Deconfined Criticality and Bosonization Duality in Easy-Plane Chern-Simons Two-Dimensional Antiferromagnets

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Two-dimensional quantum systems with competing orders can feature a deconfined quantum critical point, yielding a continuous phase transition that is incompatible with the Landau-Ginzburg-Wilson scenario, predicting instead a first-order phase transition. This is caused by the LGW order parameter breaking up into new elementary excitations at the critical point. Canonical candidates for deconfined quantum criticality are quantum antiferromagnets with competing magnetic orders, captured by the easy-plane CP^1 model. A delicate issue however is that numerics indicates the easy-plane CP^1 antiferromagnet to exhibit a first-order transition. Here we show that an additional topological Chern-Simons term in the action changes this picture completely in several ways. We find that the topological easy-plane antiferromagnet undergoes a second-order transition with quantized critical exponents. Further, a particle-vortex duality naturally maps the partition function of the Chern-Simons easy-plane antiferromagnet into one of *massless* Dirac *fermions*.

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Introduction.—It is well known that some quantum critical systems exhibit a phase structure evading the traditional Landau-Ginzburg-Wilson (LGW) theory of phase transitions [1–3]. Typical examples are two-dimensional quantum systems with competing orders, like, for instance, antiferromagnetic (AFM) and valence-bond solid (VBS) orders originating from general quantum spin models with SU(2) symmetry [3,4]. The LGW scenario predicts a first-order phase transition for such a system. However, the interplay between emergent instanton excitations (i.e., spacetime magnetic monopoles) and staggered Berry phases [4] causes the actual phase transition to become a second-order one, leading in this way to a quantum critical point separating the AFM and VBS phases. For similar reasons discussed in studies of the deconfinement transition in high-energy physics, this type of critical point has been dubbed a deconfined quantum critical point [1]. At such a critical point, order parameters on both sides of the transition fall apart into elementary particles called spinons and we speak of spinon deconfinement.

A well-studied effective theory in this context is the quantum O(3) nonlinear sigma model (NLσM),

$$\mathcal{L}_{\mathrm{NL}\sigma\mathrm{M}} = \frac{1}{2g}(\partial_\mu \boldsymbol{n})^2 + \ldots, \tag{1}$$

where $\boldsymbol{n}^2 = 1$, supplemented by instanton-suppressing terms, here symbolically represented by ellipses [1,2,5–8]. Physically, the model is an effective theory of antiferromagnets capturing the long-distance interactions, and the unit vector $\boldsymbol{n}$ is the direction of the magnetization. When tuning the coupling constant g, the system undergoes a quantum phase transition from an AFM ordered phase to a paramagnetic phase separated by a critical coupling g_c. By means of the Hopf map, $\boldsymbol{n} = z_a^* \boldsymbol{\sigma}_{ab} z_b$, where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a Pauli matrix vector, the O(3) NLσM is shown to be equivalent to the CP^1 model,

$$\mathcal{L}_{\mathrm{CP1}} = \frac{1}{g}\sum_{a=1,2} |(\partial_\mu - i a_\mu) z_a|^2 + \ldots, \tag{2}$$

where the constraint $|z_1|^2 + |z_2|^2 = 1$ holds and the gauge field is an auxiliary field given by $a_\mu = (i/2)\times \sum_a (z_a^* \partial_\mu z_a - z_a \partial_\mu z_a^*)$.

Although the gauge field a_μ is an auxiliary field at the level of field equations, it becomes dynamical when quantum fluctuations of the spinon fields z_a are accounted for, causing a Maxwell term to be generated in the low-energy regime [9]. In this context it is also interesting to consider generalizations with N complex fields, yielding an $\mathrm{O}(2N)$ symmetric version, the CP^{N-1} model. It has been recently demonstrated [10] that the large N limit in a instanton-suppressed CP^{N-1} model implies a second-order phase transition. The result agrees with the standard field theory analysis of the large N limit [9,11]. Nevertheless, lower values of N were shown numerically to exhibit a first-order phase transition, specifically for $N = 4$, 10, 15; though the $N = 2$ case remained inconclusive [10,12]. This result contrasts with the large N limit without

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instanton suppression, where a first-order phase transition occurs [13,14].

A well-studied model since the early days of DC [1,2,6,7] is the easy-plane CP^1 model with Lagrangian,

$$\mathcal{L}_{\mathrm{ep}} = \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{\mathrm{CP1}} + \frac{K}{2g^2}(|z_1|^2 - |z_2|^2)^2, \qquad (3)$$

which follows directly from the NLσM by adding the easy-plane anisotropy term, $\mathcal{L}_{\mathrm{anis}} = Kn_z^2/2g^2$, where $K > 0$. Instanton suppression in the above Lagrangian is achieved by means of a Maxwell term [2,5],

$$\mathcal{L}_{\mathrm{M}} = \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2. \qquad (4)$$

An exact particle-vortex duality transformation of the lattice Villain model version of $\mathcal{L}_{\mathrm{ep}}$ shows that the model is self-dual [1,2,5,12]. Partly on the basis of this self-duality, it was originally argued [1,2] that the easy-plane CP^1 model undergoes a second-order phase transition, featuring therefore a deconfined quantum critical point. However, it was later demonstrated numerically that the phase transition is actually a first-order one [6,7], a result that is also corroborated by renormalization group (RG) results [8].

Here we consider the topological easy-plane CP^1 Lagrangian including a Chern-Simons (CS) term, i.e., $\mathcal{L} = \mathcal{L}_{\mathrm{ep}} + \mathcal{L}_{\mathrm{CS}}$, where,

$$\mathcal{L}_{\mathrm{CS}} = i\frac{\kappa}{2}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda, \qquad (5)$$

describes a CS Lagrangian in Euclidean spacetime. For arbitrary real κ the CS action is invariant under any topologically trivial gauge transformation, since the surface term vanishes in this case. On the other hand, topologically nontrivial ones generate a surface term that does not vanish. In this case one demands the invariance of $\exp(-S_{\mathrm{CS}})$, which forces κ to be quantized, $\kappa = n/(2\pi)$, where $n \in \mathbb{Z}$ is the CS level [15,16].

The motivation for such a system is twofold. First, it is interesting to examine the case of the instanton suppression by a topological term instead of a bare Maxwell term. Second, a system with similar properties should arise in the context of chiral spin liquids [17]. Moreover, as we will elaborate later, this is of direct relevance to bilayer quantum Hall systems that have been realized experimentally.

This Letter consists of three parts. First, we perform an RG analysis of the CP^1 CS action and show that the fixed point structure implies a second-order phase transition with critical exponents depending on the CS coupling and, hence, forming a new universality class. We will see that the scaling behavior of the topological theory cannot be smoothly connected to the limit where $\kappa \to 0$. In the second part of the Letter we show that the dual model features a CS term of the form,

$$\tilde{\mathcal{L}}_{\mathrm{CS}} = -\frac{i}{2\kappa}\epsilon_{\mu\nu\lambda}(b_{1\mu} + b_{2\mu})\partial_\nu(b_{1\lambda} + b_{2\lambda}), \qquad (6)$$

with two gauge fields $b_{1\mu}$ and $b_{2\mu}$. Finally, in the third part we show that for $\kappa = 1/(2\pi)$ the duality of the second part actually corresponds to a bosonization duality [18,19] involving *massless* Dirac fermions [20].

Renormalization group analysis.—Let us start by discussing the nature of the phase transition of the easy-plane CP^1 CS model by means of RG calculations. In order to regularize the short distance behavior, we also include the Maxwell term (4) in the Lagrangian $\mathcal{L} = \mathcal{L}_{\mathrm{ep}} + \mathcal{L}_{\mathrm{CS}}$, and consider a soft constraint version of the model,

$$\mathcal{L} = \mathcal{L}_M + \mathcal{L}_{\mathrm{CS}} + \sum_{a=1,2}[|(\partial_\mu - ia_\mu)z_a|^2 + m_0^2|z_a|^2] + \frac{u}{2}(|z_1|^2 + |z_2|^2)^2 + \frac{K}{2}(|z_1|^2 - |z_2|^2)^2. \qquad (7)$$

Details of the RG calculations are presented in Supplemental Material [21]. There we show that the original theory features two IR fixed points for the renormalized dimensionless couplings $\hat{u}$, $\hat{K}$, and $\hat{e}^2$. Importantly, e^2 sets a UV scale for the renormalized dimensionless gauge coupling $\hat{e}^2$, in the sense that the IR stable fixed point $\hat{e}_*^2$ is also reached when $e^2 \to \infty$ [21]. One of the fixed points is $\mathrm{O}(2) \times \mathrm{O}(2)$ symmetric, while the second one corresponds to an emergent O(4) symmetry. Interestingly, the Abelian Higgs CS critical exponents do not belong to the XY universality class, as they are κ dependent.

An important outcome of the RG analysis is that the limit $\kappa \to 0$ with e^2 finite does not reduce to the RG equations expected for a $\mathrm{U}(1) \times \mathrm{U}(1)$ Abelian Higgs model [9]. This happens because the presence of the CS term causes the one-loop gauge field bubble in the scalar field vertex function to vanish at zero external momenta (see Supplemental Material [21] for details on this point).

From the RG analysis it follows that the correlation length critical exponents for the $\mathrm{O}(2) \times \mathrm{O}(2)$- and O(4)-symmetric IR fixed points are quantized and depend on the level of the CS term. In particular, for a level 1 CS term this yields $\nu^{O(2)\times O(2)} = 49/80 \approx 0.613$. This value is nearly the same as the one-loop result $\nu = 5/8$ of the XY universality class. For the O(4)-symmetric criticality we obtain a larger value, $\nu^{O(4)} = 2/3$, which is independent of the CS level at the one-loop order.

The anomalous dimension η_N is defined by the critical magnetization correlation function at large distances, $\langle \boldsymbol{n}(x) \cdot \boldsymbol{n}(0) \rangle \sim 1/|x|^{1+\eta_N(n)}$. For a level 1 CS term we obtain, $\eta_N^{\mathrm{O}(2)\times\mathrm{O}(2)} = 59/49 \approx 1.2$ and $\eta_N^{\mathrm{O}(4)} = 164/147 \approx 1.12$, for the $\mathrm{O}(2) \times \mathrm{O}(2)$ and O(4)-symmetric cases, respectively. This clearly shows that a new universality class emerges.

At this point the following remark is in order. Typically, DC implies considerably larger anomalous dimensions η_N as compared to the case of the LGW paradigm of phase transitions. However, it is rather rare that these values exceed unity. The leading order value in the easy-plane case without a CS term is $\eta_N = 1$ (Gaussian approximation) [1]. For the $J - Q$ model the result is $\eta_N \approx 0.35$, but the easy-plane $J - Q$ model is reported to deliver a much larger value, $\eta_N \approx 0.91$ [26]. On the other hand, the theory considered here exhibits anomalous dimensions $\eta_N > 1$. An example where this also occurs is in a lattice boson model with an emergent Z_2 gauge symmetry [27], where the anomalous dimension is numerically calculated to be $\eta \approx 1.493$.

Duality analysis.—We start the discussion of the duality transformation by changing to polar coordinates $z_a = \rho_a e^{i\theta_a}$ in the partition function of the easy-plane CS CP^1 model. After integrating ρ_2 out and assuming a strong anisotropy ($K \gg g^2$), we obtain $\rho_1^2 \approx \rho_2^2 \approx 1/2$, leading to an effective action depending only on the phase fields coupled to the gauge field,

$$S_{\text{eff}} = S_{\text{CS}} + \frac{1}{2g}\sum_{a=1,2}\int d^3x(\partial_\mu\theta_a - a_\mu)^2, \qquad (8)$$

where the CS action S_{CS} corresponds to the Lagrangian (5). The above effective action is equivalent to a two-component CS superconductor in the London limit where the amplitudes of the order parameter are constrained to be equal.

The traditional way to perform a duality transformation is to carry it out on the lattice [28]. Nevertheless, while it is a straightforward task to define a Maxwell term on the lattice [29], fundamental difficulties arise when one tries to define the CS term on the lattice. It is known to be problematic to enforce the properties of a topological continuum field theory consistently on the lattice [30–32], although recently considerable progress has been made [33–35]. For these reasons, we will restrict ourselves to performing the subsequent calculations directly in the continuum.

Even though we are working directly in the continuum, in order for the theory to be well defined at the short distances, we need to regularize it. So we include an additional Maxwell term [36]. The first step of our duality transformation introduces auxiliary fields $h_{I\mu}$, $I = 1, 2$, such that,

$$S'_{\text{eff}} = \sum_{I=1,2}\int d^3x\left[\frac{g}{2}h_{I\mu}^2 - ih_{I\mu}(\partial_\mu\theta_I - a_\mu)\right] + \frac{1}{2e^2}\int d^3x(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{\kappa}{2}\int d^3x\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda. \qquad (9)$$

To account for the periodicity of θ_I, the following decomposition in terms of longitudinal phase fluctuations and vortex gauge fields holds [28], $\partial_\mu\theta_I = \partial_\mu\varphi_I + 2\pi v_{I\mu}$, where $\varphi_I \in \mathbb{R}$ and the vorticity,

$$w_{I\mu} = \epsilon_{\mu\nu\lambda}\partial_\nu v_{I\lambda}(x) = \sum_c n_{Ic}\oint_{L_{Ic}} dy_\mu^{(c)}\delta^3(x - y^{(c)}), \qquad (10)$$

with quanta $n_{Ic} \in \mathbb{Z}$ and the integral is over a path along the cth vortex loop L_{Ic}.

Integrating out both φ_I and a_μ leads to the action,

$$\tilde{S} = \sum_{I=1,2}\int d^3x\left(\frac{g}{2}h_{I\mu}^2 + i2\pi v_{I\mu}h_{I\mu}\right) + \frac{1}{2}\int d^3x\int d^3x' D_{\mu\nu}(x - x')(h_{1\mu} + h_{2\mu})(h'_{1\nu} + h'_{2\nu}), \qquad (11)$$

where $h'_{I\mu}$ denotes dependence on x', and the propagator in momentum space,

$$D_{\mu\nu}(p) = \frac{e^2}{p^2 + e^4\kappa^2}\left(\delta_{\mu\nu} - e^2\kappa\epsilon_{\mu\nu\lambda}\frac{p_\lambda}{p^2}\right), \qquad (12)$$

is the Fourier transform of $D_{\mu\nu}(x)$. Here, the longitudinal contribution is absent due to the constraint $\partial_\mu h_{I\mu} = 0$, which appears after integrating out fields φ_I. This also leads to $h_{I\mu}$ being expressed in terms of new auxiliary fields $b_{I\mu}$ as $h_{I\mu} = \epsilon_{\mu\nu\lambda}\partial_\nu b_{I\lambda}$.

As we are interested in the case of the easy-plane CS CP^1 model, we can send $e^2 \to \infty$ after performing explicitly the calculations in Eq. (11) and obtain the following dual Lagrangian,

$$\mathcal{L}_{\text{dual}} = \sum_{I=1,2}\left[\frac{g}{2}(\epsilon_{\mu\nu\lambda}\partial_\nu b_{I\lambda})^2 + i2\pi w_{I\mu}b_{I\mu}\right] - \frac{i}{2\kappa}\epsilon_{\mu\nu\lambda}(b_{1\mu} + b_{2\mu})\partial_\nu(b_{1\lambda} + b_{2\lambda}). \qquad (13)$$

One notices that the presence of the CS term in the original model leads to the appearance of the mixed CS term anticipated in Eq. (6). Thus, the dual action (13) features gauge fields coupled to an ensemble of vortex loops $w_{I\mu}$. The latter represent the worldlines of the particles of the original model [29,37].

As mentioned earlier in the context of the original theory using a soft constraint, an IR stable fixed point for the dimensionless renormalized gauge coupling is reached as $e^2 \to \infty$. This result remains valid in the hard constraint case. In Eq. (13) $1/g$ assumes the role of e^2 of the original theory. Note that $g = \hat{g}/\Lambda$, where $\hat{g}$ is dimensionless and Λ is a UV cutoff, so the theory with a hard constraint reaches a UV nontrivial fixed point $\hat{g}_*$ as $\Lambda \to \infty$, so $g \to 0$. Thus, the duality establishes a mapping between the UV and IR regimes of the theory.

Bosonization duality.—Having obtained a bosonic dual theory, we will show now that the theory of CS easy-plane antiferromagnets is actually self-dual at criticality and leads to the bosonization duality for massless Dirac fermions. We proceed to show this by first integrating out the fields $b_{I\mu}$ in Eq. (13). This yields the dual action in terms of vortex loop fields,

$$\tilde{S} = 2\pi^2 \int d^3x \int d^3x' \tilde{D}_{\mu\nu}(x-x')(w_{1\mu}+w_{2\mu})(w'_{1\nu}+w'_{2\nu}) + \frac{\pi}{g}\int d^3x \int d^3x' \frac{(w_{1\mu}-w_{2\mu})(w'_{1\mu}-w'_{2\mu})}{|x-x'|}, \tag{14}$$

where as before we are using primes to denote the dependence on x' and $\tilde{D}_{\mu\nu}(x-x')$ in momentum space reads,

$$\tilde{D}_{\mu\nu}(p) = \frac{g\kappa^2}{2(g^2\kappa^2p^2+4)}\left(\delta_{\mu\nu} - 2\frac{\varepsilon_{\mu\nu\lambda}p_\lambda}{\kappa g p^2}\right). \tag{15}$$

Now, we will show that, similarly to the standard easy-plane theory [5], the model considered here is self-dual in the large distance regime $g^2p^2 \ll 1$. In this case the vortices $w_{1\mu}$ and $w_{2\mu}$ balance, so we can write approximately, $w_{1\mu} = w_{2\mu} \equiv w_\mu$, so that (for details, see the Supplemental Material [21]),

$$S_{\rm dual} = \int d^3x (2\pi^2 g\kappa^2 w_\mu^2 + i2\pi^2\kappa v_\mu w_\mu). \tag{16}$$

On the other hand, letting $g \to 0$ in the initial Abelian Higgs CS action (9) and integrating out $h_{2\mu}$ yields $a_\mu = \partial_\mu\theta_2$. Subsequent integration of $h_{1\mu}$ enforces $\theta_1 = \theta_2 \equiv \theta$. At the end, this yields,

$$S = \int d^3x \left(\frac{2\pi^2}{e^2}w_\mu^2 + i2\pi^2\kappa v_\mu w_\mu\right), \tag{17}$$

and therefore we obtain the duality for the partition function,

$$Z_{\rm dual}(e^2=\infty, g, \kappa) = Z[g'=0, e'^2 = 1/(g\kappa^2), \kappa]. \tag{18}$$

Underlying the above result is the duality relation between the couplings, $ge^2 = 1/\kappa^2$. For a level 1 CS term the latter reduces to $ge^2 = (2\pi)^2$, which is the Dirac quantization associated to particle-vortex duality. It is interesting to note that Eq. (18) constitutes a topological version of the "frozen superconductor" regime in the particle-vortex duality for the Abelian Higgs model in $2+1$ dimensions derived by Peskin [29] and Dasgupta and Halperin [38].

We are now ready to explore the critical dual theory which, as was discussed above, is obtained by setting $g \to 0$ in the Lagrangian (16). This yields up to an overall normalization of the partition function

$$\tilde{Z}_{\rm crit} = \sum_{\rm loops} \exp\left[i\frac{\pi\kappa}{2}\sum_{a,b} n_a n_b \times \oint_{L_a} dx_\mu^{(a)} \oint_{L_b} dx_\nu^{(b)} \epsilon_{\mu\nu\lambda}\frac{(x^{(b)}-x^{(a)})_\lambda}{|x^{(b)}-x^{(a)}|^3}\right], \tag{19}$$

where we sum over all loops L_a and L_b, not excluding $a=b$ contributions, which will turn out to be a crucial point [39,40]. For $a \neq b$ the double integral above yields a contribution $e^{i2\pi^2 N_{ab}\kappa}$, $N_{ab} \in \mathbb{Z}$, in virtue of the Gauss linking number formula [41,42]. Despite looking at first sight singular, the $a=b$ contributions are actually finite and proportional to the so-called writhe of the (vortex) loop [43–45]. The latter can be conveniently written in terms of a suitable parametrization, $x_\mu(s)$, $s \in [0,1]$, by defining the unit vector, $u_\mu(s,s') = [x_\mu(s) - x_\mu(s')]/|x(s)-x(s')|$, in which case the writhe is recast as

$$\mathcal{W}_a = \frac{1}{4\pi}\int_{L_a} ds \int_{L_a} ds' \epsilon_{\mu\nu\lambda}\frac{dx_\mu}{ds}\frac{dx_\nu}{ds'}\frac{[x_\lambda(s)-x_\lambda(s')]}{|x(s)-x(s')|^3} = \frac{1}{4\pi}\int_{L_a} ds \int_{L_a} ds' \epsilon_{\mu\nu\lambda} u_\mu \partial_s u_\nu \partial_{s'} u_\lambda. \tag{20}$$

The result is reminiscent of the point-splitting regularization employed to calculate expectation values of Wilson loops [46]. This is in agreement with Ref. [36], where it is shown that the point-splitting procedure yields the topological invariant which coincides with the writhe in theories containing a Maxwell term in addition to a CS one when $e^2 \to \infty$.

We now consider a specific case of a level 1 CS theory in the original model corresponding to $\kappa = 1/(2\pi)$. Consequently, the dual partition function at criticality (19) takes the form

$$\tilde{Z}_{\rm crit} = \sum_{\rm loops} (-1)^{N_{ab}} e^{i\pi\sum_a n_a^2 \mathcal{W}_a}. \tag{21}$$

The contribution from the linking number formula generates weight factors $(-1)^n$ in the dual model, where n is the integer. This result is reminiscent of the lack of gauge invariance of the partition function under topologically nontrivial gauge transformations in the dual model [47,48]. This result makes apparent that the considered duality corresponds to a form of bosonization akin to the one discussed by Polyakov for the CP^1 model with a CS term [39,49]. This contribution is sometimes referred to as the Polyakov spin factor [39,40,50–53]. Equation (21) relates to the representation of the partition function of a Dirac fermion in $2+1$ Euclidean dimensions in terms of loops [40,51–53], with the difference that in our case the parity anomaly factor implies that the fermions are massless [15,54–56].

As far as the writhe is concerned, it is worth recalling that it arises quite naturally in the partition function of Wilson fermions on a Euclidean cubic spacetime lattice [40]. However, the analysis of Ref. [40] and previous ones [39,50–53,57] requires massive fermions.

It is remarkable that even if the analysis above does not explicitly employ fermions, still a result that can only follow from massless fermions is obtained. To elaborate this point further we recall that a topologically nontrivial gauge transformation γ, $a_\mu \to a_\mu^\gamma$, in a continuous deformation of the gauge field, leads to the subsequent transformation of the fermion determinant $\det(\partial\!\!\!/ + i a\!\!\!/) \to (-1)^n \det(\partial\!\!\!/ + i a\!\!\!/^\gamma)$, with n being the winding number [48,54–56]. Therefore, integrating over a_μ requires accounting for redundant gauge configurations and summing over all possible winding numbers corresponding to different topological sectors in the partition function.

To further substantiate our bosonization claim, we rederived this result using the flux attachment approach to duality [19], which involves a path integral formalism corresponding to a Fourier transform for quantized fluxes. In order for this to work in our case we have to attach fluxes to both fermions and bosons. The end result is that the dual Lagrangian (13) is the bosonized version of massless Dirac fermions with half-quantized CS flux attached. (The explicit derivation can be found in the Supplemental Material [21]). Therefore, our derivation is consistent with the flux attachment technique, but in contrast to it, does not assume any conjectures as a starting point. Thus, our analysis provides yet a further check for these conjectures.

Final remarks.—We have demonstrated through RG analysis that the topological easy-plane CP^1 model undergoes a second-order phase transition. Following this result, we established a dual theory, which at criticality exhibits a parity anomaly. This occurs at the particular value of a CS coupling κ that provides topological gauge invariance. We relate that to massless Dirac fermions, thereby establishing an explicit bosonization duality [18]. Since the theory we consider here possesses a $\mathrm{U}(1)\times\mathrm{U}(1)$ symmetry, our analysis subscribes into the so-called beyond flavor bound scenario of duality [58,59].

Additionally, let us consider these results within an experimental context. The dual theory (13) with $\kappa = 1/(2\pi)$ and gauge fields rescaled as $b_{I\mu} \to b_{I\mu}/(2\pi)$ features a CS term as it occurs in the (1,1,1) quantum Hall (QH) state associated to a bilayer QH system [60–62]. As mentioned, the initial model corresponds to a two-component CS superconductor. Therefore, the duality picture discussed here naturally connects the observed resonant tunneling in bilayer QH ferromagnets [63] to a Josephson-like effect in a system that is not superconducting [64–66]. Our analysis shows that such an experimental setup represents the dual physical system to the actual easy-plane CS antiferromagnet. They belong to the same universality class so that the bilayer QH ferromagnet offers a controllable experimental system for a deconfined critical point. Moreover, in view of the connection to massless Dirac fermions established in this Letter, bilayer QH ferromagnets would, in principle, offer a platform to experimentally explore the bosonization duality in $2+1$ dimensions. It would be interesting to check whether experiments can reveal the critical behavior with quantized exponents as we predict here.

Another system of interest where our approach may (with appropriate modifications) be relevant is the topological field theory for magic-angle graphene [67], where a duality between superconductivity and insulating regimes occur.

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