Exponentially Accelerated Approach to Stationarity in Markovian Open Quantum Systems through the Mpemba Effect

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Ergodicity breaking and slow relaxation are intriguing aspects of nonequilibrium dynamics both in classical and quantum settings. These phenomena are typically associated with phase transitions, e.g., the emergence of metastable regimes near a first-order transition or scaling dynamics in the vicinity of critical points. Despite being of fundamental interest the associated divergent timescales are a hindrance when trying to explore steady-state properties. Here we show that the relaxation dynamics of Markovian open quantum systems can be accelerated exponentially by devising an optimal unitary transformation that is applied to the quantum system immediately before the actual dynamics. This initial "rotation" is engineered in such a way that the state of the quantum system no longer excites the slowest decaying dynamical mode. We illustrate our idea—which is inspired by the so-called Mpemba effect, i.e., water freezing faster when initially heated up—by showing how to achieve an exponential speeding-up in the convergence to stationarity in Dicke models, and how to avoid metastable regimes in an all-to-all interacting spin system.

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Introduction.—A strong focus of current research in many-body quantum physics is on understanding (non-equilibrium) phases of matter and transitions between them. Often associated with that are slow relaxation and divergent time correlations [1–7], which typically signal the onset of critical behavior [8–13] or the appearance of metastable dynamical regimes [14,15] near first-order phase transitions. In certain instances, the concomitant very long relaxation timescales become impractical or even detrimental when a fast approach to stationarity is desired. This is certainly the case when one is interested in studying steady-state properties [16] or, for instance, when the stationary state encodes the result of some computation [17–19]. It may also find applications in the optimization of the output of quantum engines [20–23]. In physical terms, the characteristic time needed for an open dissipative quantum system to approach stationarity is given by the lifetime τ of the slowest decaying excitation mode. A random initial pure state $|\psi\rangle$ [see Fig. 1(a)] is typically out of equilibrium and excites all dynamical decaying modes, including the slowest one. As such, it will ultimately converge to stationarity in a time proportional to τ.

In this Letter, we show that, if the slowest decaying mode is unique, one can always find an appropriate unitary operation which, once applied to the initial state $|\psi\rangle$, allows the open system to reach stationary behavior at a significantly faster pace. The idea, which is sketched in Fig. 1(b), is that the unitary operation U removes the excitation of the slowest decaying mode from the initial

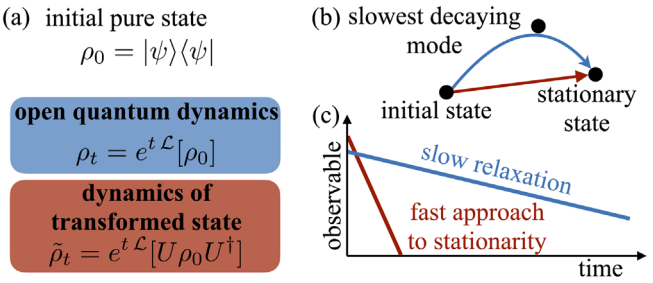

FIG. 1. Mpemba effect in a Markovian open quantum system. (a) We consider a quantum system, initially prepared in some pure state $|\psi\rangle$ and subject to Markovian open quantum dynamics. Generically, the timescale for the approach to stationarity is contained in the dynamical generator $\mathcal{L}$ and is related to the slowest decaying excitation mode. Before the time evolution starts, we apply a unitary operation U to the quantum state $|\psi\rangle$, which deexcites such a slow mode. (b) After applying the unitary operation the system dynamics is not affected by long-lived excitation and approaches the stationary state in a "more direct" way. (c) Sketch of the slow relaxation (blue line), contrasted with the accelerated one emerging after the applying the unitary (red line). The y axis is in logarithmic scale.

state $|\psi\rangle$, which achieves an exponential speeding-up [see Fig. 1(c)]. The basic mechanism underpinning our finding is reminiscent of the so-called Mpemba effect [24], which refers to the phenomenon that a hotter liquid cools at a faster rate than a colder one. Often, this and related phenomena [25–33] indeed admit a transparent physical explanation [25,27,29,32]: the state of the hotter system overlaps less with the slowest decaying modes of the cooling (dissipative) dynamics—a hypothesis which has been confirmed experimentally in a trapped colloid particle [30]. In certain instances, however, such a clear separation of timescales may not occur. Still, an anomalous relaxation toward equilibrium can be investigated by monitoring the evolution of thermodynamic quantities representing internal degrees of freedom, such as kinetic and rotational energy, kurtosis, or correlation length [26,28,31,33]. Here, we explore the analog of the Mpemba effect in Markovian open quantum systems. Using paradigmatic many-body systems of both theoretical and experimental interest we demonstrate the possibility of speeding up the approach to stationarity and to avoid long prestationary metastable regimes.

Markovian open quantum dynamics.—We first briefly discuss the fundamental elements of open quantum systems subject to Markovian dynamics [34–42]. The evolution of the density matrix ρ_t, describing the state of the quantum system, is governed by the quantum master equation $\dot{\rho}_t = \mathcal{L}[\rho_t]$ [34,36,37], where $\mathcal{L}$ is the Lindblad map

$$\mathcal{L}[X] = -i[H, X] + \sum_{\mu=1}^{N_J} \left(L_\mu X L_\mu^\dagger - \frac{1}{2}\{L_\mu^\dagger L_\mu, X\} \right). \quad (1)$$

Here, $H = H^\dagger$ is the Hamiltonian of the system, and the N_J jump operators L_μ describe the dissipative effects due to the presence of an environment. The Lindblad map $\mathcal{L}$ preserves the trace $\{\mathrm{Tr}(\mathcal{L}[X]) = 0\}$ and Hermiticity $\{(\mathcal{L}[X])^\dagger = \mathcal{L}[X^\dagger],\ \forall\ X\}$ and generates completely positive (physical) dynamics of the quantum state ρ_t.

The formal solution to the quantum master equation is given by $\rho_t = e^{t\mathcal{L}}[\rho_0]$, where the exponential of the map $\mathcal{L}$ must be interpreted as the power series. Assuming the generator to be diagonalizable, one can find the right eigenmatrices r_k such that

$$\mathcal{L}[r_k] = \lambda_k r_k. \quad (2)$$

The complex numbers λ_k are the eigenvalues of the Lindblad map. Note that, due to the Hermiticity preservation of $\mathcal{L}$, if λ_k is a complex eigenvalue, then λ_k^* must also be an eigenvalue. For the same reason, one can also show that if λ_k is real, then r_k can be chosen to be Hermitian. Associated with the map defined in Eq. (1), there is a dual map, also called the adjoint Lindblad map, which implements the evolution of observables:

$$\mathcal{L}^+[O] = i[H, O] + \sum_{\mu=1}^{N_J} \left(L_\mu^\dagger O L_\mu - \frac{1}{2}\{O, L_\mu^\dagger L_\mu\} \right).$$

This dual map $\mathcal{L}^+$ is diagonalized by the left eigenmatrices ℓ_k:

$$\mathcal{L}^+[\ell_k] = \lambda_k \ell_k. \quad (3)$$

The matrices ℓ_k are in principle different from the matrices r_k in Eq. (2). However, ℓ_k and r_k still form a basis for the space of matrices and can always be defined with the property $\mathrm{Tr}(\ell_k r_h) = \delta_{kh}$.

Since the dynamics generated by $\mathcal{L}$ are completely positive, the eigenvalues of the Lindblad map all have a nonpositive real part, $\mathrm{Re}(\lambda_k) \leq 0$. Furthermore, trace preservation enforces that at least one eigenvalue is zero, $\lambda_1 = 0$. If such an eigenvalue is nondegenerate—we will work under this assumption—the (asymptotic) stationary state of the open quantum system,

$$\rho_{\mathrm{SS}} = \lim_{t\to\infty} \rho_t, \quad (4)$$

is unique and given by the right eigenmatrix r_1. Since the left eigenmatrix associated with λ_1 is the identity $\ell_1 = 1$, one has $\mathrm{Tr}(r_1) = 1$. Finally, the matrix r_1 is guaranteed to be positive due to the complete positivity of $e^{t\mathcal{L}}$.

The spectral decomposition of $\mathcal{L}$ allows us to write the dynamics of any initial density matrix as

$$e^{t\mathcal{L}}[\rho_0] = r_1 + \sum_{k=2}^{d^2} e^{t\lambda_k} \mathrm{Tr}(\ell_k \rho_0) r_k, \quad (5)$$

where d is the dimension of the Hilbert space of the system. This decomposition shows that the matrices r_k are nothing but the excitation modes of the system, each one characterized by its decay rate $|\mathrm{Re}(\lambda_k)|$. For long times, the relevant terms are those related to the λ_k with the smallest real part in modulus. We order the eigenvalues λ_k in such a way that $|\mathrm{Re}(\lambda_2)| \leq |\mathrm{Re}(\lambda_3)| \leq \ldots \leq |\mathrm{Re}(\lambda_{d^2})|$, and we further assume that the eigenvalue λ_2 is real and unique. In this case, the timescale for relaxation is given by

$$\tau = \frac{1}{|\lambda_2|}, \quad (6)$$

and r_2 is in fact the slowest decaying excitation mode of the Markovian open quantum dynamics.

Mpemba effect.—A generic initial state will overlap with all decaying modes of Lindblad dynamics, and thus, in particular, also with the slowest one. As such, the approach to the stationary state will take place in a time which is of the order of the relaxation time (6). However, looking at Eq. (5), one sees that this timescale becomes completely irrelevant for the dynamics if $\mathrm{Tr}(\ell_2 \rho_0) = 0$. In such a case,

the state would relax at a faster rate with timescale $1/|\mathrm{Re}(\lambda_3)|$, which implies an exponential speeding-up of the convergence to stationarity. In what follows, we show how such acceleration may always be achieved when starting from an initial pure state $\rho_0 = |\psi\rangle\langle\psi|$ by performing a unitary rotation to the state before the actual time evolution takes place. This is in spirit similar to the Mpemba effect, where an initial thermal state is first heated up before the cooling dynamics is started.

Given an initial pure state $\rho_0 = |\psi\rangle\langle\psi|$, there always exists a unitary U—which depends on the state—such that

$$\mathrm{Tr}(\ell_2 U\rho_0 U^\dagger) = 0, \tag{7}$$

if the slowest decaying mode is unique. This can be shown as follows. First of all, we notice that the matrix ℓ_2 must be Hermitian since we have assumed that λ_2 is real and nondegenerate. As such we can write it in its spectral form

$$\ell_2 = \sum_{k=1}^{d} \alpha_k |\varphi_k\rangle\langle\varphi_k|,$$

where $\langle\varphi_k|\varphi_h\rangle = \delta_{kh}$. We then note that, since $\mathrm{Tr}(\ell_2 r_1) = 0$ and r_1 is positive, the set of eigenvalues α_k must contain at least two eigenvalues with the opposite sign or one equal to zero. Introducing an auxiliary orthonormal basis $\{|\psi_k\rangle\}_{k=1}^d$ for which $|\psi\rangle = |\psi_1\rangle$ (i.e., the initial state is a basis state) and using the spectral decomposition we find for the left-hand side of Eq. (7),

$$\mathrm{Tr}(\ell_2 U\rho_0 U^\dagger) = \sum_{k=1}^{d} \alpha_k \langle\psi_1|U^\dagger|\varphi_k\rangle\langle\varphi_k|U|\psi_1\rangle.$$

To simplify the construction of the unitary we divide it into two parts, $U = U_2 U_1$. The first unitary is chosen such that it maps the auxiliary basis $|\psi_k\rangle$ onto the basis $|\varphi_k\rangle$, which is simply achieved by $U_1 = \sum_k |\varphi_k\rangle\langle\psi_k|$, yielding

$$\mathrm{Tr}(\ell_2 U\rho_0 U^\dagger) = \sum_{k} \alpha_k \langle\varphi_1|U_2^\dagger|\varphi_k\rangle\langle\varphi_k|U_2|\varphi_1\rangle.$$

In the next step we construct U_2 such that the right-hand side of this expression becomes zero. Recalling that α_k are real numbers, two cases need to be considered: in case one of the α_k is zero, it is sufficient that U_2 performs a permutation of the basis $\{|\varphi_k\rangle\}$, mapping $|\varphi_1\rangle$ onto the eigenstate $|\varphi_h\rangle$ for which $\alpha_h = 0$.

In the nontrivial case, in which ℓ_2 does not have a zero eigenvalue, we can make a construction based on the following observation: the eigenvalue α_1 is a real number and can be either positive or negative. Since ℓ_2 cannot be a positive (or negative) eigenmatrix there must be an eigenvalue α_n such that $\mathrm{sign}(\alpha_n) = -\mathrm{sign}(\alpha_1)$. We then construct the Hermitean operator $F = |\varphi_1\rangle\langle\varphi_n| + |\varphi_n\rangle\langle\varphi_1|$, which we use to define the unitary

$$U(s) \coloneqq e^{-isF} = 1 + [\cos(s) - 1]F^2 - i\,\sin(s)F \tag{8}$$

where $F^2 = |\varphi_1\rangle\langle\varphi_1| + |\varphi_n\rangle\langle\varphi_n|$. Using this unitary operator we find that

$$c(s) = \mathrm{Tr}[\ell_2 U(s) U_1 \rho_0 U_1^\dagger U^\dagger(s)] = \alpha_1 \cos^2(s) + \alpha_n \sin^2(s). \tag{9}$$

The quantity $c(s)$ has the same sign as α_1 for $s = 0$, but on the other hand, it has the same sign as α_n for $s = \pi/2$. In particular, it vanishes for $\bar{s} = \arctan(\sqrt{|\alpha_1/\alpha_n|})$, so that if we take the unitary $U = U(\bar{s})U_1$, Eq. (7) is satisfied. This implies that the initial state is rotated into a state which no longer excites the slowest decaying mode and will thus relax, in general, with the timescale $1/|\mathrm{Re}(\lambda_3)|$. In particular, this means that the approach to stationarity has been exponentially accelerated by a factor $|\mathrm{Re}(\lambda_3)| - |\mathrm{Re}(\lambda_2)|$.

We have shown how to exploit the structure of the matrix ℓ_2 to find a unitary U satisfying Eq. (7). If ℓ_2 has a zero eigenvalue, e.g., if it is a "pure" (rank-1) matrix, one can take a U which maps the initial state onto an eigenstate of ℓ_2 associated with the eigenvalue zero. If ℓ_2 does not have a zero eigenvalue, e.g., it is a nonsingular matrix, then one can also exploit this property, together with the fact that a nonsingular ℓ_2 cannot be a positive matrix, to find two eigenvalues of ℓ_2 with the opposite sign. It is then sufficient to take a unitary U mapping the initial state onto an appropriate superposition of the eigenstates of ℓ_2 corresponding to such eigenvalues. Finally, we note that finding U does not necessarily require a full diagonalization of the Lindblad generator. One could represent the generator $\mathcal{L}$ as a matrix (see, e.g., Ref. [42]) and then use Krylov methods just to obtain the eigenmatrix ℓ_2.

Application to the dissipative Dicke model.—As a first application of our result, we consider the single-mode open quantum Dicke model [43,44], which is paradigmatic for the understanding of matter-light interactions and variants of which have been realized in a number of experiments [45–49]. It consists of an ensemble of two-level quantum systems, each of which is described by the spin operators $s_\alpha^{(k)} = \frac{1}{2}\sigma_\alpha^{(k)}$, with σ_α being the Pauli matrix α. The superscript k indicates the spin to which the operator belongs. These spin variables are coupled to a bosonic mode, described by annihilation and creation operators $a, a^\dagger$.

In the Markovian regime, the open quantum dynamics of the Dicke model are described by a generator of the form in Eq. (1), with Hamiltonian [43]

$$H = \Omega S_z + \omega a^\dagger a + \frac{g}{\sqrt{N}}(a + a^\dagger)S_x$$

and a single jump operator ($N_J = 1$), $L_1 = \sqrt{\kappa}a$. This latter contribution accounts for dissipative losses of excitations for the bosonic mode. While our method can also be applied to the above model, in order to simplify the numerics we make an assumption. We consider the adiabatic elimination of the bosonic mode. By performing such an approximation (see Supplemental Material [50]), the model is described solely in terms of spin degrees of freedom. The dynamics is governed by a generator of the form (1), with

$$\tilde{H} = \Omega S_z - \frac{4\omega g^2}{4\omega^2 + \kappa^2}\frac{S_x^2}{N}, \qquad \tilde{L}_1 = \frac{2|g|\sqrt{\kappa}}{\sqrt{4\omega^2 + \kappa^2}}\frac{S_x}{\sqrt{N}}. \quad (10)$$

The above dynamics conserves the total angular momentum $S^2 = S_x^2 + S_y^2 + S_z^2$. In the following we consider the largest symmetry sector, for which $S^2 = N(N+2)/4$. This subspace is formed by the $N+1$ eigenstates of the S_z operator $S_z|m\rangle = m|m\rangle$ with $m = -N/2, -N/2+1, \ldots, N/2$. As initial state ρ_0 we take a random pure state of the form $|\psi\rangle \propto \sum_m (a_m + ib_m)|m\rangle$ with a_m, b_m being uniformly distributed random numbers between 0 and 1. As shown in Fig. 2(a), the overlap of a randomly selected state $|\psi\rangle$ with the matrix ℓ_2 is generically finite. However, by tuning $U(s)$ we can find an appropriate transformation $U = U(\bar{s})U_1$ such that Eq. (7) is satisfied. For the rotated state, the overlap with ℓ_2 is thus zero, and we have an approach to stationarity governed by the decay rate $|\mathrm{Re}(\lambda_3)|$. This is clearly shown in Fig. 2(b), where we plot the Hilbert-Schmidt distance

$$\mathcal{E}_t(\rho, \rho_{\mathrm{SS}}) = \{\mathrm{Tr}(e^{t\mathcal{L}}[\rho] - \rho_{\mathrm{SS}})^2\}^{1/2}, \quad (11)$$

between the stationary state ρ_{SS} [cf. Eq. (4)] and the time-dependent state starting from the state ρ_0 as well as from $U\rho_0 U^\dagger$, respectively.

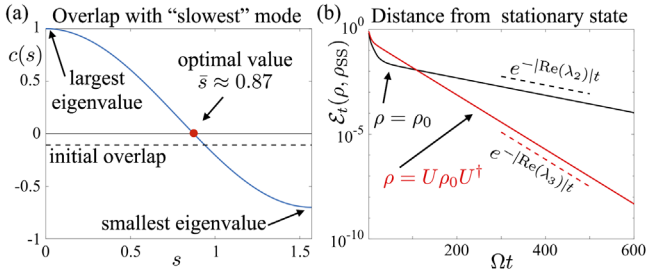

FIG. 2. Dissipative Dicke model. (a) Overlap $c(s)$ of a rotated initial random state as a function of s. According to Eq. (9), $c(s)$ can interpolate between the eigenvalue α_1 of ℓ_2—which we take here to be the largest one—and the eigenvalue α_n—which we take to be the smallest. For the choice of parameters in this figure, $\omega = 1$, $g = 1$, $\kappa = 1$ (all in units of Ω), and $N = 40$ spins, $\alpha_1 \approx 1.00$, and $\alpha_n \approx -0.70$. The optimal value for which the overlap $c(s)$ is zero is given by $\bar{s} = \arctan(\sqrt{|\alpha_1/\alpha_n|}) \approx 0.87$. The dashed line shows the overlap $\langle\psi|\ell_2|\psi\rangle$ of the initial random state with the decaying mode r_2. (b) Distance between the time-evolved state and the stationary state ρ_{SS}. We compare the case of an initial random state (black line) with the time evolution ensuing after the application of the rotation U (see main text for discussion). While in the original case, the approach to stationarity is governed by the eigenvalue λ_2, the application of U leads to an exponentially faster convergence to the steady state with the rate given by $|\mathrm{Re}(\lambda_3)|$.

Application to an all-to-all interacting spin model.—As a second application, we consider a spin model with resemblance to laser-driven interacting ensembles of Rydberg atoms [51–55]. This model allows us to demonstrate how the Mpemba effect may be used to avoid long-lasting metastable regimes [15,55,56]. From a theoretical perspective, we model the N atoms as two-level systems, exactly through the spin degrees of freedom introduced previously. The state with spin pointing up in the z direction corresponds to the atom being in the excited (Rydberg) state, while the one pointing down refers to the ground state of the atom. We consider Markovian dynamics such as the one in Eq. (1) with

$$H = \Omega S_x - \Delta S_z + \frac{V}{N}S_z^2, \qquad L_1 = \sqrt{\kappa/N}S_-,$$

where $S_- = S_x - iS_y$ is a spin ladder operator. For this model, Ω is the Rabi frequency, Δ is the laser detuning with respect to the atomic transition frequency ω_{at} while V parametrizes here the strength of the all-to-all interactions.

For certain parameter regimes, e.g., the one considered in Fig. 3, the model features a so-called metastable regime, which emerges since $|\mathrm{Re}(\lambda_2)| \ll |\mathrm{Re}(\lambda_3)|$. This means that, over a long time window during which all other decaying excitation modes have already relaxed, the mode related to $|\mathrm{Re}(\lambda_2)|$ is still relevant and keeps the system away from stationarity. In such a scenario, the accelerated relaxation achieved by applying the transformation U is even more striking since the exponential gain is by a factor of $e^{t(|\mathrm{Re}(\lambda_3)|-|\mathrm{Re}(\lambda_2)|)}$. This can be appreciated from the curves displayed in Fig. 3.

Discussion.—We have presented a general method to control the timescale for the approach to stationarity in Markovian open quantum systems, which can be considered a quantum variant of the so-called Mpemba effect. Our results show how a dramatically accelerated approach to stationarity can be achieved by applying a suitable unitary transformation to the initial state, which removes its

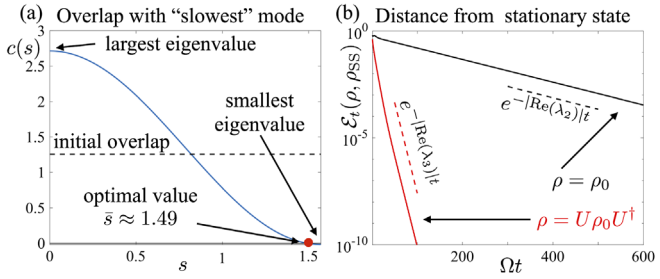

FIG. 3. All-to-all interacting spin model. (a) Overlap $c(s)$ of a rotated initial random state as a function of s. We take α_1 to be the largest eigenvalue of ℓ_2 and α_n the smallest. For the choice of parameters in this figure, $\Delta = -1$, $V = 3$, $\kappa = 1$ (all in units of Ω) and $N = 40$ spins, we have $\alpha_1 \approx 2.71$ and $\alpha_n \approx -0.02$. The optimal value $\bar{s}$ is thus $\bar{s} = \arctan(\sqrt{|\alpha_1/\alpha_n|}) \approx 1.49$, which makes the overlap $c(s)$ zero. The dashed line shows the initial overlap $\langle\psi|\ell_2|\psi\rangle$. (b) Distance between the time-evolved state and the stationary state ρ_{SS} of the interacting spin system. We compare the case of the initial random state $\rho = \rho_0$ with the one obtained after the rotation $\rho = U\rho_0 U^\dagger$. In this case, the transformation U prevents the system from entering a metastable regime which would slow down the approach to stationarity.

overlap with the slowest decaying mode. We note that the unitary operation U introduced in this work is "optimal" in the sense that it completely depopulates the slowest decaying mode. However, as discussed for instance in Ref. [27], in order to observe a Mpemba effect it would be sufficient to engineer a rotation which simply diminishes the excitation of such a slow mode. As shown in Figs. 2 and 3, this is also achieved by "nonoptimal" values of the parameter s for which the transformed overlap is smaller, in modulus, than the initial one. Considering specific many-body quantum models, it would be interesting to explore the possibility to reduce the population of the slowest decaying mode by means of a less involved unitary, for instance by implementing local and independent rotation of the different system constituents. This would not lead to an "optimal" speeding-up but would facilitate the implementation and observation of the Mpemba effect in actual experiments with open quantum many-body systems.

Finally, the method presented here can also be applied to accelerate the convergence to stationarity in periodic (Floquet) dissipative quantum dynamics. In addition, even if for non-Markovian dynamics a stationary state is, in general, not well defined, our procedure may still be applied to decrease the distance of the time-evolved quantum state from a "target" state encoded in the dissipative dynamics. We briefly discuss both applications in the Supplemental Material [50].

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