Dynamical Scaling of Surface Roughness and Entanglement Entropy in Disordered Fermion Models

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Localization is one of the most fundamental interference phenomena caused by randomness, and its universal aspects have been extensively explored from the perspective of one-parameter scaling mainly for *static* properties. We numerically study dynamics of fermions on disordered one-dimensional potentials exhibiting localization and find *dynamical* one-parameter scaling for surface roughness, which represents particle-number fluctuations at a given length scale, and for entanglement entropy when the system is in delocalized phases. This dynamical scaling corresponds to the Family-Vicsek scaling originally developed in classical surface growth, and the associated scaling exponents depend on the type of disorder. Notably, we find that partially localized states in the delocalized phase of the random-dimer model lead to anomalous scaling, where destructive interference unique to quantum systems leads to exponents unknown for classical systems and clean systems.

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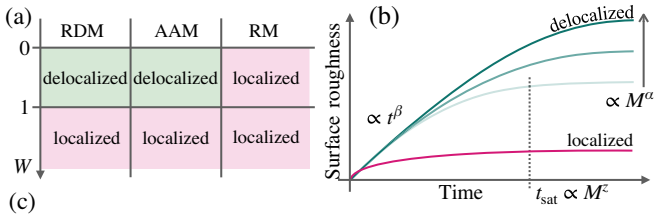
Introduction.—The Anderson localization [1] is a unique phenomenon arising from destructive interference in disordered systems. It has attracted a lot of attention in, e.g., solid-state physics, quantum optics, and classical mechanics [2–5], and has been observed in various experimental setups [6–22]. Study of the Anderson localization has significantly been put forward in light of one-parameter scaling [3,23,24], where physical quantities are scaled only by a single parameter. The example includes scaling for system-size dependence of conductance and for correlation functions at localization transition points. Despite its importance, such a one-parameter scaling has been mainly focused on static properties. Meanwhile, disorder is known to affect quantum dynamics, such as entanglement dynamics [25–33] and transport properties [34–43]. It is thus intriguing and fundamental to pursue dynamical one-parameter scaling, which can lead to a hitherto unknown classification of disordered quantum systems using their nonequilibrium properties.

It was recently found that dynamical one-parameter scaling, called Family-Vicsek (FV) scaling, appears in a clean quantum bosonic system [44]. While FV scaling was originally known in classical surface growth [45–47], we [44] found the scaling in the quantum system by introducing a "quantum surface-height operator," which represents particle-number fluctuations summed over a given length scale [see Eq. (2)]. The standard deviation of this operator, i.e., quantum surface roughness, is found to obey the Edwards-Wilkinson (diffusive) and ballistic scalings. Notably, the surface roughness is experimentally accessible in cold atomic systems using microscopes.

In this Letter, employing the surface roughness in quantum systems, we show numerical evidence that dynamical one-parameter scaling exists in one-dimensional 1)D) noninteracting fermions in a disordered potential. We use the random model (RM), the random-dimer model (RDM) [48], and the Aubry-André model (AAM) [49], which exhibit Anderson localization. The phase diagram of these models is schematically shown in Fig. 1(a) as a function of disorder strength W. Our numerical calculations find that, in the delocalized phases of the RDM and the AAM, the surface roughness obeys the FV scaling characterized by three exponents α, β, and z, as schematically shown in Fig. 1(b). Notably, we find anomalous exponents $(\alpha, \beta, \gamma) \simeq (0.352, 0.337, 1.00)$ in the RDM. We argue that the anomalous scaling is caused by numerous localized eigenstates in a delocalized phase, which are unique to quantum disordered systems. Furthermore, we find that the surface roughness is approximately proportional to the square root of the von Neumann entanglement entropy (EE), and our numerical calculation elucidates the FV-type scaling of the EE. Our finding suggests that the surface roughness can be an experimentally friendly measure for the EE. The table in Fig. 1(c) summarizes our results.

Theoretical models.—We consider noninteracting N-spinless fermions on a 1D lattice with a disordered potential. Let us denote the annihilation and creation operators on a site j by $\hat{f}_j$ and $\hat{f}_j^\dagger (j = 1, \ldots, M)$, where



(c)

Model	No. of DLES	No. of LES	FV scaling	α	β	z	$S_{\rm EE}$
RDM ($W < 1$)	$\mathcal{O}(M^{0.5})$	$\mathcal{O}(M)$	present	0.352	0.337	1.00	$t^{2\beta}$
AAM ($W < 1$)	$\mathcal{O}(M)$	$\mathcal{O}(1)$	present	0.492	0.457	1.02	$t^{2\beta}$
RM RDM ($W > 1$) AAM ($W > 1$)	Decrease with M	$\mathcal{O}(M)$	absent	...	...	...	...

FIG. 1. (a) Phase diagram for the random-dimer model (RDM), the Aubry-André model (AAM), and the random model (RM) as a function of the disorder strength W. Delocalized phases appear for $W < 1$ in the RDM and the AAM. (b) Schematic for surface-roughness growth in the localized and delocalized phases. In the delocalized phase, the growth is characterized by three power exponents α, β, and z, which, respectively, capture system-size M dependence of the saturated surface roughness, power-law growth, and a saturation time $t_{\rm sat}$ of the surface roughness. This dynamical scaling is called Family-Vicsek (FV) scaling [see Eq. (4)]. FV scaling does not emerge in the localized phase. (c) Summary of our results, including numbers of delocalized eigenstates (DLESs) and localized eigenstates (LESs) and growth laws of von Neumann entanglement entropy $S_{\rm EE}$.

M is the number of the lattice sites. Throughout this Letter, M is taken to be even. Then the Hamiltonian is given by

$$\hat{H} = -J\sum_{j=1}^{M}(\hat{f}_{j+1}^{\dagger}\hat{f}_j + \hat{f}_j^{\dagger}\hat{f}_{j+1}) + \sum_{j=1}^{M} V_j \hat{f}_j^{\dagger}\hat{f}_j, \quad (1)$$

with a hopping constant $J > 0$ and an on-site potential V_j.

We use three potentials corresponding to the RM, the RDM, and the AAM. The RM consists of a random potential with no spatial correlation, where V_j takes 0 or $V(>0)$ following the probability function $P_{\rm RM}(V_j) = (1/2)\delta(V_j) + (1/2)\delta(V_j - V)$. The potential in the RDM [48] has a spatial correlation such that the probability function is given by $P_{\rm RDM}(V_{2j-1}, V_{2j}) = (1/2)\delta(V_{2j})\delta(V_{2j-1}) + (1/2)\delta(V_{2j} - V)\delta(V_{2j-1} - V)$, with $j = 1, 2, \ldots, M/2$ [50]. The AAM has a fixed quasiperiodic structure given by $V_j = V\cos(2\pi\theta j)$ with the irrational number $\theta = (\sqrt{5}-1)/2$ [49,51,52]. We assume the periodic boundary condition for the RM and the RDM, and the open boundary condition for the AAM. In the RDM and the RM, we take ensemble averages to calculate physical quantities, and the sample number in all the calculations is $\lfloor 44000/M \rfloor$ with the floor function ($\lfloor \cdots \rfloor$).

The strength of the disorder is characterized by the dimensionless constant $W = V/(2J)$. The models have localized or delocalized phases depending on W [48,51], as shown in Fig. 1(a). In the RM, all the eigenstates are localized in the thermodynamic limit if W is nonzero. If the randomness has spatial correlation as for the RDM and the AAM, there exist delocalized phases for $W < 1$. The RDM has both delocalized eigenstates (DLESs) and localized eigenstates (LESs) in the delocalized phase, but there are no mobility edges [see Sec. I of the Supplemental Material (SM) [53]].

Surface-height operator and the roughness.—To explore dynamical one-parameter scaling, we consider the "quantum surface roughness" defined in Ref. [44]. The essential ingredient is the mathematical analogy between surface growth and one-dimensional nonlinear fluctuating hydrodynamics [59–62]. The former discusses the dynamics of the surface height $h(x,t)$ that obeys a stochastic partial differential equation, such as the Kardar-Parisi-Zhang (KPZ) equation. For the latter, recent works [59–61] found that the spatiotemporal correlation function for the sound mode $\phi(x,t)$ shows dynamical scaling similar to that for $\partial_x h(x,t)$ in the KPZ equation [59–61]. Similarly, Ref. [62] showed that, in the wave number and frequency spaces, the correlation function for the local particle number $\rho(x,t)$ in a discrete nonlinear Schrödinger equation obeys the KPZ scaling. Then one can see the correspondence between $\partial_x h(x,t)$ and the fluctuation of $\rho(x,t)$. Extending this analogy to quantum systems, we introduce the following surface-height operator [44,63]:

$$\hat{h}_j = \sum_{i=1}^{j}(\hat{f}_i^{\dagger}\hat{f}_i - \nu), \quad (2)$$

with a filling factor $\nu = N/M$. The operator represents the particle-number fluctuations summed over the subregion $[1, j]$ and can describe the particle-number fluctuations at a given length scale j. The average surface height is given by $h_{\rm av}(t) = 1/M\sum_{j=1}^{M}{\rm Tr}[\hat{\rho}(t)\hat{h}_j]$, where the density matrix $\hat{\rho}(t)$ is averaged over many realizations of the random potentials for the RM and the RDM. The following surface roughness $w(M,t)$ is defined as the standard deviations of $\hat{h}_j$:

$$w(M,t) = \sqrt{\frac{1}{M}\sum_{j=1}^{M}{\rm Tr}\{\hat{\rho}(t)[\hat{h}_j - h_{\rm av}(t)]^2\}}. \quad (3)$$

As discussed later, the surface roughness is well approximated by the particle-number fluctuations in the half of the system. This implies that the surface roughness measures the correlation between systems divided by two.

Our previous work [44] has found that the surface roughness in isolated quantum systems free from disorder exhibits the following FV scaling:

$$w(M,t) = s^{-\alpha}w(sM, s^z t) \propto \begin{cases} t^{\beta} & (t \ll t_{\rm sat}), \\ M^{\alpha} & (t_{\rm sat} \ll t), \end{cases} \quad (4)$$

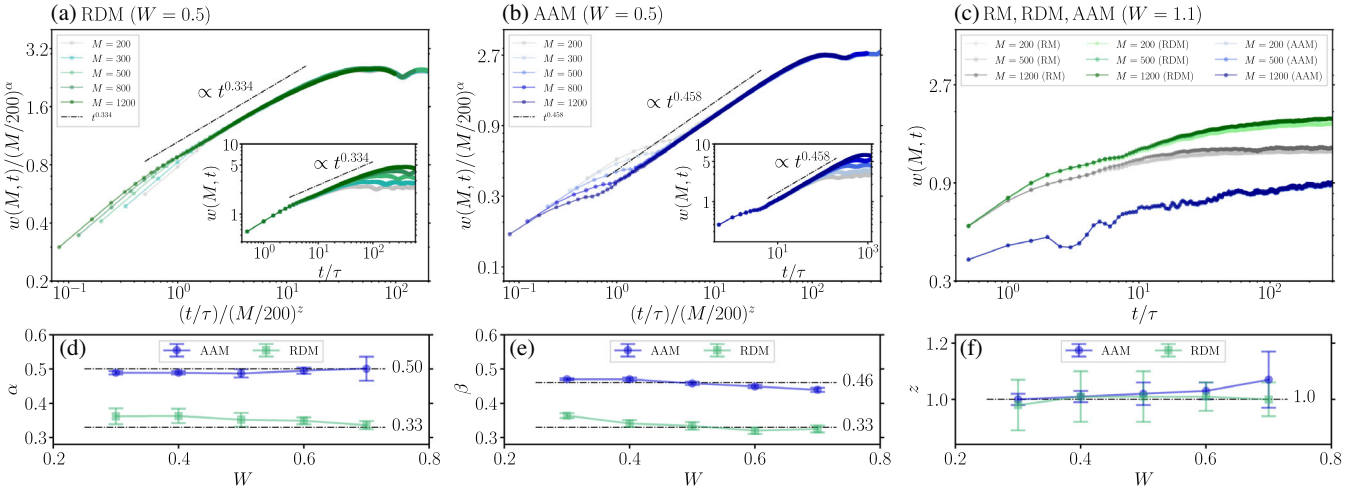


FIG. 2. Surface-roughness dynamics and FV scaling for delocalized phases of (a) RDM ($W = 0.5$) and (b) AAM ($W = 0.5$), and for (c) localized phases of RM, RDM, and AAM ($W = 1.1$). The time is normalized by $\tau = \hbar/J$. In (a) and (b), the main panels show $w(M,t)$ with $M = 200$, 300, 500, 800, and 1200 with the ordinate and the abscissa normalized by $(M/200)^{\alpha}$ and $(M/200)^{z}$, and the insets show the corresponding raw data. The delocalized phase shown in (a) and (b) exhibits clear FV scaling, whereas we find no signature of FV scaling in the localized phase in (c). (d)–(f) Dependence of α, β, and z on W, respectively, for the AAM and the RDM in the delocalized phases ($W < 1$). The numerical data used for extracting these power exponents are shown in Sec. III of the SM [53].

with a parameter s and a saturation time t_{sat}. Taking $s = 1/M$, we obtain $w(M,t) = M^{\alpha} f(t/M^{z})$ with a scaling function $f(x) = w(1,x)$. This means that the surface roughness with a different M collapses to a single curve after normalization of the ordinate and the abscissa by M^{α} and M^{z}. This dynamical one-parameter scaling was originally discussed in classical systems, and the exponents α, β, and z classify the universality of the surface-roughness dynamics [47]. The dynamical exponent z satisfies the scaling relation $z = \alpha/\beta$, and $z = 1$, $3/2$, and 2 indicate ballistic, superdiffusive, and diffusive transport. The famous classes are the Edwards-Wilkinson class [64] and the KPZ class [65], for which the scaling exponents are $(\alpha,\beta,z) = (1/2, 1/4, 2)$ and $(1/2, 1/3, 3/2)$, respectively. Our previous Letter [44] found that free fermions (hard-core bosons) without disorders have $(\alpha,\beta,z) \simeq (1/2, 1/2, 1)$.

Surface-roughness dynamics.—We numerically investigate the surface roughness to explore the FV scaling in the disordered models. Our numerical method is based on Gaussian states [66] (see also Sec. II of the SM [53]). The initial state is a staggered state $|\psi(0)\rangle = \prod_{j=1}^{N} \hat{f}_{2j}^{\dagger}|0\rangle$ with the total particle number $N = M/2$. This initial state has small surface roughness, and thus is suitable to investigate the universal aspect of the surface-roughness growth.

Figures 2(a)–2(c) show the time evolution of the surface roughness. In the delocalized phase ($W = 0.5$) of the RDM and the AAM, the surface roughness increases in time and exhibits the FV scaling (4), as shown in Figs. 2(a) and 2(b), respectively. The estimated power exponents (α,β,z) in the RDM and the AAM are (0.352,0.334,1.01) and (0.487,0.458,1.02), respectively [67]. These results clearly demonstrate that the dynamical one-parameter scaling indeed exists, even in the disordered fermion models. Notably, the exponents in the RDM are anomalous in that they are absent in classical systems and clean systems. This fact is attributed to the LESs in the delocalized phase, as discussed later. On the other hand, in the localized phase, the surface roughness is independent of the system size M and does not exhibit clear power-law growth for all the models, indicating the absence of FV scaling [see Fig. 2(c)].

We systematically investigate disorder dependence of the exponents (α,β,z) by changing W in the delocalized phases. As shown in Figs. 2(d)–2(f), we find that the exponents in the RDM and the AAM are almost independent of W. Thus, we conclude that the RDM and the AAM in the delocalized phase show FV scaling with the exponents $(\alpha,\beta,z) \simeq (0.352, 0.337, 1.00)$ and (0.492,0.457,1.02), respectively, which are obtained by averaging the exponents in Figs. 2(d)–2(f) over W. Also, we numerically investigate the dynamics starting with other initial states and find that the choice of the initial state is not important as long as the initial states have small roughness (see Sec. IV of the SM [53]).

Note that we show the numerical results only for $W \geq 0.3$. This is due to the larger localization length for smaller W, which makes it difficult to eliminate the finite-size effect. While we do not have conclusive results for the exponents for small W, we conjecture that the exponents are universal for $0 < W < 1$, in accordance with the phase diagram in Fig. 1.

The exponents in the AAM are close to $(\alpha,\beta,z) \simeq (0.500, 0.489, 1.00)$ for the noninteracting fermion model

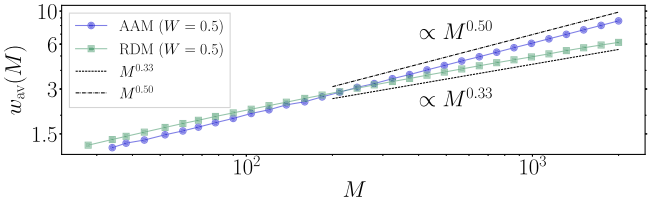


FIG. 3. Saturated surface roughness $w_{\rm av}(M)$ obtained by the approximated diagonal ensemble for the AAM and the RDM with $W = 0.5$.

without disorder [44]. This coincidence can be understood by considering the numbers of DLESs and LESs for the single-particle eigenstate of $\hat{H}$. According to Sec. I of the SM [53], the numbers of DLESs and LESs in the AAM with $W < 1$ are proportional to M and $\mathcal{O}(M^0)$, respectively. Thus, we conjecture that the effect of the remaining LESs is too weak and that the exponents are almost the same as those for fermion systems without disorder.

The situation drastically changes in the RDM with the anomalous exponent $\alpha \simeq 0.352$. According to Ref. [48] (see also Sec. I of the SM [53]), the numbers of DLESs and LESs in the RDM with $W < 1$ are proportional to $\sqrt{M}$ and M, respectively. In stark contrast to the AAM, the RDM supports many LESs even in the delocalized phase, and they can strongly affect the surface-roughness dynamics. Indeed, just from the information about the eigenstates and the initial state, we can numerically reproduce the exponent $\alpha \simeq 0.33$ and 0.5 for the RDM and the AAM, respectively, as shown in Fig. 3. In this calculation, we evaluate the saturated surface roughness $w_{\rm av}(M)$ using the approximated diagonal ensemble [68–71] (see Sec. V of the SM [53]). Since we use the same initial states for the RDM and the AAM, the result in Fig. 3 implies that the difference in α originates with the statistical property of the eigenstates. Furthermore, we can analytically derive the nonanomalous exponent $\alpha = 0.5$ for systems without LESs, i.e., disorder-free noninteracting systems (see Sec. V of the SM [53]) and systems satisfying the eigenstate thermalization hypothesis [68]. All our findings support our argument that the anomalous scaling in the RDM is attributed to the limited number of DLESs and the large number of LESs.

Entanglement entropy and surface roughness.—We find that the surface roughness is related to von Neumann EE through a nontrivial relation. The EE quantifies quantum entanglement in a pure state in a system divided into two subsystems. Here, we divide the M-site system into subsystems $A = \{j | 1 \le j \le M/2\}$ and $B = \{j | M/2 < j \le M\}$ and define the reduced density matrix $\hat{\rho}_{\rm re}(t) = {\rm tr}_B[\hat{\rho}_{\rm pure}(t)]$, where $\hat{\rho}_{\rm pure}(t)$ is a density matrix for a single realization of the disordered models. Then the EE is calculated by $S_{\rm EE}(M,t) = -\overline{{\rm Tr}_A[\hat{\rho}_{\rm re}(t)\log\hat{\rho}_{\rm re}(t)]}$, where the overline denotes the ensemble average in the RDM.

To derive the relation between $S_{\rm EE}(M,t)$ and $w(M,t)$, we assume that (i) $h_{\rm av}(t) \simeq 0$, (ii) $w(M,t)^2 \simeq {\rm Tr}\{\hat{\rho}(t)[\hat{h}_{M/2} - h_{\rm av}(t)]^2\}$, and (iii) $\sum_{j=1}^{M/2} {\rm Tr}[\hat{\rho}(t)\hat{n}_j] \simeq \nu M/2$. The validity of these assumptions is numerically confirmed in Sec. VI of the SM [53]. Assumptions (i) and (ii) lead to

$$w(M,t)^2 \simeq {\rm Tr}\left[\hat{\rho}(t)\left(\sum_{j=1}^{M/2}\hat{f}_j^\dagger\hat{f}_j - \frac{M\nu}{2}\right)^2\right]. \tag{5}$$

Equation (5) indicates that $w(M,t)^2$ can be approximated by the particle-number fluctuation in the half of the system from the averaged number $\nu M/2$. Thus, both $w(M,t)^2$ and $S_{\rm EE}(M,t)$ have information about the correlation between the divided systems A and B. We then find the following relation (see Sec. VI of the SM [53]):

$$S_{\rm EE}(M,t) \simeq 3w(M,t)^2, \tag{6}$$

where we use (iii) and the additional assumption that eigenvalues of the correlation matrix ${\rm Tr}[\hat{\rho}_{\rm pure}(t)\hat{f}_i^\dagger\hat{f}_j](i,j \in A)$ are uniformly distributed between zero and unity. Note that Refs. [72,73] discuss relations similar to Eq. (6) for ground states of free-fermion models, but not for dynamics.

Substituting Eq. (6) into Eq. (4), we obtain the following FV-type scaling in the delocalized phases:

$$S_{\rm EE}(M,t) = s^{-2\alpha}S_{\rm EE}(sM, s^z t) \propto \begin{cases} t^{2\beta} & (t \ll t_{\rm sat}), \\ M^{2\alpha} & (t_{\rm sat} \ll t). \end{cases} \tag{7}$$

Figure 4 shows time evolutions of $S_{\rm EE}(M,t)$ in the RDM and the AAM with $W = 0.5$. Our numerical results clearly reveal that the EE obeys the FV-type scaling (7). The insets of Fig. 4 verify Eq. (6), showing that the relation works quite well, especially in the early stages of the dynamics. Although they deviate from one another in the late stages, the FV-type scaling still holds with the expected exponent $(2\alpha, 2\beta, z)$.

This finding suggests that the surface roughness may become a possible measure for entanglement and its universal scaling. Furthermore, we rigorously prove in Sec. VI of the SM [53] that if the bipartite number fluctuation ${\rm Tr}[\hat{\rho}(t)\hat{h}_{M/2}^2]$ [74–76] with assumption (iii) exhibits power-law growth t^β, $S_{\rm EE}(M,t)$ also grows as $t^{2\beta}$ in the thermodynamic limit (and vice versa).

Finally, we comment on the entanglement dynamics studied in view of the surface roughness. Using quantum circuit models, Nahum and co-workers [77,78] showed that the EE obeys the KPZ equation. This means that the EE itself behaves as the surface height, which differs from our result in Eq. (6). The difference may be attributed to the models used in the previous and our works because they have different conserved quantities, which can lead to the distinct longtime dynamics. We also stress that FV scaling was not observed in Refs. [77,78] in that they did not examine saturation of the fluctuations of the EE.

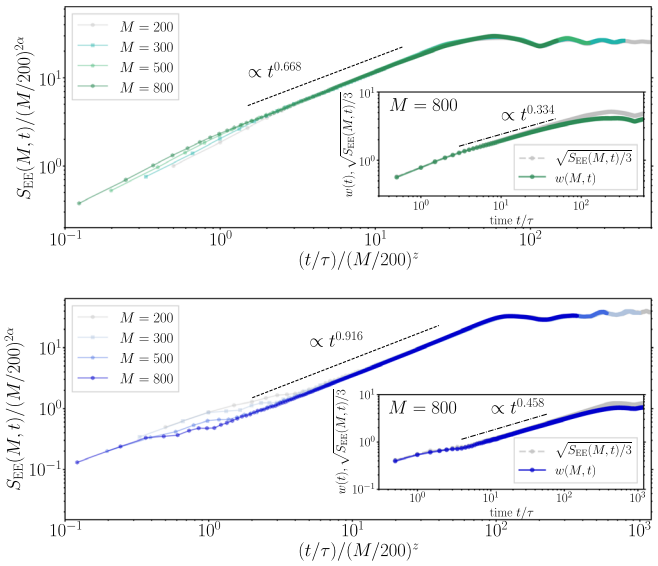


FIG. 4. FV scaling for von Neumann EE. $S_{\rm EE}(M,t)$ for the RDM (upper panel) and the AAM (lower panel), with $W = 0.5$ and $M = 200$, 300, 500, and 800, where the time is normalized by $\tau = \hbar/J$ and the ordinate and the abscissa are normalized by $(M/200)^{2\alpha}$ and $(M/200)^{z}$ with the exponents α and z obtained in Fig. 2. Insets: time evolutions of $\sqrt{S_{\rm EE}(M,t)/3}$ and $w(M,t)$ for $M = 800$, which confirm the success of Eq. (6) in the early stages of the dynamics.

Conclusion and outlook.—We have numerically found the dynamical one-parameter scaling of surface roughness and entanglement entropy in disordered fermion models, including anomalous scaling arising from partial quantum localization [79]. Our study opens an unexplored avenue for pursuing the unexpected relation between Anderson localization and surface growth physics through the FV scaling and the EE. From this viewpoint, it is interesting to investigate the universality class of the FV scaling for many-body localization [29–31,39–43,80–85] and the Anderson localization with long-range interactions.

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