Catalytic Quantum Teleportation

Patryk Lipka-Bartosik and Paul Skrzypczyk
H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, United Kingdom

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In this work, we address fundamental limitations of quantum teleportation—the process of transferring quantum information using classical communication and preshared entanglement. We develop a new teleportation protocol based upon the idea of using ancillary entanglement catalytically, i.e., without depleting it. This protocol is then used to show that catalytic entanglement allows for a noiseless quantum channel to be simulated with a quality that could never be achieved using only entanglement from the shared state, even for catalysts with a small dimension. On the one hand, this allows for a more faithful transmission of quantum information using generic states and fixed amount of consumed entanglement. On the other hand, this shows, for the first time, that entanglement catalysis provides a genuine advantage in a generic quantum-information processing task. Finally, we show that similar ideas can be directly applied to study quantum catalysis for more general problems in quantum mechanics. As an application, we show that catalysts can activate so-called passive states, a concept that finds widespread application, e.g., in quantum thermodynamics.

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Introduction.—Quantum entanglement leads to correlations between distant particles that cannot be explained by any classical mechanism [1–3]. This intricate phenomenon is nowadays seen as an indispensable resource with an enormous number of modern applications. One of the most important applications of entanglement is quantum teleportation [4], a communication task that uses a pair of maximally entangled qubits $\langle qq\rangle$ and two bits of communication $[c \to c]$ to simulate a noiseless quantum channel $[q \to q]$:

$$\langle qq\rangle + 2[c \to c] \geq [q \to q]. \tag{1}$$

The significance of the protocol is best evidenced by its widespread applicability in various areas of quantum information [5–8], computation [9–11], and even general relativity [12–15]. Quantum teleportation has been realized in laboratories using a variety of different technologies, including photonic qubits [16–21], optical modes [22–24], nuclear magnetic resonance [25], atomic ensembles [26–28], trapped atoms [29–31], or solid-state systems [32–34].

Realistic teleportation protocols use generic entangled states, and, therefore, the quantum channels they simulate are inevitably noisy. In terms of the teleportation inequality (1), this means that substituting $\langle qq\rangle$ with a generic bipartite state $\langle\rho\rangle$ leads to a quantum channel that is no longer noiseless and has to be replaced with a general teleportation channel $\mathcal{N}$. A central problem of fundamental and practical significance is engineering teleportation protocols that simulate as faithfully as possible noiseless quantum channels, as measured by a natural figure of merit, the average fidelity of teleportation [35].

The teleportation inequality (1) is perhaps the best evidence for the resourcelike nature of entanglement, as it guarantees that simulating a noiseless quantum channel always consumes a pair of maximally entangled qubits. Therefore, it is reasonable to expect that, in a general protocol, teleportation fidelity can be increased only at the expense of using more entanglement. Interestingly, quantum mechanics allows for a very bizarre use of entanglement, one that is already helpful without entanglement being consumed or degraded. This surprising phenomenon is called *quantum catalysis* and was introduced in Ref. [36], further analyzed in Refs. [37–43], and subsequently adapted to many physical settings [44–60]. Quantum catalysis demonstrates that access to a special entangled state (the catalyst) allows two distant parties to manipulate their entanglement in a way that would be otherwise impossible. Importantly, the catalyst is not consumed during the process, so that the parties can repeat their task again or use it for another purpose. This makes catalysis a particularly interesting extension of the standard paradigm of local operations and classical communication (LOCC). Indeed, quantum catalysis can be viewed as a paradigm shift that leads to the ultimate limits of quantum protocols under fixed resources. Since the catalyst appearing in a catalytic protocol is not depleted, it does not contribute to the overall balance of consumed resources.

In this work, we are interested in finding the ultimate such limit of quantum teleportation. More specifically, we ask what is the best teleportation fidelity that can be achieved when consuming a given entangled state and using an arbitrary amount of entanglement catalytically? We show that this natural extension of the standard teleportation protocol allows quantum channels with much larger teleportation fidelity to be achieved or, equivalently, for the transfer of quantum information much more reliably. More formally, we show that using a quantum catalyst one can achieve teleportation fidelity equal to a regularization of the standard teleportation fidelity. This quantifier is then shown to be strictly larger than the standard teleportation fidelity for a wide range of pure states, therefore uncovering new limits for quantum teleportation under fixed resources. This can also be interpreted as providing the first example where quantum catalysis is successfully used in a generic information processing task and opens up the prospects for much further investigation into the power and generality of quantum catalysis, beyond what had been appreciated up until now. In this line, we show that our methods can be adapted beyond quantum teleportation and quantum information.

Framework.—In what follows, we will be interested in scenarios involving two distant parties (say, Alice and Bob) who are allowed to use LOCC. We say that $\mathcal{E} \in \mathrm{LOCC}(A:B)$ if it can be written as a sequence of quantum channels applied locally by A and B, intertwined with classical communication. To quantify entanglement, we will use the entanglement fraction [61], which is defined as the maximal overlap with a maximally entangled state, that is,

$$f(\rho) \coloneqq \max_{\mathcal{E}} \langle \phi^+_{AB} | \mathcal{E}(\rho_{AB}) | \phi^+_{AB} \rangle \quad \text{such that } \mathcal{E} \in \mathrm{LOCC}(A:B), \tag{2}$$

where $|\phi^+_{AB}\rangle = \sum_{i=1}^{d} |i\rangle_A |i\rangle_B / \sqrt{d}$ denotes a maximally entangled state shared between A and B.

Standard quantum teleportation.—Before presenting our main results, let us briefly recall the task of quantum teleportation [4]. In its most general form, the protocol involves two spatially separated parties, Alice and Bob, who share an arbitrary quantum state ρ_{AB} of dimension $d_A \times d_B$. A third party, often called a referee, provides Alice with a quantum state φ_R of dimension d_R which is unknown to both parties. The goal set before Alice and Bob is to transfer the unknown state from one party to another, using only local operations and classical communication, i.e., quantum channels $\mathcal{T} \in \mathrm{LOCC}(RA:B)$, and shared entanglement. Under this condition, all possible states which can be achieved in Bob's lab can be written as

$$\rho'_B = \mathrm{tr}_{RA} \mathcal{T}(\varphi_R \otimes \rho_{AB}). \tag{3}$$

The above protocol can be viewed equivalently as a process of establishing a quantum channel between Alice and Bob that maps the input state φ_R to the output ρ'_B. The goal of quantum teleportation is then to simulate a noiseless quantum channel between Alice and Bob, i.e., an identity map $\mathrm{id}_{A\to B}$. The quality of teleportation, or, equivalently, the fidelity of the resulting teleportation channel, can be quantified using the average fidelity of teleportation [35] (or simply "fidelity of teleportation") defined as

$$\langle F \rangle_\rho \coloneqq \max_{\mathcal{T}} \int \langle \varphi | \mathrm{tr}_{RA} \mathcal{T}(\varphi_R \otimes \rho_{AB}) | \varphi \rangle d\varphi \quad \text{such that } \mathcal{T} \in \mathrm{LOCC}(RA:B). \tag{4}$$

The integral in Eq. (4) is computed over a uniform distribution of all pure input states $\varphi = |\varphi\rangle\langle\varphi|$ according to a normalized Haar measure $\int d\varphi = \mathbb{1}$. It can be easily verified that $0 \le \langle F \rangle_\rho \le 1$ for all density operators ρ. Furthermore, the case $\langle F \rangle_\rho = 1$ corresponds to perfect teleportation from Eq. (1) and is possible if and only if ρ is maximally entangled. In practice, the fidelity of teleportation will always be less than one. Furthermore, when the shared state is separable, the corresponding teleportation protocol is said to be "classical," and fidelity of teleportation is bounded by $\langle F \rangle_c \coloneqq 2/(d_R + 1)$. Importantly, it was shown in Ref. [61] that fidelity of teleportation (4) is related with entanglement fraction (2) via

$$\langle F \rangle_\rho = \frac{f(\rho) d_R + 1}{d_R + 1}. \tag{5}$$

In what follows, we will focus on this quantity and show that catalysts allow one to increase the entanglement fraction without consuming any additional entanglement.

Results.—Let us start by describing a catalytic extension of the general quantum teleportation protocol. Then we display a main theorem that gives a lower bound on its performance and show that the bound is tight enough to demonstrate a sharp advantage with respect to the standard teleportation protocol. We conclude with a simple generalization of these methods that can be used to address catalytic advantages in more general settings.

Catalytic quantum teleportation.—Assume that Alice and Bob, in addition to their shared state ρ_{AB}, have access to a quantum system CC' prepared in a state $\omega_{CC'}$. This additional system is distributed such that Alice has access only to C and Bob only to its C' part. Alice is then given an unknown quantum state φ_R, and the parties perform a protocol $\mathcal{T} \in \mathrm{LOCC}(RAC:BC')$ which now acts on both systems they share and the input system. Moreover, for the protocol to be catalytic, we demand that $\mathcal{T}$ does not modify the catalyst. Notably, we do allow the catalyst to become correlated with ρ_{AB} during the process, and in Supplemental Material [62] we show that these correlations can be made arbitrarily small in trace distance, at the expense of using larger catalysts. The final state of Bob's subsystem at the end of the catalytic teleportation protocol reads

$$\rho'_B = \mathrm{tr}_{RACC'}[\mathcal{T}(\varphi_R \otimes \rho_{AB} \otimes \omega_{CC'})]. \tag{6}$$

The quality of the protocol can be quantified similarly as in the case of standard teleportation, i.e., using the fidelity of teleportation (4). Since we have the freedom to choose the catalyst, we define the fidelity of catalytic teleportation $\langle F_{\mathrm{cat}}\rangle_\rho$ as

$$\begin{aligned} \langle F_{\mathrm{cat}}\rangle_\rho = \max_{\mathcal{T},\omega} \int \langle\varphi|\mathrm{tr}_{RACC'}\mathcal{T}(\varphi_R\otimes\rho_{AB}\otimes\omega_{CC'})|\varphi\rangle d\varphi \\ \text{such that } \mathrm{tr}_{RAB}\mathcal{T}(\varphi_R\otimes\rho_{AB}\otimes\omega_{CC'}) = \omega_{CC'}, \\ \mathcal{T}\in\mathrm{LOCC}(RAC{:}BC'), \\ \omega_{CC'}\geq 0, \qquad \mathrm{tr}[\omega_{CC'}] = 1. \end{aligned} \tag{7}$$

Let us now define a regularization of the entanglement fraction from Eq. (2), which we will denote by $f_{\mathrm{reg}}(\rho)$ and whose significance will soon become evident, namely,

$$f_{\mathrm{reg}}(\rho) \coloneqq \lim_{n\to\infty}\frac{f_n(\rho^{\otimes n})}{n}, \tag{8}$$

where $f_n(\sigma)$ is the solution to

$$\begin{aligned} f_n(\sigma) \coloneqq \max_{\mathcal{E}}\sum_{i=1}^n \langle\phi^+|\mathrm{tr}_{/i}\mathcal{E}(\sigma)|\phi^+\rangle \\ \text{such that } \mathcal{E}\in\mathrm{LOCC}(A_1\ldots A_n{:}B_1\ldots B_n), \end{aligned} \tag{9}$$

where $\mathrm{tr}_{/i}(\cdot)$ is the partial trace performed over particles $1\ldots i-1, i+1\ldots n$. Notice that by taking a suboptimal guess $\mathcal{E}=\mathcal{E}_1\otimes\mathcal{E}_2\otimes\ldots\otimes\mathcal{E}_n$ with $\mathcal{E}_1=\mathcal{E}_2=\cdots=\mathcal{E}_n$ we can infer that $f_{\mathrm{reg}}(\rho)\geq f(\rho)$ for all quantum states ρ. With the above definitions, we are now ready to present our main result.

Theorem 1: The fidelity of catalytic teleportation satisfies

$$\langle F_{\mathrm{cat}}\rangle_\rho \geq \frac{f_{\mathrm{reg}}(\rho)d_R+1}{d_R+1}. \tag{10}$$

In other words, there is a protocol $\mathcal{T}\in\mathrm{LOCC}(RAC{:}BC')$ and a catalyst $\omega_{CC'}$ that achieves the bound (10).

Proof.—We will sketch the proof of Theorem 1 (see Supplemental Material [62] for a formal derivation). We start by constructing the catalyst and a subroutine $\mathcal{T}_{\mathcal{E}}$ that increases entanglement fraction of ρ_{AB}. Then we use this preprocessed state to perform standard teleportation $\mathcal{T}'$. The total protocol then reads $\mathcal{T}=\mathcal{T}'\circ\mathcal{T}_{\mathcal{E}}$.

Let $n\geq 2$ be a finite natural number and denote $C\coloneqq C_2\ldots C_nM$ and $C'\coloneqq C_2'\ldots C_n'M$, where M is a classical register. Moreover, let $\mathcal{E}\in\mathrm{LOCC}(AC{:}BC')$ be a channel (yet to be determined) and denote $\sigma^{n-i}\coloneqq\mathrm{tr}_{1\ldots i}\mathcal{E}(\rho^{\otimes n})$, where $\mathrm{tr}_{1\ldots i}(\cdot)$ denotes partial trace over the first i copies of $\rho^{\otimes n}$. Consider the following catalyst, introduced in Ref. [63]:

$$\omega_{CC'} = \frac{1}{n}\sum_{i=1}^n \underbrace{\rho^{\otimes i}\otimes\sigma^{n-i}}_{C_2C_2'\ldots C_nC_n'}\otimes|i\rangle\langle i|_M. \tag{11}$$

This catalyst is a state of $n-1$ quantum registers, each of dimension d, and a classical register of dimension n. For a given value i of the classical register, the remaining $n-1$ quantum registers contain i copies of the shared bipartite state ρ and an $n-i$-partite state σ^{n-i} that is the marginal of $\mathcal{E}(\rho^{\otimes n})$.

Let us label for clarity $A_1\equiv A$ and $A_i\equiv C_i$ for $2\leq i\leq n$ and similarly for B_i and $BC_2\ldots C_n$. The joint state of the resource and the catalyst, $\rho_{AB}\otimes\omega_{CC'}$, is presented in Fig. 1(a) for the exemplary case with $n=5$. The initial protocol $\mathcal{T}_{\mathcal{E}}$ can be summarized as follows. (1) Apply $\mathcal{E}\in\mathrm{LOCC}(AC{:}BC')$ to the nth pair using M as the control [see Fig. 1(b)]. (2) Relabel $|i\rangle_M\to|i+1\rangle_M$ for $i<n$ and $|n\rangle_M\to|1\rangle_M$ [see Fig. 1(c)]. (3) Relabel $A_1B_1\to A_iB_i$ for all i in M [see Fig. 1(d)]. (4) Discard the catalyst CC'. As a result, the system and the catalyst transform into

$$\begin{aligned} \rho_{AB}\to\rho_{AB}^{(n)} &= \mathrm{tr}_{CC'}\mathcal{T}_{\mathcal{E}}(\rho_{AB}\otimes\omega_{CC'}) \\ &= \frac{1}{n}\sum_{i=1}^n \mathrm{tr}_{/i}\mathcal{E}(\rho_{AB}^{\otimes n}), \end{aligned} \tag{12}$$

$$\omega_{CC'}\to\omega_{CC'}' = \mathrm{tr}_{AB}\mathcal{T}_{\mathcal{E}}(\rho_{AB}\otimes\omega_{CC'}) = \omega_{CC'}. \tag{13}$$

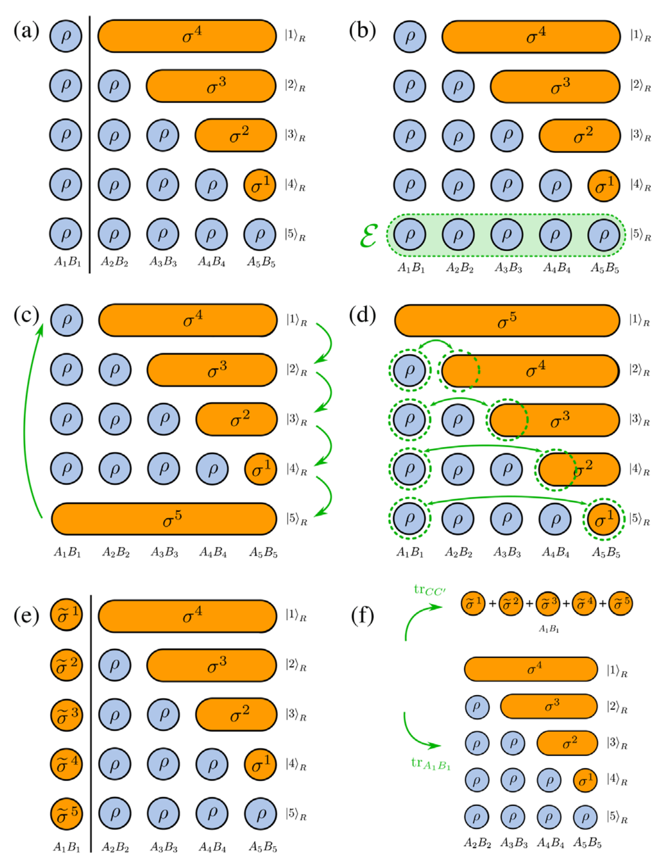

FIG. 1. The catalytic subroutine that uses a noisy entangled state as a catalyst to enhance entanglement fraction. (a)–(e) describe subsequent steps of the protocol and (f) the final state of the main system and the catalyst. The catalyst remains unchanged as the system is transformed into a state with a higher entanglement fraction.

Next, we apply the standard teleportation scheme for noisy states [64]. Taking $\{U_a^{\mathrm{A}}\}$ for $a \in \{1, \ldots, d^2\}$ to be the set of generalized Pauli operators with respect to basis $\{|i\rangle^{\mathrm{A}}\}$, we can summarize $\mathcal{T}'$ as follows. (1) Twirl ρ_{AB} into an isotropic state:

$$\rho_{AB}^{(n)} \to f(\rho_{AB}^{(n)})\phi_{AB}^{+} + [1 - f(\rho_{AB}^{(n)})]\phi_{AB}^{\perp}, \qquad (14)$$

where $\phi^{\perp} = (\mathbb{1} - \phi^{+})/(d^2 - 1)$. (2) Perform teleportation on $RA \to B$: (a) Alice measures RA using a positive operator-valued measure with elements:

$$M_a^{RA} = (\mathbb{1} \otimes U_a)\phi_{RA}^{+}(\mathbb{1} \otimes U_a^{\dagger}); \qquad (15)$$

(b) Alice communicates outcome a to Bob; (c) Bob applies $U_a^{\dagger}(\cdot)U_a$ to his share of the state. The fidelity of teleportation in this process reads

$$\frac{f(\rho_{AB}^{(n)})d_R + 1}{d_R + 1}. \qquad (16)$$

Notice that so far the channel $\mathcal{E}$ was arbitrary, and so we can now optimize $\mathcal{T} = \mathcal{T}' \circ \mathcal{T}_{\mathcal{E}}$ over all feasible channels $\mathcal{E} \in \mathrm{LOCC}(AC\!:\!BC')$. Taking the limit $n \to \infty$ and using $\lim_{n\to\infty} f(\rho_{AB}^{(n)}) = f_{\mathrm{reg}}(\rho_{AB})$ leads to Eq. (10). ∎

The regularized entanglement fraction appears difficult to compute, in general. However, for large n, one can use typicality arguments to find a wide range of states for which the lower bound in Eq. (10) still demonstrates a significant advantage over standard teleportation.

Demonstrating catalytic advantage in teleportation.—Our reasoning so far was valid for arbitrary bipartite density operators ρ_{AB}. In this section, we will restrict our attention to pure states $\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$ and use typicality arguments to infer that the presented protocol for catalytic teleportation leads to a generic advantage over the standard teleportation protocol. Interestingly, this is a consequence of an essential property of catalysis: that certain catalysts amplify typical properties of states, even at the level of a single copy. This property of catalysts has been recently employed in Refs. [52–54,57,60].

Lemma 1: The regularized entanglement fraction $f_{\mathrm{reg}}(\psi_{AB})$ for pure states ψ_{AB} satisfies

$$f_{\mathrm{reg}}(\psi_{AB}) \geq \max_{\psi'} f(\psi'_{AB}) \qquad (17)$$

$$\text{such that } S(\rho_A) \geq S(\rho'_A), \qquad (18)$$

where $\rho_A = \mathrm{tr}_B \psi_{AB}$ and $\rho'_A = \mathrm{tr}_B \psi'_{AB}$ and $S(\rho) = -\mathrm{tr}\rho \log \rho$ is the Shannon entropy.

We now apply Lemma 1 to show that catalytic teleportation outperforms standard teleportation for a wide range of generic quantum states.

Example.—As a simple example, let us consider teleporting a three-dimensional quantum system ($d_R = 3$) using a singlet. In this case, the state shared between Alice and Bob can be written as $\psi_{AB} = \sum_{i=1}^{3} \sqrt{\lambda_i}|i\rangle_A|i\rangle_B$, with Schmidt coefficients $\lambda_1 = 1/2$, $\lambda_2 = 1/2$, and $\lambda_3 = 0$. Its entanglement fraction is equal to $f(\psi_{AB}) = (\sum_{i=1}^{3} \sqrt{\lambda_i})^2/3 = 2/3$, and, therefore, its fidelity of teleportation reads

$$\langle F \rangle_{\psi} = 0.75, \qquad (19)$$

which is also larger than the classical threshold $\langle F_c \rangle = 1/2$.

Let us now analyze the protocol for catalytic teleportation. In this case, the relevant benchmark is the fidelity of catalytic teleportation (7) whose lower bound can be found using Lemma 1. To compute it, we choose the optimizer in Eq. (17) to be the state ψ'_{AB} with Schmidt coefficients $\lambda'_1 = x$ and $\lambda'_2 = \lambda'_3 = (1 - x)/2$, where x is the unique solution to $h(x) = x \log 2$ (which is $x \approx 0.77$) and $h(x) = -x \log x - (1 - x) \log(1 - x)$. This is a feasible choice, since the entropies of marginals of ψ_{AB} and ψ'_{AB} are both equal to $\log 2$. According to Lemma 1, the regularized entanglement fraction can be lower bounded by the entanglement fidelity of ψ'_{AB}; therefore, $f_{\mathrm{reg}} \geq f(\psi'_{AB}) \approx 4/5$. Using Theorem 1, we can then infer that

$$\langle F_{\mathrm{cat}} \rangle \geq 0.85, \qquad (20)$$

which is roughly 13% larger than the best fidelity that could ever be obtained when using ψ_{AB} alone. Interestingly, this simple example is not a singular case: There are, in fact, many entangled states whose performance in teleportation can be catalytically enhanced. To show this in Fig. 2, we used Lemma 1 and numerically computed the lower bound on the catalytic advantage $\eta(\psi) \coloneqq (\langle F_{\mathrm{cat}} \rangle - \langle F \rangle)/\langle F \rangle$. In Supplemental Material [62], we further show that a similar advantage is present when using a small catalyst (qutrit). In that case, the enhancement is around 2.5% of what can be achieved using ψ_{AB} only.

Beyond quantum teleportation.—The catalytic subroutine $\mathcal{T}_{\mathcal{E}}$ we used to prove Theorem 1 can be used to address more general problems, beyond increasing the entanglement fraction, in various paradigms—other than LOCC. Let us mention the general idea, postponing the details and an explicit application to Supplemental Material [62].

Let us for simplicity focus on the case of a single party S and let $\mathcal{E}$ be any channel from a class of channels $\mathscr{C} \subseteq$ CPTP acting on S. Moreover, let O be an arbitrary observable on S. Our goal is to minimize (or maximize) the expectation of O in the state ρ under the available class of operations $\mathscr{C}$. Define

$$\mathcal{R}(\rho) \coloneqq \min_{\mathcal{E}\in\mathscr{C}} \mathrm{tr}[\mathcal{E}(\rho)O]. \qquad (21)$$

Let us also define an analogous quantity for when many copies of ρ are processed collectively, i.e.,

$$\mathcal{R}_{\mathrm{col}}(\rho) \coloneqq \min_{\mathcal{E}\in\mathscr{C}} \frac{1}{n} \mathrm{tr}[\mathcal{E}(\rho^{\otimes n})O^{\otimes n}], \qquad (22)$$

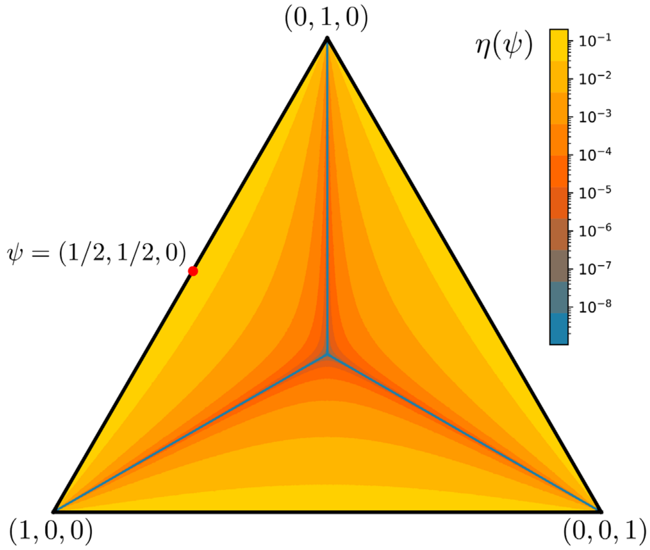

FIG. 2. The catalytic advantage $\eta(\psi)$ in quantum teleportation. The triangle describes the space of all pure states of two qutrits. In particular, each point $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ corresponds to a unique (up to local unitaries) state with Schmidt coefficients $\{\lambda_i\}$ for $1 \leq i \leq 3$. The red point corresponds to the example from the main text.

where now $\mathcal{E}$ is a collective operation that acts on n copies of ρ and $O^{\otimes n} \equiv \sum_{i=1}^{n} \mathbb{1}_{/i} \otimes O_i$. Very often, $\mathcal{R}_{\mathrm{col}}(\rho) < \mathcal{R}(\rho)$; i.e., processing multiple copies collectively is strictly better than processing them one by one. Interestingly, the same improvement in manipulation abilities can be achieved when using only a single copy of ρ and a suitably chosen catalyst. This is the content of our next theorem.

Theorem 2: Let ρ be a quantum state and $\mathcal{D} \in \mathscr{C}$. Then there is a quantum state ω such that

$$\min_{\mathcal{D} \in \mathscr{C}} \mathrm{tr}[\mathcal{D}(\rho \otimes \omega)(O \otimes \mathbb{1})] = \mathcal{R}_{\mathrm{col}}(\rho), \tag{23}$$

and, moreover,

$$\mathrm{tr}_2[\mathcal{D}(\rho \otimes \omega)] = \omega. \tag{24}$$

Notice that by taking $S = AB$, $\mathscr{C} = \mathrm{LOCC}(A\!:\!B)$, and $O = \phi_{AB}^{+}$ we obtain the catalytic subroutine from the previous section. Interestingly, the reasoning presented above is much more general, and to demonstrate this in Supplemental Material [62] we apply Theorem 2 to the problem of work extraction in quantum thermodynamics. As a consequence, it can be shown that catalysis unlocks the energy contained in a passive state under arbitrary classes of operations, therefore generalizing the main result from Ref. [49].

Discussion.—We have introduced an extension of the standard teleportation protocol, to the case when Alice and Bob use entangled states in a catalytic way. We showed that, when arbitrary catalysts are allowed, the teleportation fidelity can be lower bounded by a regularization of the standard teleportation fidelity. We then showed that this regularized quantifier is strictly larger than the standard teleportation fidelity, therefore demonstrating a genuine catalytic advantage for a wide range of quantum states.

Quantum teleportation is one of many information-theoretic protocols whose performance depends directly on the entanglement fraction of the used resource. Our new methods (in particular, the catalytic subroutine) can be, therefore, directly applied to study other protocols whose performance is quantified using entanglement fraction (see, e.g., [65–68]).

The generalized version of our catalytic subroutine, in a certain sense, allows collective effects to be incorporated at a single-copy level, using appropriately chosen catalysts. Since quantum advantages generally result from the ability of processing many quantum states simultaneously, we hope that the methods described here will lead to interesting extensions of quantum protocols that enjoy the performance of collective processing but using only a few copies of the resource.

Finally, we believe that catalysis can lead to interesting extensions of standard quantum resource theories. Since entanglement fraction can be viewed as one of the Renyi entropies, it is plausible to expect that correlated catalysis can be used to selectively increase other Renyi entropies. This can potentially lead to better performances in various operational tasks, ranging from standard discrimination [69–73] up to more exotic variants thereof [74].

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