High-Efficiency Arbitrary Quantum Operation on a High-Dimensional Quantum System

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The ability to manipulate quantum systems lies at the heart of the development of quantum technology. The ultimate goal of quantum control is to realize arbitrary quantum operations (AQUOs) for all possible open quantum system dynamics. However, the demanding extra physical resources impose great obstacles. Here, we experimentally demonstrate a universal approach of AQUO on a photonic qudit with the minimum physical resource of a two-level ancilla and a $\log_2 d$-scale circuit depth for a d-dimensional system. The AQUO is then applied in a quantum trajectory simulation for quantum subspace stabilization and quantum Zeno dynamics, as well as incoherent manipulation and generalized measurements of the qudit. Therefore, the demonstrated AQUO for complete quantum control would play an indispensable role in quantum information science.

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Behind the flourish of quantum technology [1,2] is the mature quantum control technique [3,4], which allows one to manipulate the quantum states of a physical system with unprecedented precision. The conquests in the quantum domain proceed from arbitrary quantum state preparation [5–7] to arbitrary quantum gates [8–11], but are mostly limited to closed quantum systems. However, isolated quantum systems do not exist in practice. On one hand, quantum systems are surrounded by the environment and unavoidably exposed to noise. On the other hand, the control and readout of the quantum systems are necessary for quantum tasks, requiring communication with external systems. Therefore, practical quantum systems are open, and their states should be described by density operators, and physical processes acting on the quantum systems are generally described by quantum operations [1] as $\mathcal{E}(\rho_s) = \sum_j E_j \rho_s E_j^\dagger$, where ρ_s is the density matrix of the quantum system with d dimensions and E_j is the Kraus operator ($\sum_j E_j^\dagger E_j = I_{d\times d}$ with $I_{d\times d}$ being the d-dimensional identity matrix).

Great attention has already been drawn for realizing arbitrary quantum operations (AQUOs) and universal quantum control of open quantum systems [12–15]. Although direct control of the interaction between the quantum system and the environment to realize certain open quantum system dynamics has been demonstrated [16,17], such an analog approach for AQUO is experimentally challenging due to the lack of the capability for arbitrary Hamiltonian engineering. Quantum operations of a d-dimensional system could also be realized digitally by unitary gates acting on a dilated Hilbert space. For example, rank-2 AQUOs are experimentally demonstrated in a trapped-ion system by using a two-dimensional ancilla, and it allows for the dissipative quantum state preparation and the simulation of dynamical maps [18,19]. However, at least m ancillary degrees of freedom are required for simulating the environment and realizing a rank-m quantum operation [1] ($m \leq d^2$, see Supplemental Material, Ref. [20]), imposing tremendous resource overhead. Very recently, Shen *et al.* theoretically proved an efficient scheme to realize AQUOs with an arbitrary rank by adaptive control with only one two-dimensional ancilla and a $\log_2 d$-scale circuit depth [15].

Here, we experimentally demonstrate AQUOs on a qudit with a dimension $d = 4$. With the assistance of only one ancilla qubit, we validate the adaptive control scheme [13,15] through high-fidelity universal unitary gates and real-time feedback control in a superconducting quantum circuit. Based on the AQUOs, we experimentally simulate the quantum trajectory of the qudit, and show the important applications of the AQUOs in subspace stabilization and the quantum Zeno effect [37,38]. Complete quantum control is illustrated by implementing a quantum task, including state preparation, incoherent quantum information processing, and detection through a rank-16 symmetric informationally complete positive operator-valued measure (POVM) [33]. Our results on complete control of quantum systems could be easily generalized to other experimental platforms, such as trapped-ion [39] and phononic [40] systems, and open up new possibilities in exploring quantum computation, quantum simulation, and quantum metrology [41–43].

Figure 1(a) sketches the architecture for realizing the AQUO on an arbitrarily high-dimensional quantum system, which consists of a two-level ancilla qubit coupling with the system and a classical register that communicates with the ancilla. The elementary quantum circuit of the AQUO is shown in Fig. 1(b) and only requires three types of operations: a unitary gate U on the composite of the quantum system and the ancilla (total dimension of $2d$), a projective measurement of the ancilla including an extraction of the outcome, and reset of the ancilla after the measurement. For each implementation of the elementary quantum circuit, we have the state of the composite after the unitary gate as $U(\rho_s \otimes |g\rangle\langle g|)U^\dagger = E_0\rho_s E_0^\dagger \otimes |g\rangle\langle g| + E_1\rho_s E_1^\dagger \otimes |e\rangle\langle e| + E_0\rho_s E_1^\dagger \otimes |g\rangle\langle e| + E_1\rho_s E_0^\dagger \otimes |e\rangle\langle g|$. Here, $|g\rangle$ and $|e\rangle$ are the ground and the excited states of the ancilla, respectively, and E_j is the Kraus operator according to the ancilla measurement outcome ($j \in \{0, 1\}$ corresponding to $\{|g\rangle, |e\rangle\}$). By choosing an appropriate U and tracing out the ancilla regardless of the outcome of the measurement, a rank-2 AQUO

$$\mathcal{E}(\rho_s) = E_0\rho_s E_0^\dagger + E_1\rho_s E_1^\dagger \quad (1)$$

can be realized. From the aspect of open quantum system dynamics, the ancilla plays two roles in the AQUO. First, the ancilla can be treated as a quantum dice, i.e., a quantum random number generator for mimicking noise and inducing stochastic quantum jumps of the system [44]. Second, by monitoring the quantum system through the ancilla, the entropy of the system could be dumped through the ancilla into the classical register. For example, a quantum error correction operation could restore the pure quantum state from a mixed state [1].

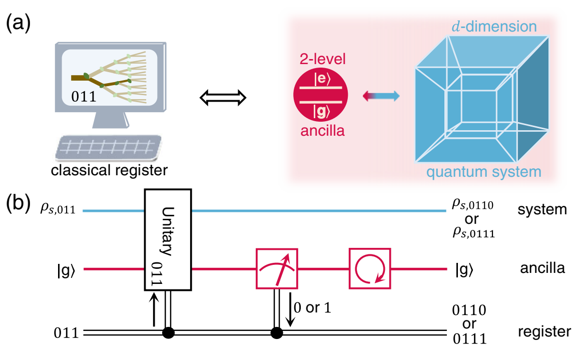

FIG. 1. Scheme of arbitrary quantum operation (AQUO) on a high-dimensional quantum system with minimum resource. (a) The architecture: A d-dimensional quantum system is controlled by only one two-level ancilla qubit and a classical register. (b) The elementary quantum circuit for AQUO. A unitary gate is chosen according to the digits in the classical register and acted on the composite of the quantum system and the ancilla. The ancilla is projectively measured with binary outcomes that are extracted by the register, and then the ancilla is reset to the ground state. The circuits can be implemented repetitively with the measurement results of the ancilla stored in the register for the next implementation of a new unitary gate.

To experimentally demonstrate the AQUOs of a high-dimensional quantum system, we carry out the architecture in Fig. 1(a) via a superconducting quantum circuit, which consists of a transmon qubit and a high-quality microwave cavity [34,45–47]. The cavity provides a photonic qudit by exploring its infinitely large Hilbert space of Fock states, and the AQUO on the qudit is realized through the transmon qubit serving as the ancilla and a field programmable gate array (FPGA) serving as the classical register [34]. The FPGA not only records the measurement outcomes of the ancilla, but also executes classical logic in real time and adaptively sends appropriate control pulses to the composite cavity-transmon system for arbitrary unitary gates [8,11]. All required gates on the system could be implemented with high fidelities and negligible latency, thus allowing the repeating and cascading of the quantum circuits (see Supplemental Material, Ref. [20]).

We first test the rank-2 AQUOs in the spirit of stochastic quantum trajectory simulation [44]. Under the continuous monitoring by a Markovian environment, a quantum system evolves stochastically conditional on the projection outcome of the environment, which has been widely studied in theory by the Monte-Carlo method [44]. Interpreting the ancilla as a monitor to the system and a quantum dice, we can simulate the stochastic quantum trajectories of the qudit in experiments via AQUOs. By repetitively erasing the ancilla, the ancilla and the register together act as an infinitely large Markovian environment to the system without information backflow. In such a manner, we experimentally simulate two quantum trajectories on a photonic qudit ($d = 4$): an odd-parity subspace stabilization and quantum Zeno blockade (QZB) due to two-photon dissipation. The corresponding quantum operations on the truncated space of Fock states are depicted in Fig. 2(a).

To stabilize the odd-parity subspace of the photonic qudit, we engineer proper unitary gates to achieve the two Kraus operators: E_0 reserves the odd-parity subspace span $\{|1\rangle, |3\rangle\}$, and E_1 converts the even-parity subspace to the odd-parity subspace $\{|0\rangle \to |1\rangle, |2\rangle \to |3\rangle\}$. Because of the spontaneous decay of the cavity, single-photon losses of the photonic qudit will convert quantum states in the odd-parity subspace to the even-parity subspace, and eventually destroy the quantum information stored in the qudit. By repetitively implementing the odd-parity subspace stabilization operations [Fig. 2(b)], we expect that any state trajectories in the odd-parity subspace would have a longer lifetime. To compromise with the operation error in practice, which is mainly induced by the ancilla's decoherence, we choose an optimal interval time (12.5 μs)

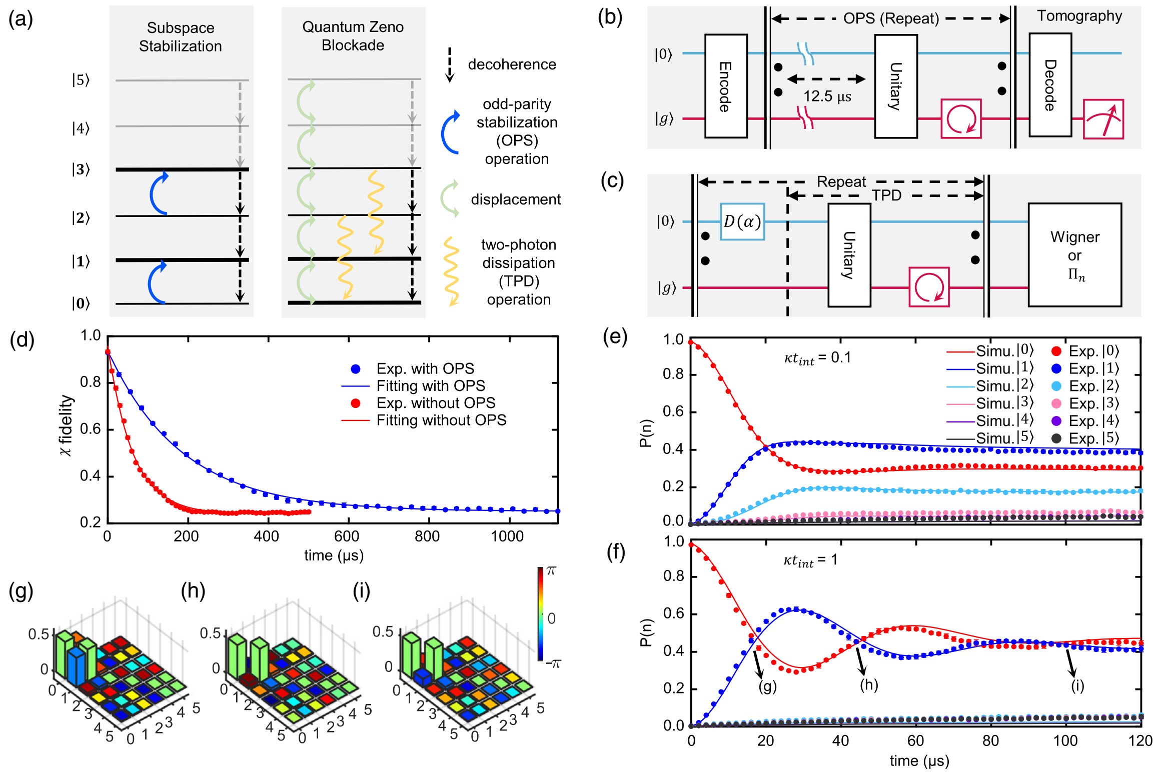

FIG. 2. Experimental quantum trajectory simulation through AQUO. (a) The energy-level transition diagrams of rank-2 AQUOs, with the left and the right panels representing two distinct schemes for the odd-parity subspace stabilization (b), (d) and quantum Zeno blockade (c), (e)–(i), respectively. (b) Experimental sequence for the subspace stabilization by odd-parity stabilization (OPS) operations. (c) Experimental sequence for the quantum Zeno blockade, consisting of two-photon dissipation (TPD) and displacement $[D(\alpha)]$ operations and Wigner tomography or photon-number measurement Π_n. (d) Results of the χ matrix fidelity decay in the odd-parity subspace. The process fidelity decay times $T_1 = 184.3$ μs and 61.9 μs for the system with and without repetitive OPS operations, respectively. (e)–(f) Time evolutions of Fock state populations ($|0\rangle$ to $|5\rangle$) for effective TPD rates of $\kappa/2\pi = 8$ kHz (e) or $\kappa/2\pi = 80$ kHz (f), a displacement value $\alpha = -0.1i$, and a repetition interval $t_{\mathrm{int}} = 2$ μs. Dots and lines present the experimental and the numerical results, respectively. (g)–(i) Density matrices reconstructed by Wigner tomography of the system at evolution times of 16 μs, 44 μs, and 100 μs in (f), respectively.

between two adjacent AQUOs. To characterize the performance of the stabilization operation, we carry out process tomography in the odd-parity subspace averaged over an ensemble of trajectories and evaluate the χ fidelity by comparing with an ideal qubit encoded in $\{|1\rangle, |3\rangle\}$. Figure 2(d) shows the exponentially decaying χ fidelities that demonstrate that the coherence time of the quantum state in the odd-parity subspace is boosted by approximately 3 times with the stabilization operations (184.3 μs), compared with that without the operations (61.9 μs).

Significantly different dynamics are unraveled with a two-photon dissipation operation realized by engineering two Kraus operators (see Supplemental Material, Ref. [20]) that effectively induce jumps of Fock states $|n\rangle \to |n-2\rangle$ for $n \geq 2$ [Fig. 2(a)]. A continuous implementation of such a quantum operation induces the quantum Zeno effect [37,38] that blockades the transitions from energy levels $\{|0\rangle, |1\rangle\}$ to other higher levels and thus realizes an equivalent two-level system in a harmonic oscillator. The QZB is revealed when exciting the cavity with coherent drives through a displacement gate on the cavity. By alternatively implementing the engineered quantum operation and the displacement gate with a repetition interval of $t_{\mathrm{int}} = 2$ μs [Fig. 2(c)], the evolution of the qudit shows the Rabi oscillation dynamics that mimics a two-level atom under a coherent drive, as expected for QZB [Fig. 2(f)]. The populations of the qudit are confined in the lowest two levels, whose combined population is 0.942 at the evolution time $t = 20$ μs when $\kappa t_{\mathrm{int}} = 1$. For a smaller κ ($\kappa t_{\mathrm{int}} = 0.1$), the QZB is weaker with a higher population leakage to other states, as shown in Fig. 2(e). The detailed density matrices are shown in Figs. 2(g)–2(i). The QZB protocol can be extended to a larger Zeno subspace with Kraus operators that induce multiphoton jumps [48]. A non-Markovian environment could also be realized with the memory effect of an ancilla by partially resetting the ancilla to the ground state or selecting the unitary gate depending on the previous ancilla measurement outcomes [48].

The extension of the AQUO to rank-m ($m \leq d^2$) could be implemented by introducing an m-dimensional ancilla.

However, this approach demands large physical resource overhead. Instead, as proposed in Ref. [15], a rank-m AQUO could be realized by an n-step ($n = \lceil \log_2 m \rceil$) cascading of the elementary quantum circuits in Fig. 1(b) via adaptive control and a recycling strategy of a two-level ancilla. Regardless of the measurement outcomes of the ancilla, the achieved quantum operation is

$$\mathcal{E}(\rho_s) = \sum_{r_1, r_2, \ldots, r_n} E_{n,r_n}^{(r_{n-1},\ldots,r_1)}, \ldots, E_{1,r_1} \rho_s E_{1,r_1}^{\dagger}, \ldots, E_{n,r_n}^{\dagger(r_{n-1},\ldots,r_1)}, \tag{2}$$

where $E_{k,r_k}^{(r_{k-1},\ldots,r_1)}$ denotes the Kraus operator due to the unitary $U_k^{(r_{k-1},\ldots,r_1)}$ at the kth step according to the outcomes of all previous steps $r_{k-1}, \ldots, r_1$ with binaries $r_k \in \{0, 1\}$. Such an architecture is universal for arbitrarily high-dimensional quantum systems, and greatly saves the hardware overhead.

Figure 3(a) provides a binary-tree illustration of the scheme, where the system evolves along the branches in the n-layer binary tree according to the ancilla outcomes $r_n, \ldots, r_1$, and each bifurcation (leaf) represents an elementary rank-2 AQUO. So, there are n measurements of the ancilla in sequence, which require $2^n - 1$ unitary gates and produce 2^n outcomes. For the qudit with $d = 4$ considered in this Letter, $n \equiv 2\log_2 d = 4$ layers are enough for full-rank ($m = d^2 = 16$) AQUOs. All d^2 Kraus operators $E_k = D_{ghij} C_{ghi} B_{gh} A_g$ can be realized through the appropriate choice of operators at each step $\{A_g, B_{gh}, C_{ghi}, D_{ghij}\}$ (see Supplemental Material, Ref. [20]), where g, h, i, j are the outcomes and $k = (ghij)_2$ is the binary representation.

In addition to the arbitrary state preparation and manipulation of quantum information through Eq. (2), the AQUO can also be translated to POVM $M_k = E_k^{\dagger} E_k$ when recording the outcome k with a probability $p_k = \mathrm{Tr}[E_k \rho_s E_k^{\dagger}]$ [1,15,35]. We next implement a quantum information processing task as an example to illustrate the application allowed by the powerful tool of AQUOs.

Figure 3(b) depicts the protocol for manipulating the quantum coherence of a qudit. The experimental procedure includes the state preparation, strictly incoherent operations (SIO) [36], and a SIC POVM [33] for output state tomography, and all steps are realized through AQUOs. The maximum coherent state $|\psi_{\mathrm{mcs}}\rangle = \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle + |3\rangle)$ and the maximally mixed state $\rho_{\mathrm{mms}} = \frac{1}{4} I_{4\times 4}$ are chosen as examples and prepared with fidelities $\mathcal{F} = 92.5\%$ and 98.8%, respectively, as shown in Figs. 3(c) and 3(d). The quantum coherences of the two experimental states are quantified by the relative entropy of coherence [36] $\mathcal{C} = 1.754$ and 0.018, respectively. The coherence of the system can be manipulated by the AQUO. Here we demonstrate two SIOs with ranks of 2 and 4, respectively (see Supplemental Material, Ref. [20]), which could also be used to measure the generalized parity of a bosonic mode. The performances of the two SIOs are tested by evaluating the fidelity and quantum coherence for the output of $|\psi_{\mathrm{mcs}}\rangle$. Here, the SIC POVM for a $d = 4$ dimensional system

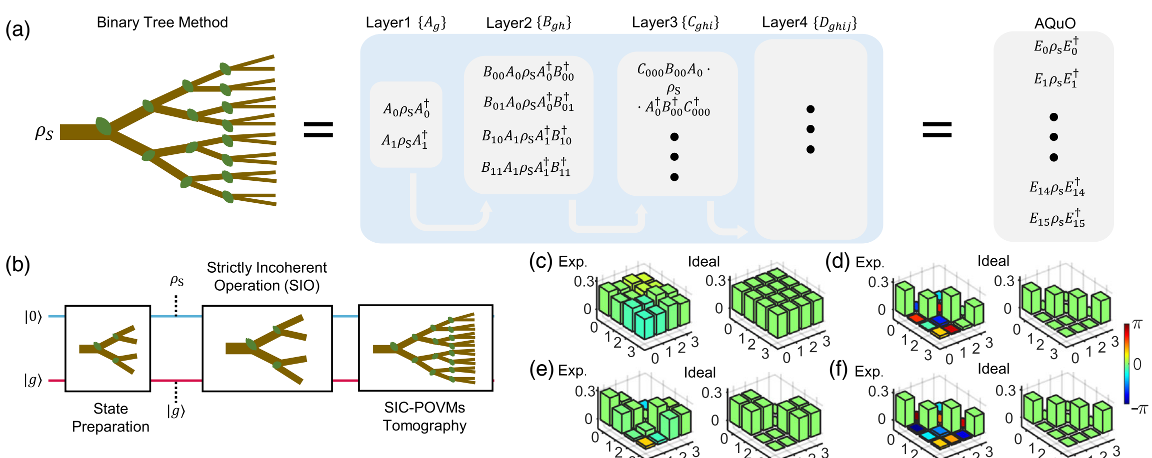

FIG. 3. AQUO with adaptive control of the ancilla. (a) The binary-tree illustration of the full-rank AQUO construction for a photonic qudit with $d = 4$. Each leaf represents an implementation of a rank-2 AQUO with a given unitary gate on the composite of the quantum system and the ancilla, and the evolution of the system follows one of the two branches according to the binary measurement outcomes of the ancilla (0 or 1). For a four-layer binary tree, there are four measurements that produce $2^4 = 16$ possible outcomes (0000 to 1111), and eventually up to 16 Kraus operators could be realized by $2^4 - 1$ unitaries (leaves). (b) An example quantum protocol for the manipulation of quantum coherence, which consists of arbitrary state preparation, SIO, and output state tomography through a symmetric informationally complete positive operator-valued measure (SIC POVM). (c) and (d) Density matrices of the photonic qudit with fidelities of 92.5% and 98.8% for the maximum coherent state $|\psi_{\mathrm{mcs}}\rangle$ and the maximally mixed state ρ_{mms}, respectively. (e) and (f) Density matrices of the photonic qudit after implementing two SIOs (rank-2 and rank-4) on $|\psi_{\mathrm{mcs}}\rangle$, with the output state fidelity of the photonic qudit being 92.2% and 99.1%, respectively.

contains 16 elements, corresponding to a rank-16 quantum operation, and thus is realized by a four-layer binary tree [Fig. 3(a)]. More details about the SIC POVM construction can be found in the Supplemental Material, Ref. [20].

The results of the quantum coherence manipulation are shown in Figs. 3(e) and 3(f). For the rank-2 SIO, the output state $\mathcal{F} = 92.2\%$ and $\mathcal{C} = 0.694$, while for the rank-4 SIO, the output state $\mathcal{F} = 99.1\%$ and $\mathcal{C} = 0.016$, proving a better coherence erasing ability of the rank-4 SIO. Note that the state tomography of the photonic qudit through a SIC POVM can be completed in several minutes, in sharp contrast to several hours required by the conventional Wigner tomography method in our experiment (see Supplemental Material, Ref. [20]), manifesting the advantage of AQUOs in practical quantum tasks.

Our AQUO can be extended to the case of coupling one control qubit to two manipulated cavities simultaneously, which may be of importance to realize dissipative evolution for generating and stabilizing various nontrivial two-body [49–55] and many-body states [56–59]. The AQUO can be easily implemented in other spin-oscillator systems, such as the trapped-ion system [39] and the hybrid superconducting-phononic system [40]. Importantly, the AQUO provides a unified framework of open quantum system control. For example, the recently demonstrated quantum error corrections that rely on the ancilla-induced Markovian dissipation for the correction [60–62] can be considered as rank-2 AQUOs, and the repeated adaptive phase estimations to create the Gottesman-Kitaev-Preskill state [63] resemble higher-rank AQUOs. This Letter therefore presents a significant conceptual advance in quantum technology that is beneficial for quantum information processing [1,42], quantum simulation [64,65], and quantum precision measurement [41].

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[20] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.090504 for a discussion of experimental device and setup, basics about quantum operations, experimental construction of AQuOs, imperfections, and state reconstruction and SIC-POVMs, which include Refs. [1,8,10,11,15,17,21–36].

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