. However, they affect the critical behavior quite differently. The mass term (5) is expected to be relevant at the CH transitions, since it drastically changes the long-distance properties of the gauge-field correlations. In particular, the Coulomb phase disappears in the presence of a photon mass. Therefore, as soon as the perturbation is turned on ($r > 0$), the system is expected to flow out of the charged AH FP. On the other hand, the quadratic terms P_L and P_A , cf. Eq. (6), may be interpreted as the result of the Fadeev-Popov procedure for a gauge fixing [2], being related to the Lorentz ($\partial_\mu A_\mu = 0$) and axial ($\mathbf{n} \cdot \mathbf{A} = 0$) gauge fixing [37], respectively. If they are the only GSB perturbations present in the model, they are expected to be irrelevant for gauge-invariant correlations (more precisely, their presence does not change gauge-invariant expectation values). However, as we shall see below, they play a role, when they are added to the action together with the mass term (5), as they make the limit $r \rightarrow 0$ well defined.

Relevance of the GSB perturbations.—To characterize the strength of the perturbation P_M , we compute the corresponding RG dimension $y_r > 0$. This exponent provides information on how to scale r to keep GSB effects small. Indeed, when the correlation length ξ increases,

approaching the continuum limit, one should decrease r faster than ξ^{-y_r} to ensure that GSB effects are negligible. We estimate y_r by finite-size scaling (FSS) analyses of Monte Carlo (MC) data. We consider the correlation function $\langle \text{Tr} Q_x Q_y \rangle$ of the operator Q_x defined in Eq. (4) and the corresponding second-moment correlation length ξ . We consider RG-invariant quantities R , such as $R_\xi = \xi/L$ and the Binder parameter $U = \langle \mu_2^2 \rangle / \langle \mu_2 \rangle^2$, where $\mu_2 = \sum_{x,y} \text{Tr} Q_x Q_y$. At continuous transitions driven by the parameter J , they are expected to behave as [30]

$$R(L, J, g_0) \approx f_R(X) + O(L^{-\omega}), \quad X = (J - J_c)L^{1/\nu}, \quad (7)$$

where ν is the length-scale critical exponent, and $\omega > 0$ is the exponent controlling the leading scaling corrections. It is also useful to consider the FSS relation [38]

$$U = F_U(R_\xi) + O(L^{-\omega}), \quad (8)$$

where F_U is a universal function independent of any normalization. To estimate y_r , we consider the behavior of the RG-invariant quantities R in the presence of the GSB term (5). In the large- L limit, we expect [39]

$$R(L, J, g_0, r) \approx \mathcal{F}_R(X, Y), \quad Y = rL^{y_r}, \quad (9)$$

which holds provided [40] that $y_r > 1/\nu$, where ν is the thermal exponent of the gauge model [along the CH transition line, we have $1/\nu = 1.387(6)$, $1.247(12)$ for $N = 15$, 25 , respectively]. Equation (9) is the usual FSS relation for a multicritical point in systems with a global symmetry. However, in the present case, its validity is not obvious, given that the mass term P_M is not well defined in the ($r = 0$) gauge-invariant noncompact theory: averages of the mass term can only be computed in the presence of a maximal gauge fixing [37,41], such as the axial (using C^* conditions) or Lorentz ones. For these reasons, we consider three different actions with GSB terms

$$M1: S_1 = S_{\text{AH}} + P_M, \quad (10)$$

$$M2: S_2 = S_{\text{AH}} + P_L + P_M, \quad (11)$$

$$M3: S_3 = S_{\text{AH}} + P_M \quad \text{with} \quad A_{x,3} = 0, \quad (12)$$

where $M2$ can be associated with the Lorentz gauge, and $M3$ is defined imposing the axial gauge. We expect Eq. (9) to be well defined in models $M2$ and $M3$, while its validity in model $M1$ is instead not clear.

Numerical estimates of the RG dimensions.—We performed MC simulations for $N = 15$ and $N = 25$ along the CH transition line (estimates of the critical points and exponents can be found in Ref. [25]) for the three models $M1 - M3$; see the Supplemental Material [36] for details. The results confirm that P_M is relevant. Indeed, for fixed r ,

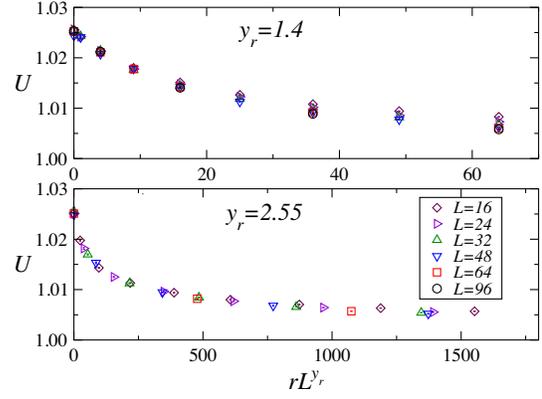


FIG. 1. Data of U at the critical point $J_c \approx 0.295\,515$ of the NCAH model for $N = 25$ and $g_0^2 = 2.5$, as a function of $Y = rL^{y_r}$. Results for models $M1$ (without gauge fixing, top) and $M3$ (axial gauge, bottom).

there is a clear departure from the gauge-invariant ($r = 0$) critical behavior. In Fig. 1 we show results for $N = 25$ at the critical point. The exponent y_r is estimated by fitting the data at J_c to Eq. (9), setting $X = 0$. We obtain $y_r = 1.4(1)$ for $M2$ [for both $a = 1$ and $a = 10$, cf. Eq. (6)] and $y_r = 2.55(5)$ for $M3$. We also mention that if we apply Eq. (9) to U computed in $M1$ without gauge fixing, we obtain the effective estimate $y_r \approx 1.4$, see top of Fig. 1, confirming the relevance of P_M along the CH transition line. Analogous results are obtained for $N = 15$, in particular, $y_r = 2.55(10)$ for $M3$. The exponent y_r turns out to depend on the gauge fixing, indicating that the gauge fixing influences the RG properties of the mass perturbation. Apparently, gauge-dependent modes, which are controlled by the gauge fixing term, are crucial in determining the effects of the photon mass term. Note that y_r is quite large, therefore the corresponding GSB perturbation must decrease rapidly with L —faster than L^{-y_r} —to keep GSB effects under control.

Critical behaviors in the presence of finite GSB terms.—We now address the behavior of the NCAH model in the presence of a finite GSB term such as the photon mass. Also for finite r we expect a transition at a finite value $J_c(r)$, with $J_c(r = 0) = J_c$, where J_c is the CH transition point in the gauge-invariant model. Since the charged fixed point is unstable with respect to P_M , we expect the transition to belong to a different universality class, which should only depend on the global symmetry of the model. Although the global symmetry group for $r > 0$ is $U(N)$, we will now argue that continuous transitions at $J_c(r)$ are characterized by a larger $O(2N)$ invariance group. We note that, since gauge fields are not expected to be relevant for $r \neq 0$, one can use the standard Landau-Ginzburg-Wilson (LGW) approach [27–30] to predict the critical behavior. Since the gauge symmetry is broken, z_x represents the microscopic order-parameter field. Therefore, the LGW basic field is an N -component complex vector $\Psi(x)$. The

Lagrangian is the sum of the kinetic term $|\partial_\mu \Psi|^2$ and of the most general $U(N)$ -invariant quartic potential

$$\mathcal{L}_{\text{LGW}} = \partial_\mu \Psi^* \cdot \partial_\mu \Psi + w \Psi^* \cdot \Psi + \frac{u}{4} (\Psi^* \cdot \Psi)^2. \quad (13)$$

It is easy to check that \mathcal{L}_{LGW} is actually $O(2N)$ invariant. Indeed, there are no dimension 2 and 4 $U(N)$ -invariant operators that break the $O(2N)$ symmetry. The lowest-dimension operators that are not $O(2N)$ symmetric have dimension 6 close to four dimensions—for instance, $(\text{Im} \Phi^* \cdot \partial_\mu \Phi)^2$ —and thus they are expected to be irrelevant at the 3D $O(2N)$ FP. Therefore, the critical behavior of generic vector systems with global $U(N)$ invariance (without gauge symmetries) is expected to belong to the $O(2N)$ universality class, implying an effective enlargement of the global symmetry of the critical modes (restricted only to the critical region).

The above analysis can be extended to lattice AH models with compact gauge fields (CAH), using the link variables $\lambda_{x,\mu} \in U(1)$ and the pure gauge action $S_\lambda = -g_0^{-2} \sum_{x,\mu \neq \nu} \text{Re} \lambda_{x,\mu} \lambda_{x+\hat{\mu},\nu} \bar{\lambda}_{x+\hat{\nu},\mu} \bar{\lambda}_{x,\nu}$ in Eq. (2). Unlike NCAH models, CAH models with $N \geq 2$ present only two phases, separated by a disorder-order transition line where gauge correlations are not critical [31]. Since the scalar fields turn out to be the only critical degrees of freedom, the effective description of the transitions is provided by the $SU(N)$ -invariant LGW Φ^4 theory with a matrix gauge-invariant order parameter, corresponding to Q_x^{ab} in Eq. (4) [31,42]. For $N = 2$ this LGW theory has a stable $O(3)$ vector FP, thus predicting $O(3)$ continuous transitions [30] for any gauge coupling $g_0 > 0$, including $g_0 \rightarrow \infty$ [for $g_0 \rightarrow 0$, instead, the model becomes equivalent to the $O(4)$ vector model]. This has been also confirmed numerically [31]. Gauge invariance can be broken by adding $P_M = -r \sum_{x,\mu} \text{Re} \lambda_{x,\mu}$, which plays the role of a photon mass for $\lambda_{x,\mu}$ close to 1. When the gauge symmetry is effectively broken (as discussed in Ref. [43], this requires r to be sufficiently large), the critical behavior should be described by the LGW theory (13), which predicts that continuous transitions belong to the $O(4)$ vector universality class.

The RG predictions at fixed r are confirmed by numerical results for both NCAH and CAH models. In Fig. 2 we plot U versus R_ξ for the NCAH model with $N = 25$ and $r = 1$. The data around the critical point $J_c(r)$ are expected to converge to a universal curve, cf. Eq. (8), which can be compared with the analogous curves of models that belong to known universality classes. The data approach the $O(2N)$ vector universal curve [obtained using an appropriate operator that corresponds to Q_x^{ab} in the $O(2N)$ model [31]], confirming the LGW RG argument. For the CAH model with $N = 2$, we observe an asymptotic $O(4)$ vector behavior for $r = 1$ and $r = 2.25$, in agreement with the general arguments [36].

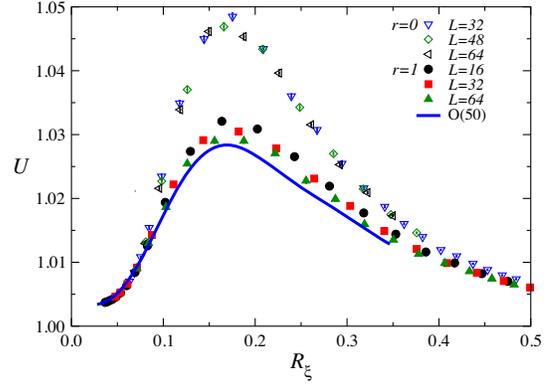


FIG. 2. Estimates of U versus R_ξ for the model $M1$ with $N = 25$, $g_0^2 = 2.5$, $r = 1$. We also report results for the gauge-invariant model ($r = 0$) and the $O(50)$ vector model (full line, obtained by large- L extrapolations of MC data for the appropriate spin-2 correlations, see Ref. [31] and the Supplemental Material [36]). The results for $r = 1$ appear to converge toward the $O(50)$ universal curve, consistent with $O(L^{-\omega})$ corrections with $\omega \approx 1$, supporting the RG prediction reported in the text.

Various classes of GSB perturbations.—On the basis of the results presented in this Letter, we may distinguish three classes of GSB perturbations. (i) First, there are GSB perturbations that are relevant at the stable FP of the lattice gauge-invariant theory. They drive the system out of criticality and may give rise to a different critical behavior. The photon mass term (5) plays this role along the CH line in the NCAH model. (ii) A second class corresponds to gauge fixings and GSB perturbations like those appearing in Eq. (6). If they are the only GSB terms present in the model, they are irrelevant: gauge-invariant observables are unchanged. However, if they are present together with some relevant GSB perturbation, they play a role: the RG flow close to the charged FP depends both on the gauge fixing and on the relevant perturbation. This may be due to the fact that a gauge fixing is needed to make non-gauge-invariant correlations well defined in the gauge-invariant theory or to the role of gauge-dependent modes that are sensitive to gauge fixings. (iii) There are GSB perturbations associated with RG operators with negative RG dimensions, whose effects are suppressed in the critical (continuum) limit.

When the added GSB perturbations are relevant, the lattice system may develop a different critical behavior or continuum limit. This is the case of the NCAH model with a photon mass term, which has a global $U(N)$ invariance. Quite interestingly, the transitions in this model belong to the $O(2N)$ vector universality class, with an effective enlargement of the global symmetry at the transition. This symmetry enlargement is expected in any model in which the GSB perturbation is relevant and it preserves the global $U(N)$ symmetry.

Cyclic RG flow.—It is worth noting that the above results lead to a peculiar RG flow; see Fig. 3 for a sketch in the

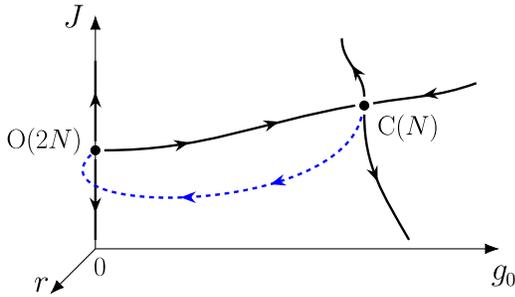


FIG. 3. Sketch of the cyclic RG flow of the NCAH model in the space of the parameters J , g_0 , and r , showing an unusual loop between the $O(2N)$ and the charged FP $C(N)$.

coupling space (J, g_0, r) . For $g_0^2 \rightarrow 0$, the gauge fields are frozen, and the model is equivalent to the $O(2N)$ vector model, whose critical behavior is controlled by the corresponding $O(2N)$ FP. If the gauge interactions are turned on, i.e., one sets $g_0 > 0$ keeping $r = 0$, the system flows toward the charged FP of the AH field theory, which is stable for any $0 < g_0^2 \lesssim 4$. Finally, if a photon mass is added, i.e., one sets $r > 0$, since the charged FP is unstable under this perturbation, the RG flow goes back to the $O(2N)$ FP, which is now stable, independent of g_0 and r . This RG behavior can be hardly reconciled with an irreversibility of the RG flow, analogous to that generally associated with the monotonic properties implied by the c theorem of 2D critical systems [44,45], see also Refs. [46–51] for similar proposals in 3D systems [52].

Conclusions.—In conclusion, we have studied the effect of GSB perturbations on the critical behavior—or, equivalently, the continuum limit—of gauge-invariant theories. The behavior at the charged FP turns out to be more complicated than that observed when global symmetries are broken. In particular, we observe apparent violations of universality. For instance, the RG dimension of the same GSB perturbation appears to depend on local gauge-fixing conditions, a result that, we believe, should be further investigated. Moreover, GSB perturbations give rise to unexpected phenomena, like the cyclic RG flow sketched in Fig. 3.

Several extensions are called for, to achieve a satisfactory understanding of the problem and to identify its universal features, such as the study of other lattice gauge theories—in particular, it would be interesting to extend the analysis to the non-Abelian gauge groups—and of other classes of GSB perturbations, for example, preserving residual discrete gauge subgroups (such approximations may be useful for analog simulations). It would also be important to rephrase and extend the present results to quantum Hamiltonian systems [53] (see Refs. [54,55] for recent works addressing issues related to GSB effects and the approach to the continuum limit).

Numerical simulations have been performed on the CSN4 cluster of the Scientific Computing Center at INFN-PISA.

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