Stability of Time-Reversal Symmetry Protected Topological Phases

Tian-Shu Deng,[1,*] Lei Pan,[1,*] Yu Chen,[2,†] and Hui Zhai[1,‡]
[1]*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*
[2]*Graduate School of China Academy of Engineering Physics, Beijing 100193, China*

(Received 2 October 2020; revised 12 March 2021; accepted 13 July 2021; published 17 August 2021)

In a closed system, it is well known that the time-reversal symmetry can lead to Kramers degeneracy and protect nontrivial topological states such as the quantum spin Hall insulator. In this Letter, we address the issue of whether these effects are stable against coupling to the environment, provided that both the environment and the coupling to the environment also respect time-reversal symmetry. By employing a non-Hermitian Hamiltonian with the Langevin noise term and utilizing the non-Hermitian linear response theory, we show that the spectral functions for Kramers degenerate states can be split by dissipation, and the backscattering between counterpropagating edge states can be induced by dissipation. The latter leads to the absence of accurate quantization of conductance in the case of the quantum spin Hall effect. As an example, we demonstrate this concretely with the Kane-Mele model. Our study can also include interacting topological phases protected by time-reversal symmetry.

DOI: 10.1103/PhysRevLett.127.086801

Time-reversal symmetry (TRS) protected topological phases, such as the TRS protected topological insulator (TI) in two and three dimensions, are intriguing states of matter that have been extensively studied in the past two decades. Unlike the quantum Hall effect, the topological classifications of these states require the presence of TRS [1]. With TRS, the nontrivial topological properties have been firmly established in a closed system [2,3]. A natural question is whether the concept of TRS protected topology, as well as its physical consequences such as quantized conductance, still holds in the presence of coupling to the environment. This is certainly a very important issue, because, for any practical applications of these topological materials, it is inevitable that the materials should be coupled to an external environment.

A natural expectation is that protection from TRS is guaranteed if the Hamiltonian of the system, the environment, and the coupling between the environment and the system all obey TRS. However, this expectation was recently challenged by McGinley and Cooper [4]. They show explicitly that the coupling to the environment can lead to decoherence between two Kramers doublet states even though the coupling and the environment both obey TRS. Their argument is deeply rooted in the fact that, even if an isolated system obeys TRS, its subsystem can behave as seemingly violating the TRS. Actually, this fact plays a key role in thermalization of a closed quantum system. In quantum thermalization, considering a pure state of a closed system whose evolution equations obey the TRS, the evolution of its subsystem can undergo an irreversible process that loses information and reach thermalization with the rest of the system acting as a bath [5,6].

Without loss of generality, we consider an open quantum system coupled to the environment through a pair of operators $\hat{\mathcal{O}}$ and $\hat{\mathcal{O}}^\dagger$, and both operators obey TRS but do not have to be Hermitian. Nevertheless, the entire Hamiltonian, including the system, the system-environment coupling, and the environment itself, is Hermitian and obeys TRS. By treating the degrees of freedom of the environment with the Markovian approximation, the open quantum system can be described by a non-Hermitian Hamiltonian with a Langevin noise term, which ensures the quantum mechanical commutative relation and preserves the trace of the density matrix [7]. This non-Hermitian Hamiltonian can be generally written as

$$\hat{H} = \hat{H}_0 - i\gamma\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}} + \hat{\mathcal{O}}^\dagger\hat{\xi} + \hat{\xi}^\dagger\hat{\mathcal{O}}, \tag{1}$$

where γ is the dissipation strength. $\hat{H}_0$ is the Hermitian Hamiltonian of the system itself, and it also obeys TRS. Here we should emphasize that $\hat{H}_0$ does not have to be a noninteracting one [7]. $\hat{\xi}$ is the Langevin noise operators that satisfy $\langle\hat{\xi}(t)\hat{\xi}^\dagger(t')\rangle = 2\gamma\delta(t-t')$ and $\langle\hat{\xi}^\dagger(t)\hat{\xi}(t')\rangle = \langle\hat{\xi}(t)\hat{\xi}(t')\rangle = \langle\hat{\xi}^\dagger(t)\hat{\xi}^\dagger(t')\rangle = 0$. All the calculation done with this non-Hermitian calculation should be accompanied by averaging over the Langevin noise term in the end. In Ref. [12], we have developed a non-Hermitian linear response theory. This theory starts with the equilibrium state of $\hat{H}_0$ and treats dissipation order by order, which determines how an equilibrium system responds to weak dissipation. To implement the non-Hermitian linear response theory, we should introduce an interaction picture which separates out the dissipation term from the system term. For instance, in the interaction picture, we should define $\hat{\mathcal{O}}^I(t) = e^{i\hat{H}_0 t}\hat{\mathcal{O}}e^{-i\hat{H}_0 t}$, and similar definitions for other operators with upper scribe I.

Summary of results.—We consider generally a pair of Kramers degenerate eigenstates of $\hat{H}_0$, say, $|\Psi\rangle_1$ and $|\Psi\rangle_2$. When we specifically consider a TRS protected TI, they can be chosen as a pair of degenerate edge states located at the same edge. We denote the Hilbert space formed by these two states as $\mathcal{H}_K$. In this Letter, by studying the linear response of the density matrix, the Green's function, and the matrix element of a local impurity potential, respectively, we obtain three main results as summarized below. (i) *Loss of coherence*.—Suppose that initially the quantum state is a pure state in $\mathcal{H}_K$ as $|\Psi\rangle = \alpha_1|\Psi_1\rangle + \alpha_2|\Psi_2\rangle$, where $\alpha_{i=1,2}$ are two constants and the initial density matrix is given by $\hat{\rho}_K(0) = |\Psi\rangle\langle\Psi|$. By turning on the dissipation, the quantum state evolves under $\hat{H}$, and the density matrix becomes $\hat{\rho}(t)$. By projecting onto $\mathcal{H}_K$ space by the projection operator $\hat{\Pi}_K$, one can obtain the projected density matrix $\hat{\rho}_K(t) = 1/\mathcal{N}\hat{\Pi}_K\hat{\rho}(t)\hat{\Pi}_K$, with normalization factor $\mathcal{N} = \mathrm{Tr}[\hat{\Pi}_K\hat{\rho}(t)\hat{\Pi}_K]$. We define $\delta\hat{\rho}_K(t) = \hat{\rho}_K(t) - \hat{\rho}_K(0)$. We show that $\delta\hat{\rho}_K(t)$ is not proportional to $\hat{\rho}_K(0)$. (ii) *Break of degeneracy.*—With dissipation, the retarded Green's function in $\mathcal{H}_K$ space is a two-by-two matrix $\mathcal{G}$ with the matrix elements defined as

$$\mathcal{G}_{ij} = -i\Theta(t)\langle\{\hat{c}_i(t), \hat{c}_j^\dagger(0)\}\rangle, \tag{2}$$

where $\hat{c}_i$ and $\hat{c}_i^\dagger$ are annihilation and creation operators, respectively, corresponding to eigenstates $|\Psi_i\rangle$ of $\hat{H}_0$ and $\hat{c}_i(t) = e^{i\hat{H}t}\hat{c}_i e^{-i\hat{H}t}$. We show that $\mathcal{G}_{ij}$ is no longer proportional to an identity matrix in the $\mathcal{H}_K$ space. (iii) *Presence of backscattering*.—For a local impurity potential $\hat{V}$, we consider the matrix element of this impurity potential between two Kramers states, i.e., $V_{ij}^0 = \langle\Psi_i|\hat{V}|\Psi_j\rangle$. Suppose, without dissipation, this matrix element is identically zero for $i \neq j$. This can be satisfied, for instance, when $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are, respectively, the left-moving and the right-moving edge states of a quantum spin Hall state. With dissipation, we need to consider

$$V_{ij}(t) = \langle\Psi_i|\hat{V}(t)|\Psi_j\rangle, \tag{3}$$

where $\hat{V}(t) = e^{i\hat{H}t}\hat{V}e^{-i\hat{H}t}$. We show that $V_{ij}(t) \neq 0$ for $i \neq j$.

Result (i) naturally leads to $S_v(t) \neq S_v(t=0)$, where $S_v(t) = -\mathrm{Tr}\hat{\rho}_K(t)\log\hat{\rho}_K(t)$ is the von Neumann entropy. That is to say, for $t > 0$, the entropy becomes nonzero, and the system loses its phase coherence. This is consistent with the result presented in Ref. [4]. Results (ii) and (iii) are the central results of this work. With result (ii), we can further plot the spectrum function $A(\omega)$, which shows two split peaks. This means the lack of Kramers degeneracy for a non-Hermitian open system even though the coupling to environment also respects the TRS. Result (iii) is directly related to the two-dimensional quantum spin Hall. Quantized conductance is the hallmark of quantum spin Hall, due to the forbidden backscattering between the left- and the right-moving edge states. Therefore, the presence of backscattering means that the conductance of a quantum spin Hall state is not perfectly quantized. This physics has been qualitatively discussed in Refs. [4,13]. This is perhaps one of the reasons that accuracy of quantization observed in quantum spin Hall samples so far [13–16] is far less than that observed in quantum Hall samples, in addition to other possible explanation such as the inelastic scatterings [14,17,18] and electromagnetic noise-induced scattering [19]. These results are essentially due to the irreversible nature of the bath and bear a lot of similarity to the H-theorem in statistical mechanics. In other words, these results can be viewed as the manifestations of the H-theorem in the TRS protected topology. Time-reversal symmetry can also be spontaneously broken in the dynamical evolution, as pointed out in recent papers [20,21].

Application of Schur's lemma.—Before proceeding into the details of the derivation, we should emphasize that these results essentially rely on the TRS being an antiunitary symmetry. In other words, if the symmetry that protects the topological phase is a unitary symmetry, the phenomena (i)–(iii) described above should not occur. Mathematically, the difference is rooted in the celebrated Schur's lemma in the group theory [22]. Schur's lemma says that, for a unitary group, if an operator $\hat{M}$ commutes with all elements in the group, then this operator, in an irreducible representation, has to be proportional to an identity matrix. Nevertheless, when Schur's lemma is applied to an antiunitary group, not only the operator $\hat{M}$ has to commute with all elements in the group, but also the operator $\hat{M}$ has to be a Hermitian operator, and then this operator is proportional to an identity matrix in an irreducible representation [23]. As we will see below, when the Hermitian condition and respecting the antiunitary symmetry condition cannot be satisfied simultaneously, this operator is generally no longer proportional to identity. This is the key mathematical reason responsible for the difference between the unitary symmetry protection and the antiunitary symmetry protection.

To be more concrete, we will give two examples that will be used below.

The first example is about $\hat{\Pi}_K\hat{\mathcal{O}}^I(t)\hat{\mathcal{O}}^{\dagger,I}(t)\hat{\Pi}_K$. By using the fact that the states in $\mathcal{H}_K$ are degenerate states of $\hat{H}_0$, we have $\hat{\Pi}_K e^{\pm i\hat{H}_0 t} = e^{\pm i\hat{H}_0 t}\hat{\Pi}_K = e^{\pm iE_0 t}\hat{\Pi}_K$, and, therefore, $\hat{\Pi}_K\hat{\mathcal{O}}^I(t)\hat{\mathcal{O}}^{\dagger,I}(t)\hat{\Pi}_K = \hat{\Pi}_K\hat{\mathcal{O}}\hat{\mathcal{O}}^\dagger\hat{\Pi}_K$. Note that $\hat{\Pi}_K\hat{\mathcal{O}}\hat{\mathcal{O}}^\dagger\hat{\Pi}_K$ is Hermitian and time-reversal symmetric. It is also important to note that the Hilbert space $\mathcal{H}_K$ of two Kramers degenerate states forms an irreducible representation of the TRS; thus, the projection $\hat{\Pi}_K$ enforces the restriction to an irreducible space of TRS. Therefore, this term obeys Schur's lemma, and, consequently, it is proportional to identity. The same holds for $\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t)\hat{\mathcal{O}}^I(t)\hat{\Pi}_K$.

The second example is about $\hat{\Pi}_K\hat{\mathcal{O}}^I(t)\hat{\Pi}_K$ and $\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t)\hat{\Pi}_K$. It can also be shown that $\hat{\Pi}_K\hat{\mathcal{O}}^I(t)\hat{\Pi}_K = \hat{\Pi}_K\hat{\mathcal{O}}\hat{\Pi}_K$. Because $\hat{\mathcal{O}}$ is time-reversal symmetric but is generally not Hermitian, this term does not satisfy Schur's lemma for the antiunitary TRS. The same holds for $\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t)\hat{\Pi}_K$. However, for unitary symmetry, because Schur's lemma does not require the operator being Hermitian, all these operators are proportional to identity in an irreducible space if they obey the unitary symmetry. Hence, the unitary symmetry protected topological states are stable against coupling to environment.

Loss of coherence.—Here, we consider density matrix in the interaction picture $\hat{\rho}(t) = \hat{\mathcal{U}}(t)\hat{\rho}_K(0)\hat{\mathcal{U}}^\dagger(t)$ with $\hat{\mathcal{U}}(t) = e^{i\hat{H}_0 t}e^{-i\hat{H}t}$, and, by expanding $\hat{\rho}(t)$ to the leading order of γ and averaging the noise, we can obtain

$$\hat{\rho}(t) - \hat{\rho}_K(0) = 2\gamma\int_0^t\Big(-\frac{1}{2}\{\hat{\mathcal{O}}^{\dagger,I}(t')\hat{\mathcal{O}}^I(t'),\hat{\rho}_K(0)\} + \hat{\mathcal{O}}^I(t')\hat{\rho}_K(0)\hat{\mathcal{O}}^{\dagger,I}(t')\Big)dt', \tag{4}$$

where the second term results from averaging over the Langevin noise. By projecting back to $\mathcal{H}_K$, the first term in the rhs in Eq. (4) can be written as

$$\{\hat{\Pi}_K\hat{\mathcal{O}}^I(t')\hat{\mathcal{O}}^{\dagger,I}(t')\hat{\Pi}_K,\hat{\rho}_K(0)\}, \tag{5}$$

and the second term in the rhs in Eq. (4) can be written as

$$[\hat{\Pi}_K\hat{\mathcal{O}}^I(t')\hat{\Pi}_K]\hat{\rho}_K(0)[\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t')\hat{\Pi}_K]. \tag{6}$$

With the two examples discussed above, we can conclude that Eq. (5) is proportional to $\hat{\rho}_K(0)$ but Eq. (6) is not proportional to $\hat{\rho}_K(0)$. Hence, $\delta\hat{\rho}_K(t)$ is not proportional to $\hat{\rho}_K(0)$, and the entropy changes.

Break of degeneracy.—Here, we apply the linear response theory to the Green's function defined in Eq. (2) and consider that the Kramers doublet are both occupied by a pair of fermions. Similar as the discussion above, we consider $\delta\mathcal{G}_{ij} = \mathcal{G}_{ij} - \mathcal{G}_{ij}^{(0)}$, where

$$\mathcal{G}_{ij}^{(0)} = -i\Theta(t)\langle\{\hat{c}_i^I(t),\hat{c}_j^{\dagger,I}(0)\}\rangle \tag{7}$$

and $\mathcal{G}_{ij}^{(0)}$ is the Green's function without dissipation. It is easy to see that $\mathcal{G}_{ij}^{(0)} \propto \delta_{ij}$. What we need to show is that $\delta\mathcal{G}_{ij}$ is not proportional to δ_{ij}. We can also expand $\delta\mathcal{G}_{ij}$ to the leading order of γ. We shall not show the full expression of this term here [7]. Generally speaking, there are two types of terms in the leading-order expansion. One kind of term includes, for instance,

$$\int_0^t\langle\hat{c}_j^{\dagger,I}(0)[\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t_1)\hat{\mathcal{O}}^I(t_1)\hat{\Pi}_K]\hat{c}_i^I(t)\rangle dt_1, \tag{8}$$

which involves $\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t_1)\hat{\mathcal{O}}^I(t_1)\hat{\Pi}_K$. The other kind of term includes, for instance,

$$\int_0^t\langle\hat{c}_j^{\dagger,I}(0)[\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t_1)\hat{\Pi}_K]\hat{c}_i^I(t)[\hat{\Pi}_K\hat{\mathcal{O}}^I(t_1)\hat{\Pi}_K]\rangle dt_1, \tag{9}$$

which involves $\hat{\Pi}_K\hat{\mathcal{O}}^I(t_1)\hat{\Pi}_K$ and $\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t_1)\hat{\Pi}_K$. With the two examples discussed above, we can also see that the first kind of term is still proportional to δ_{ij}, but the second kind of term is not. Hence, up to the leading order of γ, $\mathcal{G}$ is already not an identity matrix, and the spectrum is split.

Presence of backscattering.—Here, we consider the matrix element defined in Eq. (3). Similarly, we define $\delta V_{ij} = V_{ij}(t) - V_{ij}^0$, and we expand δV_{ij} to the leading order of γ [7]. Here, as a typical example, we focus on one of the terms that are similar to the ones discussed above, which reads

$$\int_0^t\langle\Psi_i|[\hat{\Pi}_K\hat{\mathcal{O}}^{\dagger,I}(t_1)\hat{V}^I(t)\hat{\mathcal{O}}^I(t_1)\hat{\Pi}_K]|\Psi_j\rangle dt_1 = \int_0^t\langle\Psi_i|[\hat{\Pi}_K\hat{\mathcal{O}}^\dagger e^{i\hat{H}_0(t-t_1)}\hat{V}e^{-i\hat{H}_0(t-t_1)}\hat{\mathcal{O}}\hat{\Pi}_K]|\Psi_j\rangle dt_1. \tag{10}$$

Here, we should note a difference between the discussion here and the two cases above. The above two results can both be proved within the Kramers degenerate space $\mathcal{H}_K$. However, if in this case we are restricted in the $\mathcal{H}_K$ space, V^I is an identity matrix that commutes with $\hat{H}_0$. Thus, Eq. (10) becomes

$$\int_0^t\langle\Psi_i|[\hat{\Pi}_K\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}\hat{\Pi}_K]|\Psi_j\rangle dt_1, \tag{11}$$

where $\hat{\Pi}_K\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}\hat{\Pi}_K$ is Hermitian and obeys TRS. One can show that this holds for other terms in the leading-order expansion of δV_{ij}. Therefore, restricted in $\mathcal{H}_K$ space, it is an identity matrix and cannot induce backscattering.

Hence, we should consider the $\hat{V}$ operator out of $\mathcal{H}_K$ space, where $\hat{V}$ is no longer represented as identity and, in general, does not commute with $\hat{H}_0$. Then, it is easy to see that the operator in the square brackets in Eq. (10) does not respect TRS, although it is a Hermitian one. Therefore, this term does not obey Schur's lemma and is not proportional to identity. Once not an identity matrix, nothing guarantees this term to be a diagonal matrix, and, generically, the off-diagonal matrix elements exist, which lead to $\delta V_{ij} \neq 0$ for $i \neq j$. Similar discussions can be applied to other terms in the first-order expansion of δV_{ij}. Taking i and j as a pair of degenerate counter-propagating edge states of a quantum spin Hall, we have now established the presence of backscattering and the absence of perfect quantization of conductance.

Example: The Kane-Mele model with dissipation.—Here, we use the celebrated Kane-Mele model on a honeycomb lattice for two-dimensional quantum spin Hall to illustrate these three results more concretely [24]. For this model, we have

$$\hat{H}_0 = J\sum_{\langle i,j\rangle,s}\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + i\lambda_{\mathrm{SO}}\sum_{\langle\langle i,j\rangle\rangle,s,s'}\nu_{ij}\hat{c}^\dagger_{i,s}\sigma^z_{ss'}\hat{c}_{j,s'} + i\lambda_R\sum_{\langle i,j\rangle,s,s'}\hat{c}^\dagger_{i,s}(\boldsymbol{\sigma}\times\boldsymbol{d}_{ij})^z_{ss'}\hat{c}_{j,s'} + \lambda_\nu\sum_{i,s}\xi_i\hat{c}^\dagger_{i,s}\hat{c}_{i,s}, \quad (12)$$

where i and j are the site index, s and s' are the spin index, and $\boldsymbol{\sigma}$ are the Pauli matrices. The first term is the nearest-neighbor hopping with strength J. The second term is a spin-orbit coupling between second-neighbor hopping, with $\nu_{ij} = \pm 1$ and strength λ_{SO}. The third term is the nearest Rashba term with strength λ_R, where $\boldsymbol{d}_{ij}$ is the vector connecting i and j sites. The last term is a staggered potential with $\xi_i = \pm 1$ for different sublattices and strength λ_ν. We choose the parameters such that the model is in the topological nontrivial insulator state. Moreover, the spin-rotational, the mirror, and the reflectional symmetries are all explicitly broken such that the degeneracy can come only from the time-reversal symmetry.

We introduce the coupling operator $\hat{O}$ either defined on site i as

$$\hat{O} = i\sum_{s,s'}\hat{c}^\dagger_{i,s}\sigma^y_{ss'}\hat{c}_{i,s'}, \qquad \hat{O}^\dagger = -i\sum_{s,s'}\hat{c}^\dagger_{i,s}\sigma^y_{ss'}\hat{c}_{i,s'} \quad (13)$$

or defined on a nearest-neighboring link $\langle i, j\rangle$ as

$$\hat{O} = \sum_s \hat{c}^\dagger_{i,s}\hat{c}_{j,s}, \qquad \hat{O}^\dagger = \sum_s \hat{c}^\dagger_{j,s}\hat{c}_{i,s}. \quad (14)$$

It is easy to see that the operators defined above obey TRS and are not Hermitian. In the numerical simulation, we include a number of $\hat{O}$ operators defined above located at the edge of an sample, and the number of coupling operators is denoted by M. We note that the discussion above can be generalized straightforwardly to the cases with more coupling operators.

Here, we numerically diagonalize the Kane-Mele model on a $N_x \times N_y$ sample, with an open boundary condition along $\hat{x}$ and a periodical boundary condition along $\hat{y}$. Here, we should emphasize that, in order to obey TRS, the operators $\hat{O}$ have to be a quadratic fermion operator, and, therefore, the total Hamiltonian contains four-fermion terms and cannot be solved by diagonalizing a quadratic matrix. Therefore, even though the spectrum of $\hat{H}_0$ can be obtained exactly, the effects of dissipation still need to be computed by the non-Hermitian linear response theory, and the numerical results are shown in Fig. 1. We take two edge states of $\hat{H}_0$ with the same energy and located at the same edge as the Kramers degenerate states $|\Psi_i\rangle$ ($i = 1,\ 2$). First, starting with an initial pure state $|\Psi\rangle = 1/\sqrt{2}|\Psi_1\rangle + 1/\sqrt{2}|\Psi_2\rangle$, we determine the evolution of the density matrix, with which we compute the time dependence of the von Neumann $S_v(t)$ shown in Fig. 1(a). One can see that the entropy increases linearly in time, with

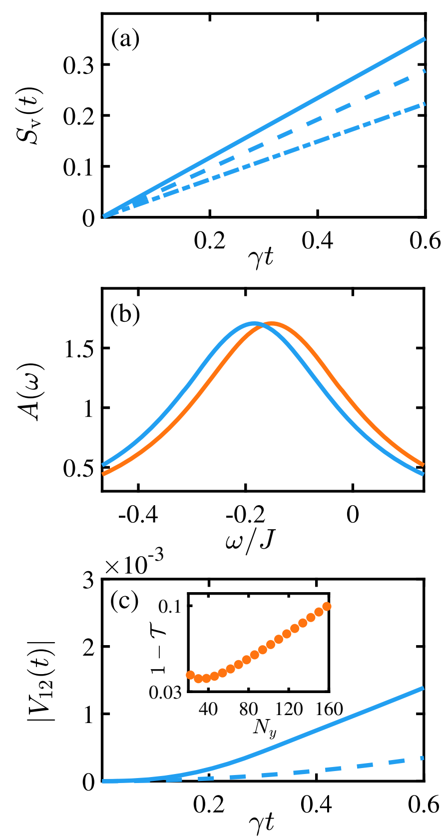

FIG. 1. (a) The von Neumann entropy $S_v(t)$ as a function of time. Here, $M = 20$ (solid line), 16 (dashed line), and 12 (dotted-dashed line), respectively. (b) The spectral function $A(\omega)$ for two Kramers degenerate states with dissipation, with $M = 20$. Without dissipation, the eigenenergies of these two states are degenerate and equal to $-0.17J$. (c) Time evolution of the matrix element of the impurity potential $V_{12}(t)$ between two degenerate edge states with $M = 20$. The solid line includes contributions from all states, and the dashed line includes only contributions from edge states. The dissipation strength γ is taken as $0.2J$, and other parameters in the Kane-Mele model are chosen as $\lambda_{\mathrm{SO}}/J = 0.06$, $\lambda_R/J = 0.05$, and $\lambda_\nu/J = 0.1$. The impurity strength V is taken as $V = J$. Aside from the inset, the size of honeycomb lattice is set as $N_x = 8$ and $N_y = 30$. The inset shows the transmission coefficient as a function of N_y with $N_x = 8$, where the impurity density is fixed at around 0.05 and the number of coupling operators is $M = N_y$.

a larger slope for larger M. Second, the spectral function $A(\omega)$ for these two states is shown in Fig. 1(b). One can see that the peaks of two spectral functions are split. Third, we compute the backscattering matrix element of an on-site impurity potential between these two edge states. Here, we plot the results with contributions from all states, as well as results with contributions from edge states only, which show both edge and bulk states contribute to the nonzero matrix elements. With the backscattering matrix element, we can also estimate the transmission coefficient [7], and the system size dependence of the transmission coefficient is shown in the inset in Fig. 1(c). It shows that the transmission coefficient can deviate from unity when the system size is beyond a certain threshold, resulting in a finite deviation of the conductance from the quantized value.

Remarks.—In summary, we have discussed how a system responds to dissipations, with TRS imposed on both the system and the environment, as well as the coupling operators between them. The main results are the absence of Kramers degeneracy and the absence of accurate quantization of conductance for TRS protected TI. We also recover the results reported in Ref. [4] on losing of phase coherence. However, different from Ref. [4], we employ a non-Hermitian Hamiltonian formalism for open quantum system with the Markovian approximation to the environment, and the way we impose TRS is also different. Different from many works on non-Hermitian physics in the recent literature, our non-Hermitian Hamiltonian contains Langevin noise term to ensure unitarity. In fact, the above discussions show that the Langevin noise terms play a crucial role, because most terms violating Schur's lemma are essentially from the Langevin noise average. We should also emphasize that the way we impose the TRS symmetry leads to quartic non-Hermitian terms. This also makes our model different from those considered in recent works on topological classification of the non-Hermitian Hamiltonian, where the models under consideration are always quadratic [25,26].

We thank Zhong Wang, Yun Li, Chaoming Jian, Hong Yao, and Pengfei Zhang for helpful discussions. This work is supported by Beijing Outstanding Young Scientist Program (H. Z.), National Natural Science Foundation of China Grant No. 11734010 (H. Z. and Y. C.), National Natural Science Foundation of China under Grant No. 11604225 (Y. C.), Ministry of Science and Technology of the People's Republic of China under Grant No. 2016YFA0301600 (H. Z.), and Beijing Natural Science Foundation (Z180013) (Y. C.).

[*]These authors contributed equally to this work.
[†]ychen@gscaep.ac.cn
[‡]hzhai@tsinghua.edu.cn

[1] C.-K. Chiu, J. C. Y. Teo, A. P. Schnyder, and S. Ryu, Rev. Mod. Phys. **88**, 035005 (2016).
[2] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[3] X. L. Qi and S. C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
[4] M. McGinley and N. R. Cooper, Nat. Phys. **16**, 1181 (2020).
[5] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Adv. Phys. **65**, 239 (2016).
[6] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. **91**, 021001 (2019).
[7] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.127.086801 for (i) the equivalence between the non-Hermitian Hamiltonian and the total Hamiltonian of the whole system; (ii) the full expression of $\mathcal{G}_{ij}^{(1)}$; (iii) the full expression of the matrix element V_{ij} to the leading order of γ; (iv) the calculation of the transmission coefficient; and (v) application of our results to interacting topological phases, which includes Refs. [8–11].
[8] V. Fatemi, S. Wu, Y. Cao, L. Bretheau, Q. D. Gibson, K. Watanabe, T. Taniguchi, R. J. Cava, and P. Jarillo-Herrero, Science **362**, 926 (2018).
[9] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Phys. Rev. Lett. **59**, 799 (1987).
[10] F. Pollmann, E. Berg, A. M. Turner, and M. Oshikawa, Phys. Rev. B **85**, 075125 (2012).
[11] Z. Wang, Q. Li, W. Li, and Z. Cai, Phys. Rev. Lett. **126**, 237201 (2021).
[12] L. Pan, X. Chen, Y. Chen, and H. Zhai, Nat. Phys. **16**, 767 (2020).
[13] A. Roth, C. Brüne, H. Buhmann, L. W. Molenkamp, J. Maciejko, X. L. Qi, and S. C. Zhang, Science **325**, 294 (2009).
[14] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X. L. Qi, and S. C. Zhang, Science **318**, 766 (2007).
[15] L. Du, I. Knez, G. Sullivan, and R. R. Du, Phys. Rev. Lett. **114**, 096802 (2015).
[16] S. Wu, V. Fatemi, Q. D. Gibson, K. Watanabe, T. Taniguchi, R. J. Cava, and P. Jarillo-Herrero, Science **359**, 76 (2018).
[17] T. L. Schmidt, S. Rachel, F. von Oppen, and L. I. Glazman, Phys. Rev. Lett. **108**, 156402 (2012).
[18] J. I. Väyrynen, M. Goldstein, and L. I. Glazman, Phys. Rev. Lett. **110**, 216402 (2013).
[19] J. I. Väyrynen, D. I. Pikulin, and J. Alicea, Phys. Rev. Lett. **121**, 106601 (2018).
[20] P. Gao, Y. P. He, and X. J. Liu, Phys. Rev. B **94**, 224509 (2016).
[21] M. McGinley and N. R. Cooper, Phys. Rev. Lett. **121**, 090401 (2018).
[22] E. P. Wigner, *Group Theory and Its Application to the Quantum Mechanics of Atomic Spectra* (Academic Press, New York, 1959).
[23] J. O. Dimmock, J. Math. Phys. (N.Y.) **4**, 1307 (1963).
[24] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).
[25] Z. Gong, Y. Ashida, K. Kawabata, K. Takasan, S. Higashikawa, and M. Ueda, Phys. Rev. X **8**, 031079 (2018).
[26] K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Phys. Rev. X **9**, 041015 (2019).