Quasiparticle Energy Relaxation in a Gas of One-Dimensional Fermions with Coulomb Interaction

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We consider a system of charged one-dimensional spin-$\frac{1}{2}$ fermions at low temperature. We study how the energy of a highly excited quasiparticle (or hole) relaxes toward the chemical potential in the regime of weak interactions. The dominant relaxation processes involve collisions with two other fermions. We find a dramatic enhancement of the relaxation rate at low energies, with the rate scaling as the inverse sixth power of the excitation energy. This behavior is caused by the long-range nature of the Coulomb interaction.

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The Tomonaga-Luttinger liquid theory is widely used to describe low-energy properties of interacting fermions in one dimension [1]. It is based on the model of interacting fermions with linear dispersion, which admits an exact solution. The resulting excitation spectrum is that of a system of noninteracting bosons [2]. This idealization is appropriate in the low-energy limit. Importantly, this model is free of inelastic scattering, and thus it cannot describe relaxation of the system toward equilibrium.

Recent theoretical progress has shown the importance of the nonlinear corrections to the spectrum because they affect response functions and enable quasiparticle relaxation [3,4]. Experiments with one-dimensional conductors support these findings. In particular, the behavior of the response functions was probed in Refs. [5,6], equilibration rates for hot electrons and holes were measured in Ref. [7], and peculiar features of the relaxation of very hot electrons were observed in Ref. [8]. These experiments have demonstrated the crucial role of the curvature of the spectrum of electrons.

Significant theoretical progress has been achieved in the case of weakly interacting fermions with a quadratic spectrum [3,9]. In one dimension, pair collisions result in identical sets of momenta before and after scattering. As a result, the decay of quasiparticles is controlled by three-particle scattering processes [10]. For quasiparticles with energies near the Fermi level, the two types of processes shown in Fig. 1 should be considered. In the initial state, the scattering processes of type (a) [shown in Fig. 1(a)] have one particle with the opposite sign of momentum from the other two, whereas all three particles are near the same Fermi point for the processes of type (b) [shown in Fig. 1(b)]. Due to the conservation laws, the final states of the three particles are in the same configuration as the initial ones. It is worth noting that the processes of type (b) are allowed only at finite temperature T, whereas those of type (a) bring about the relaxation of quasiparticles even at $T = 0$ [9].

The relaxation of quasiparticles in the system of spin-$\frac{1}{2}$ fermions with weak Coulomb repulsion was considered in Ref. [11]. At zero temperature, a quasiparticle with the energy ϵ above the Fermi level decays with the rate $\tau^{-1} \propto \epsilon^2$ [12]. At finite temperatures, this result applies as long as $\epsilon \gg \sqrt{T\mu}$, where μ is the chemical potential of the Fermi gas. At energies below $\sqrt{T\mu}$, the quasiparticle relaxation rate was found to have only a weak dependence on energy: $\tau^{-1} \propto \ln^2(\mu/\epsilon)T$. Both rates are due to the processes shown in Fig. 1(a) [13].

It is important to note that in Ref. [11], the Coulomb interaction was assumed to be screened at small momentum transfers by a nearby gate, which enabled the authors to neglect the contribution of type (b) processes to the relaxation rate. In this Letter, we show that type (b) processes lead to a dramatically different behavior in the unscreened case. We found that at quasiparticle energies below $\sqrt{T\mu}$, it gives the dominant contribution to the

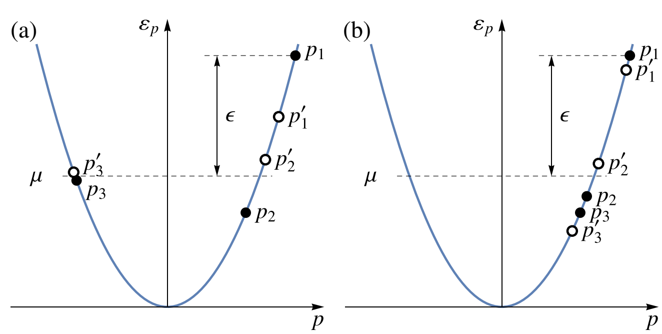

FIG. 1. Different scattering mechanisms that contribute to relaxation of quasiparticles in a one-dimensional system of weakly interacting fermions. At $T = 0$, only processes of type (a) are allowed. At nonzero temperature, processes of type (b) are responsible for dominant contribution to relaxation rate at energies $\epsilon \ll \sqrt{T\mu}$.

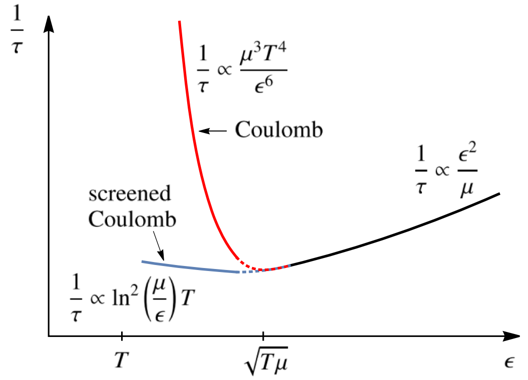

FIG. 2. Sketch of energy dependence of quasiparticle relaxation rate for spin-$\frac{1}{2}$ fermions with Coulomb (this work) and screened Coulomb (Ref. [11]) interactions. In former case, there is rapid increase of rate at energies below $\sqrt{T\mu}$ as opposed to a gradual logarithmic rise in the latter case. A similar sharp increase of the relaxation rate at low energies also occurs for holes.

relaxation rate, which behaves as $\tau_b^{-1} \propto \mu^3 T^4/\epsilon^6$; see Fig. 2 [12]. This implies a drastic enhancement of the rate as the quasiparticle excitation energy ϵ drops below the characteristic energy $\sqrt{T\mu}$, which is in contrast to the weak energy dependence of $\tau^{-1} \propto \ln^2(\mu/\epsilon)T$ for the screened case [11]. This behavior is qualitatively different from that of quasiparticles in most other systems of fermions, where the relaxation rate decreases at lower energies. For example, in three-dimensional Fermi liquids, $\tau^{-1} \propto \epsilon^2$ [19].

We study a one-dimensional system of fermions with quadratic dispersion $\varepsilon_p = p^2/2m$ and a weak two-body interaction. In the second quantization, the latter is described by

$$\hat{V} = \frac{1}{2L} \sum_{\substack{p_1,p_2,q \\ \sigma_1,\sigma_2}} V(q) \hat{a}^\dagger_{p_1+q,\sigma_1} \hat{a}^\dagger_{p_2-q,\sigma_2} \hat{a}_{p_2,\sigma_2} \hat{a}_{p_1,\sigma_1}. \quad (1)$$

Here, $\hat{a}$ and $\hat{a}^\dagger$ are the fermionic spin-$\frac{1}{2}$ operators obeying the standard anticommutation relations, L is the system size, and $V(q)$ is the Fourier transform of the two-body interaction potential. For electrons in a quantum wire, the latter has the Coulomb form $U(x) = e^2/|x|$ that should be cut off at short distances by the width of the wire w. Here, e denotes the electron charge. At small momenta $|q| \ll \hbar/w$, the Fourier transform of the interaction potential is $V(q) = 2e^2 \ln(\hbar/|q|w)$.

Let us consider a right-moving quasiparticle well above the Fermi level, i.e., with energy $\epsilon = \varepsilon_p - \mu \gg T$, where p denotes the quasiparticle momentum. Such an energetic quasiparticle, on average, loses its energy due to collisions with other quasiparticles, and thus drifts toward the Fermi level. The relaxation proceeds predominantly via three-particle scattering processes where the other two quasiparticles are near the Fermi level. In this case, the rate of energy change of the initial quasiparticle is given by

$$\dot{\epsilon} = \frac{1}{2} \sum_{\substack{p_1>p_2>p_3 \\ p_1'>p_2'>p_3'}} (\varepsilon_{p_1'} - \varepsilon_{p_1}) W^{p_1',p_2',p_3'}_{p_1,p_2,p_3} \times n_{p_2} n_{p_3} (1-n_{p_1'})(1-n_{p_2'})(1-n_{p_3'}) \delta_{p,p_1}. \quad (2)$$

Here, $W^{p_1',p_2',p_3'}_{p_1,p_2,p_3}$ is the scattering rate of the three fermions with momenta p_1, p_2, and p_3 into p_1', p_2', and p_3' summed over all spin indices, whereas n_p denotes the Fermi distribution function. The prefactor $\frac{1}{2}$ in Eq. (2) compensates for the summation over the spin of the initial quasiparticle. The main focus of this Letter is the quasiparticle relaxation that arises due to processes shown in Fig. 1(b). In this case, all the momenta that participate in the sum of Eq. (2) are positive.

The conservation laws of momentum and energy enable us to estimate the momentum change of the initial quasiparticle in a three-particle collision. For quadratic dispersion, we find

$$p_1 - p_1' = \frac{(p_3' - p_3)(p_3' - p_2)}{p_1 - p_2'}. \quad (3)$$

For the typical processes shown in Fig. 1(b), the momenta p_2, p_3, and p_3' are near the Fermi point, and $|p_3' - p_3|$, $|p_3' - p_2| \sim T/v_F$, where v_F is the Fermi velocity. In combination with the momentum conservation law, this yields

$$v_F|p_1 - p_1'| \sim \frac{T^2}{\epsilon} \ll \epsilon. \quad (4)$$

Thus, for type (b) processes, both the initial and final states have one highly excited quasiparticle, whereas the other two are always near the Fermi level. This enables us to identify the fermion at p_1' as a new state of the initial quasiparticle after the scattering event. Equation (2) shows how the energy of this quasiparticle changes with time. We define

$$\frac{1}{\tau} = -\frac{\dot{\epsilon}}{\epsilon} \quad (5)$$

as the energy relaxation rate. In this Letter, we distinguish it from the quasiparticle decay rate, which is obtained by omitting $(\varepsilon_{p_1'} - \varepsilon_{p_1})$ in Eq. (2).

For the processes shown in Fig. 1(a), after the scattering event, the two right-moving quasiparticles have energies on the order of ϵ [11]. This is qualitatively different from the case of type (b) processes, where only one quasiparticle in the final state has energy well above T. In Ref. [11], the definition of the energy relaxation rate equivalent to Eqs. (2) and (5) was applied to account for the effect of a finite temperature on the relaxation due to the processes of type (a). This means that out of the two right-moving quasiparticles with energies much greater than T, the one with the higher momentum was identified as a new state of the initial quasiparticle.

The scattering rate entering Eq. (2) can be found using Fermi's golden rule, where the matrix element is obtained in the second-order perturbation theory in the interaction given by Eq. (1) [10,20]. In order to take advantage of the conservation laws, we express the momenta p_1, p_2, and p_3 in terms of the new variables P, $\mathcal{E}$, and α as

$$p_j = \frac{1}{3}P - 2\sqrt{\frac{m\mathcal{E}}{3}}\cos\left(\alpha - \frac{2\pi j}{3}\right), \qquad j = 1, 2, 3. \tag{6}$$

Here, $P = p_1 + p_2 + p_3$ is the total momentum of three particles, whereas $\mathcal{E} = \varepsilon_{p_1} + \varepsilon_{p_2} + \varepsilon_{p_3} - P^2/6m$ is their total energy in the center-of-mass frame [21]. There are analogous formulas for the primed momenta. The conservation laws dictate that collisions do not affect P and $\mathcal{E}$, thus only changing the angle variable, $\alpha \to \alpha'$. This observation dictates the general form of the three-particle scattering matrix element

$$W_{p_1,p_2,p_3}^{p_1',p_2',p_3'} = \Theta(\mathcal{E},\alpha,\alpha')\delta(\mathcal{E}-\mathcal{E}')\delta_{P,P'}. \tag{7}$$

Starting with a general expression for $W_{p_1,p_2,p_3}^{p_1',p_2',p_3'}$ [10,20], after a somewhat tedious calculation, we obtain Eq. (7) with

$$\Theta = \frac{2592\pi e^8}{\hbar L^4 \mathcal{E}^2}\ln^2\left(\frac{\hbar^2}{mw^2\mathcal{E}}\right)\frac{f(\alpha+\alpha') + f(\alpha-\alpha')}{[\cos(3\alpha) - \cos(3\alpha')]^2}, \tag{8}$$

$$f(\theta) = \left[\sum_{j=1}^{3}\sin\left(\frac{\theta}{2}+\frac{2\pi j}{3}\right)\ln\left|\sin\left(\frac{\theta}{2}+\frac{2\pi j}{3}\right)\right|\right]^2. \tag{9}$$

This result applies to any three-particle scattering process, provided that $\ln(\hbar^2/mw^2\mathcal{E}) \gg 1$. The latter condition takes the forms $\hbar/wp_F \gg 1$ and $\hbar v_F/w\epsilon \gg 1$ for the processes of types (a) and (b), respectively. Here, $p_F = \sqrt{2m\mu}$ is the Fermi momentum.

We begin our evaluation of the relaxation rate of a quasiparticle with the energy $\epsilon = \varepsilon_p - \mu$ via type (b) processes by analyzing Eq. (2). The distribution functions at low temperature severely constrain the configurations of momenta, which give significant contribution to $\dot{\epsilon}$. In the zero temperature limit, we have $p_2, p_3 \to p_F$ corresponding to $\mathcal{E}^* = \epsilon^2/6\mu$ and $\alpha^* = 5\pi/3$; see Eq. (6). We account for the deviations of p_2, p_3, p_2', and p_3' from p_F and of p_1' from p_1 at finite temperature in the leading order in small parameters $\varrho = (\mathcal{E}-\mathcal{E}^*)/\mathcal{E}^*$, $\sigma = \alpha - \alpha^*$, and $\sigma' = \alpha' - \alpha^*$. Function (8) is only weakly dependent on ϱ, which we can therefore neglect, leading to

$$\Theta = \frac{4608\pi e^8\mu^2}{\hbar L^4\epsilon^4}\ln^2\left(\frac{\hbar v_F}{w\epsilon}\right)\left[\frac{\ln^2|\sigma-\sigma'|}{(\sigma+\sigma')^2} + \frac{\ln^2|\sigma+\sigma'|}{(\sigma-\sigma')^2}\right]. \tag{10}$$

Equation (10) is singular at $\sigma = \pm\sigma'$, which corresponds to the nullification of the energy denominators in the initial expression of the second-order perturbation theory for the scattering rate (7). For type (b) processes, these singularities lead to a divergent quasiparticle decay rate, which is defined by omitting the energy difference $(\varepsilon_{p_1'} - \varepsilon_p)$ in the right-hand side of Eq. (2). However, the energy relaxation rate given by Eqs. (5) and (2) is well defined.

We are now in a position to evaluate the rate of quasiparticle energy change $\dot{\epsilon}$ using Eq. (2). Converting the sum into an integral over the variables P, ϱ, σ, and their primed versions, we first perform the integrations that involve the δ functions and then integrate over ϱ. The remaining integral over σ and σ' is an antisymmetric function, and thus nullifies the rate if one approximates $n_{p_1'}$ by n_p. Accounting for the leading-order deviation in the distribution function of p_1' results in a term proportional to $\sigma^2 - \sigma'^2$ [22]. In combination with the energy difference in Eq. (2), which is also proportional to $\sigma^2 - \sigma'^2$, it regularizes the singularities arising from Eq. (10). For the resulting relaxation rate, we eventually obtain [22]

$$\frac{1}{\tau_b} = \frac{64\pi}{5\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4\ln^2\left(\frac{\hbar v_F}{w\epsilon}\right)\ln^2\left(\frac{\epsilon}{T}\right)\frac{\mu^3 T^4}{\epsilon^6}. \tag{11}$$

Equation (11) is our main result. We now compare it with the energy relaxation rate due to the competing type (a) processes [11].

Unlike the processes shown in Fig. 1(b), the ones of Fig. 1(a) contribute to quasiparticle relaxation even at $T = 0$. In this case, the quasiparticle decay rate is well defined despite the singularities in Eq. (10). It is given by [11]

$$\frac{1}{\tau_a} \sim \frac{1}{\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4\ln^2\left(\frac{\hbar}{wp_F}\right)\ln^2\left(\frac{\mu}{\epsilon}\right)\frac{\epsilon^2}{\mu}. \tag{12}$$

The evaluation of the decay rate at finite temperatures is plagued by the singularities of Eq. (10). Instead, the energy relaxation rate (5) can be studied. At $T \gg \epsilon^2/\mu$, the result

$$\frac{1}{\tau_a} \sim \frac{1}{\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4\ln^2\left(\frac{\hbar}{wp_F}\right)\ln^2\left(\frac{\mu}{\epsilon}\right)T \tag{13}$$

was found in Ref. [11]. It is worth mentioning that at $T = 0$, the energy relaxation rate has the same form as the quasiparticle decay rate (12), albeit with a different numerical prefactor [22,23]. A comparison of Eqs. (11)–(13) shows that the quasiparticles with energies $\epsilon \gg \sqrt{T\mu}$ decay with the rate (12), whereas at $T \ll \epsilon \ll \sqrt{T\mu}$, our result (11) gives the dominant contribution [12]. For unscreened Coulomb interaction, we conclude that the contribution (13) is always subdominant.

We now briefly discuss the relaxation of a hole, which represents the absence of a fermion in the Fermi sea. Because they propagate at speeds below the Fermi velocity, holes are stable excitations at zero temperature. At nonzero temperatures, they drift toward the Fermi level as a result of scattering off other excitations. At $\epsilon_h \gg T$, where $\epsilon_h = \mu - \varepsilon_p$ denotes the energy of the hole, the corresponding rate of energy change and the relaxation rate can be obtained from the expressions analogous to Eqs. (2) and (5). In Eq. (2), one should properly order the summation indices and replace the quasiparticle distribution function n_p, the dispersion ε_p, and ϵ, respectively, by the corresponding quantities for holes: $1 - n_p$, $-\varepsilon_p$, and ϵ_h. For type (b) processes, the evaluation parallels the one for particles and results in the relaxation rate (11), with ϵ replaced by ϵ_h.

Holes can also relax due to processes that involve quasiparticles near both Fermi points; see Fig. 3. Since the left-moving pair has a characteristic momentum $|p_3 - p_3'| \lesssim T/v_F$, from Eq. (3), we find the energy change of the hole

$$\Delta\epsilon_h = \frac{p_1' + p}{2m}(p_1' - p_1) \lesssim \min\left(\epsilon_h, \frac{T\mu}{\epsilon_h}\right). \quad (14)$$

At $\epsilon_h \gg \sqrt{T\mu}$, we have $\Delta\epsilon_h \ll \epsilon_h$, i.e., the hole loses a small fraction of its energy in a three-particle collision. For such deep holes, we can define the rate of energy change $\dot{\epsilon}_h$ and the relaxation rate τ_h^{-1} using the approach analogous to that of Eqs. (2) and (5) for particlelike excitations. The rate of energy change of a hole is given by [22]

$$\dot{\epsilon}_h = -\frac{1}{\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4 \ln^2\left(\frac{\hbar}{w p_F}\right) T^2 F\left(\frac{p}{p_F}\right), \quad (15)$$

where

$$F(a) = \frac{2}{\pi}\left(\ln\frac{1-a^2}{4} + a\ln\frac{1+a}{1-a}\right)^2 \frac{a^2}{(1-a^2)^3}. \quad (16)$$

Equation (15) is valid for deep holes, i.e., for $\epsilon_h = \mu - \varepsilon_p \gg \sqrt{T\mu}$. In the special case $\sqrt{T\mu} \ll \epsilon_h \ll \mu$ corresponding to deep holes near the Fermi level, from Eq. (15),

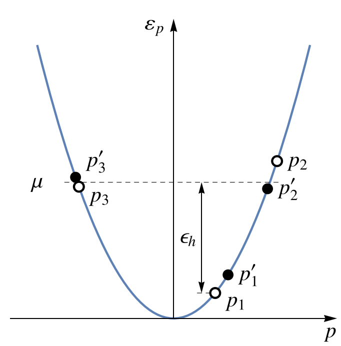

FIG. 3. Dominant scattering mechanism that contributes to relaxation of a deep hole, i.e., at $\epsilon_h \gg \sqrt{T\mu}$.

we find $\tau_h^{-1} \propto \mu T^2/\epsilon_h^2$ [12]. This result is consistent with the corresponding expression given in Ref. [11]. We note that Eq. (15) was obtained to the leading order in low temperature, which limits its applicability to $p \gg \sqrt{mT}$. An accurate expression for smaller p is obtained by multiplying Eq. (15) by $1 + mT/p^2$ [22].

Equation (13) for the energy relaxation rate due to the processes shown in Fig. 1(a) [11] and our Eq. (11) for relaxation due to the processes of Fig. 1(b) are applicable to both particles and holes. In particular, they apply to shallow holes with energies in the range $T \ll \epsilon_h \ll \sqrt{T\mu}$ [22]. Comparing the obtained results, we find that the relaxation of deep holes occurs primarily due to processes shown in Fig. 3. In this case, Eq. (15) gives the dominant contribution to their rate of energy change. In contrast, the relaxation of shallow holes with energies in the range $T \ll \epsilon_h \ll \sqrt{T\mu}$ is controlled by processes shown in Fig. 1(b). Their relaxation rate is given by Eq. (11) with ϵ replaced by ϵ_h, whereas the corresponding rate of energy change follows from Eq. (5).

In this Letter, we studied quasiparticles with energies $\epsilon \gg T$. This condition was important for the applicability of the approach based on Eq. (2), which assumes that the initial state of momentum p is not thermally populated. At $\epsilon \sim T$, one must account for the effect of the thermal population of the state p, which can be achieved in a Boltzmann equation description. An order of magnitude estimate of the typical relaxation rate of the distribution function in the latter approach can be obtained by extrapolating the rate (11) to $\epsilon \sim T$,

$$\frac{1}{\tau_b} \sim \frac{1}{\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4 \ln^2\left(\frac{\hbar v_F}{wT}\right)\frac{\mu^3}{T^2}. \quad (17)$$

Unlike most other systems of fermions, in our case, the relaxation rate increases at lower temperatures. This can be attributed to the long-range nature of Coulomb interaction, which results in a singularity of the interaction potential at zero momentum, and thus enhances scattering at small momentum transfer [20].

The fact that the relaxation rate (17) increases at $T \to 0$ raises an important question of the applicability of the picture of fermionic quasiparticles and holes used in this Letter. Indeed, at sufficiently low temperature, one may expect to reach the regime where the standard assumption of $\hbar/\tau_b \ll T$ is violated. In this case, the uncertainty of the energy of a typical quasiparticle $\delta\epsilon \sim \hbar/\tau_b$ is comparable to or larger than the energy itself, $\epsilon \sim T$, and the quasiparticles are no longer well defined. In addition, in systems of weakly interacting spin-$\frac{1}{2}$ fermions, the well-known phenomenon of spin-charge separation [1,24] results in the breakdown of the fermionic quasiparticle description. As a result, only the excitations with sufficiently high energies can be treated as quasiparticles [11]. For an excitation with energy $\epsilon \sim T$ in a system with long-range interactions, the condition of Ref. [11] can be presented in the form $T \gg p_F V(T/v_F)/\hbar$. For Coulomb interactions, this yields

$$T \gg T^* = \mu \frac{e^2}{\hbar v_F} \ln\left(\frac{\hbar^2 v_F}{w p_F e^2}\right). \qquad (18)$$

Our results are obtained under the assumptions that the interactions are weak ($e^2/\hbar v_F \ll 1$) and the width of the channel is small ($w p_F/\hbar \ll 1$). In this case, Eq. (18) ensures that the condition $\hbar/\tau_b \ll T$ is also satisfied.

A promising experimental setup to test our results is that of Ref. [7]. It consists of a grounded quantum wire connected to two spatially separated electrodes. One electrode is used to inject the current of the particles within a band of momenta, whereas the other collects the current in the same band. In the experiment [7], the collected current of particlelike excitations was greater than the injected one. This was interpreted to occur due to relatively fast relaxation of quasiparticles, creating more excitations in the energy window of the collected current, which enabled one to estimate the quasiparticle relaxation rate [3,11]. For a similar setup with unscreened Coulomb interactions, we expect the experimentally observed relaxation rate to be dramatically enhanced in the energy window between T and $\sqrt{T\mu}$.

The relaxation rate (11) was derived for pure Coulomb interaction. The effect of screening of the interaction by a gate will not modify Eq. (11) in the range of energies $\epsilon_d \ll \epsilon \ll \sqrt{T\mu}$, where $\epsilon_d = \hbar v_F/d$ and d is the distance to the gate. At $\epsilon \ll \epsilon_d \ll \sqrt{T\mu}$, the relaxation rate still rises with decreasing ϵ as [22]

$$\frac{1}{\tau_b} = \frac{576\pi}{5\hbar}\left(\frac{e^2}{\hbar v_F}\right)^4 \ln^2\left(\frac{d}{w}\right) \ln^2\left(\frac{\epsilon_d}{\epsilon}\right) \frac{\mu^3 T^4}{\epsilon^2 \epsilon_d^4}. \qquad (19)$$

Up to the numerical coefficient and logarithmic factors, Eq. (19) is consistent with the estimate of Ref. [11]. It is important to note that the rate of Eq. (19) is much larger than the rate controlled by the processes of Fig. 1(a). Therefore, our conclusions do not change qualitatively in the case of the interaction screened by a gate.

In summary, we have studied the rate of energy relaxation for quasiparticles and holes in a weakly interacting one-dimensional system of fermions with Coulomb repulsion. Compared to the case of screened interaction, we have found that the scattering processes shown in Fig. 1(b) lead to a dramatic enhancement of the quasiparticle relaxation rate at low energies, $\tau^{-1} \propto \epsilon^{-6}$ at $T \ll \epsilon \ll \sqrt{T\mu}$; see Fig. 2. A similar enhancement also holds for shallow holes. For deep holes, we have obtained their energy relaxation at arbitrary momenta; see Eq. (15).

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[21] Equivalently, $\mathcal{E} = [(p_1 - p_2)^2 + (p_1 - p_3)^2 + (p_2 - p_3)^2]/6m$.

[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.086803 for the details.

[23] For this reason, we use the same notation for both decay and relaxation rates.

[24] I. E. Dzyaloshinskii and A. I. Larkin, Correlation functions for a one-dimensional Fermi system with long-range interaction (Tomonaga model), Sov. Phys. JETP **38**, 202 (1974).