Experimental Single-Copy Entanglement Distillation

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The phenomenon of entanglement marks one of the furthest departures from classical physics and is indispensable for quantum information processing. Despite its fundamental importance, the distribution of entanglement over long distances through photons is unfortunately hindered by unavoidable decoherence effects. Entanglement distillation is a means of restoring the quality of such diluted entanglement by concentrating it into a pair of qubits. Conventionally, this would be done by distributing multiple photon pairs and distilling the entanglement into a single pair. Here, we turn around this paradigm by utilizing pairs of single photons entangled in multiple degrees of freedom. Specifically, we make use of the polarization and the energy-time domain of photons, both of which are extensively field tested. We experimentally chart the domain of distillable states and achieve relative fidelity gains up to 13.8%. Compared to the two-copy scheme, the distillation rate of our single-copy scheme is several orders of magnitude higher, paving the way towards high-capacity and noise-resilient quantum networks.

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Entanglement lies at the heart of quantum physics, reflecting the quantum superposition principle between remote subsystems without a classical counterpart. In addition to its fundamental importance, entanglement is an essential resource for most quantum information applications [1], and its distribution between remote parties provides the basis for quantum communication [2], distributed quantum computing [3], and eventually the quantum internet [4]. For entanglement distribution, photons are almost ideal carriers of quantum states. It is possible to create close-to-maximally entangled photon pairs at high rates in the laboratory [5–8], and recent efforts are venturing out of laboratory environments [9–12], bringing the ultimate goal of a global quantum network within reach. Nonetheless, noise and interaction with the environment are unavoidable, leading to decoherence [13] and the degradation of entanglement.

To counteract such detrimental noise effects, entanglement distillation, also referred to as entanglement purification, was introduced [14,15]. In entanglement distillation, two copies of a noisy entangled state are employed to distill a single copy with a higher degree of entanglement, a process which can be cascaded until ultimately a pure Bell state is reached. Its implementation is based on two-photon controlled NOT (CNOT) gates [see Fig. 1(a)], and facilitates, for example, quantum repeaters [16–18]. Experimental implementations of entanglement distillation, however, face two challenges: first, high optical losses in realistic scenarios lead to low transmission probabilities of a single photon pair [9,12], let alone the transmission of multiple photon pairs, which substantially limits the distillation rate. Second, the required two-photon CNOT gate cannot be deterministically realized with passive linear optics [19–22]. Therefore, all demonstrations of photonic two-copy entanglement distillation

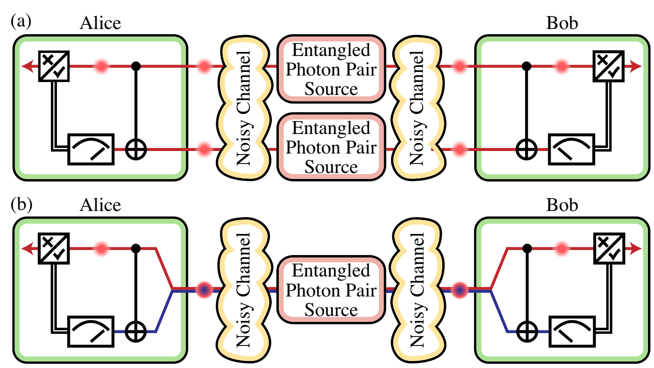

FIG. 1. Schematics of an elementary entanglement distillation step. (a) Two-copy entanglement distillation in which both Alice and Bob apply a CNOT gate between the two single photons they receive. The control photon pair is successfully purified if the measurement outcomes of the target qubits are correlated. (b) Single-copy entanglement distillation employs two entangled subspaces encoded in DOF of a single photon pair. The CNOT gate is now applied between the two DOF and the successful distillation of the control (upper) DOF is heralded by the measurement outcomes of the target (lower) DOF. The classical channel is omitted.

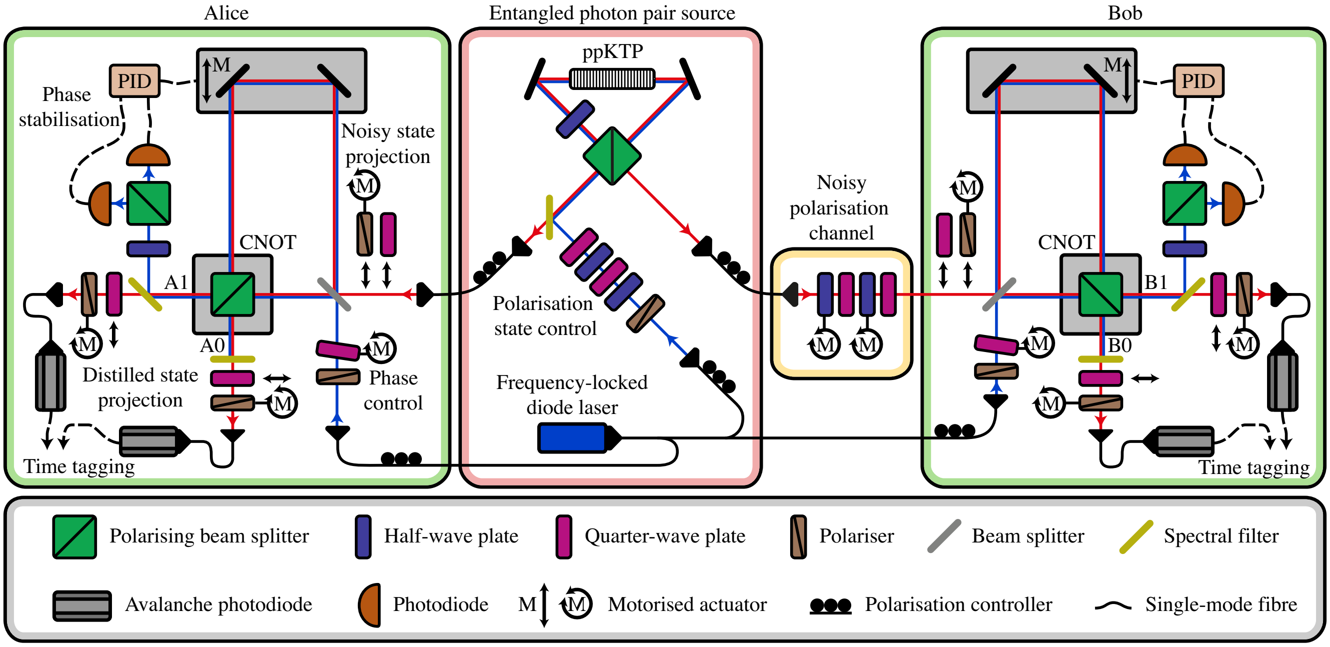

FIG. 2. Experimental setup for single-copy entanglement distillation in polarization and energy-time. Pairs of entangled photons are created in a periodically poled potassium titanyl phosphate (ppKTP) crystal placed in a Sagnac interferometer. Polarization entanglement is produced by bidirectionally pumping the crystal in the interferometer and overlapping the resulting photon-pair modes, while a narrow-line width pump laser gives rise to energy-time entanglement. The entangled photons are single-mode coupled and guided to Alice and Bob, where the distillation is carried out by interfering the polarization and the energy-time domain of single photons with a modified Franson interferometer, stabilized by a proportional integral derivative (PID) controller. The quantum state is either measured before (noisy state projection) or after (distilled state projection) the interferometer and the photons collected in paths A0, A1, B0, and B1 are detected and time tagged.

[18,23–25] are based on a parity check rather than on a genuine CNOT gate.

To overcome these problems, single-copy entanglement distillation was proposed, which harnesses entanglement in different degrees of freedom (DOF) of a single photon pair [26], known as hyperentanglement [27–29]. Instead of operating on two photons, each carrying a qubit, the CNOT gate now acts on two qubits encoded in different DOF of a single photon [see Fig. 1(b)]. Importantly, the CNOT gate between two DOF can be realized deterministically with linear optics [30,31], which has also been used in a recent purification implementation [32]. Furthermore, hyperentanglement is readily available in spontaneous parametric down-conversion (SPDC) [27,33] as well as in other photon-pair creating processes [34,35], and serves as a versatile experimental platform, featuring enhanced communication channel capacity [31,36,37]. Notably, the polarization and the energy-time DOF are particularly robust quantum information carriers and have been distributed over free-space [9,38,39] and long-distance fiber [12,40] links, marking them as ideal candidates for future in-field applications. On the other hand, the originally proposed spatial encoding [26] is less noise resilient outside of a protected laboratory environment [41,42].

Here, we report the first experimental implementation of a single-copy distillation protocol exploiting hyperentanglement in the field-tested polarization and energy-time degrees of freedom. We overcome the two principal limitations in standard distillation schemes: the probabilistic nature of multiphoton interactions and the low success rates due to two-pair transmission, both obliterating entangled-photon rates in any realistic scenario. By introducing different noise scenarios at finely tuned noise levels, we thoroughly test our distillation scheme and successfully recover entanglement and state fidelity. We beat the standard two-copy distillation rate by several orders of magnitude and thus unlock quantum communication in unprecedented noise regimes.

Our experimental platform consists of an entangled photon-pair source, which is connected to the spatially separated communicating parties Alice and Bob via 12 m-long single-mode fibres [Fig. 2]. Alice and Bob each have a distillation setup at their disposal and can characterize their part of the entangled quantum state prior to or after the distillation step. We obtain polarization entanglement in our photon-pair source by superposing a SPDC process in the clockwise and the counterclockwise direction of a Sagnac interferometer [43,44], creating a superposition of an H-polarized and a V-polarized photon pair. Energy-time entanglement, on the other hand, arises from energy conservation in the SPDC process driven by a temporally coherent pump field, leading to a potentially large superposition of temporal modes $|t_i t_i\rangle$ of the photon pairs [45]. Considering a two-dimensional subspace of the energy-time domain, the resulting hyperentangled state we produce is close to

$$|\mathbf{\Phi}^+\rangle = \frac{1}{2}[(|\mathrm{H},\mathrm{H}\rangle + |\mathrm{V},\mathrm{V}\rangle) \otimes (|t_L, t_L\rangle + |t_S, t_S\rangle)], \quad (1)$$

where both subspaces are entangled in a Φ^+ Bell state. In order to access the temporal modes at Alice and Bob, we map both photons to the path domain in a Franson interferometer [46] by employing two Mach-Zehnder interferometers with a temporal imbalance of $t_L - t_S$ between the long (L) and the short (S) arm. This mapping is probabilistic due to the randomness of the employed 50:50 beam splitters. Owing to their indistinguishability, only those photon pairs arriving simultaneously at the outputs of the interferometers exhibit quantum interference, requiring postselection on coincidences [47]. The polarizing beam splitter at the output ports of the interferometer constitutes the single-photon CNOT gate. Depending on the polarization state (control qubit) of the photon, the path (target qubit) of the photon is either left unchanged or acted upon with an exclusive OR gate. In analogy with the two-copy distillation protocol [14], the polarization state is successfully distilled if the measurement outcomes in the computational path basis are correlated. For our experimental setup Fig. 2, this implies that only photons collected in paths A0 and B0 or A1 and B1 are postselected (see Supplemental Material [48] for experimental methods).

In order to fully characterize the performance of the protocol, we separately introduce different error types in the polarization channel and the energy-time channel. All trace-preserving errors on a qubit can be decomposed into the so-called error basis. Apart from the identity σ_0, this operator basis consists of the Pauli operators σ_x (bit-flip error), σ_z (phase-flip error) and σ_y (bit-phase-flip error), corresponding to rotations about the three Bloch sphere axes. The error basis is therefore capable of transforming the Φ^+ Bell state into any mixture of Bell states ("Bell-diagonal states") by means of one-sided transformations. We utilize this fact in the polarization domain by inserting four waveplates in Bob's channel ("Noisy polarization channel"), which introduce arbitrary superpositions in the error basis, and we generate mixed states by averaging over different wave plate settings. The noisy polarization channel therefore results in a mixture of the Φ^+_{pol} state and an erroneous state $\rho^{\mathrm{err}}_{\mathrm{pol}}$. We implement the noisy channel for the energy-time domain by gradually changing the coincidence window, leading to a mixture of the Φ^+_{e-t} state and an erroneous state $\rho^{\mathrm{err}}_{e-t} = (|\Psi^+\rangle\langle\Psi^+|_{e-t} + |\Psi^-\rangle\langle\Psi^-|_{e-t})/2$ (see Supplemental Material [48] for details on the noise control). Owing to the independent manipulation of the polarization and the energy-time domain, the resulting family of mixed states is still a product state $\rho^{\mathrm{noisy}}_{\mathrm{pol}} \otimes \rho^{\mathrm{noisy}}_{e-t}$, with

$$\rho^{\mathrm{noisy}}_{\mathrm{pol}} = F^{\mathrm{noisy}}_{\mathrm{pol}}|\Phi^+\rangle\langle\Phi^+|_{\mathrm{pol}} + (1 - F^{\mathrm{noisy}}_{\mathrm{pol}})\rho^{\mathrm{err}}_{\mathrm{pol}},$$
$$\rho^{\mathrm{noisy}}_{e-t} = F^{\mathrm{noisy}}_{e-t}|\Phi^+\rangle\langle\Phi^+|_{e-t} + (1 - F^{\mathrm{noisy}}_{e-t})\rho^{\mathrm{err}}_{e-t}.$$

As we are now able to experimentally produce mixed entangled states, we employ them as input to our distillation procedure. The success of the distillation protocol can be evaluated by comparing the noisy and distilled state with each other. For this purpose, we utilize the quantum state fidelity F to the Φ^+ state before and after the distillation. The fidelity is an easily measurable quantity and a good estimator of entanglement [49]. At first, we measure the fidelity $F^{\mathrm{noisy}}_{\mathrm{pol}} = \langle\Phi^+|\rho^{\mathrm{noisy}}_{\mathrm{pol}}|\Phi^+\rangle_{\mathrm{pol}}$ of the noisy polarization state $\rho^{\mathrm{noisy}}_{\mathrm{pol}}$ to the Φ^+_{pol} state and similarly the fidelity F^{noisy}_{e-t} of the noisy energy-time state $\rho^{\mathrm{noisy}}_{e-t}$. After the distillation step

$$\rho^{\mathrm{noisy}}_{\mathrm{pol}} \otimes \rho^{\mathrm{noisy}}_{e-t} \overset{\mathrm{distillation}}{\longrightarrow} \rho^{\mathrm{distill}}_{\mathrm{pol}},$$

the fidelity $F^{\mathrm{distill}}_{\mathrm{pol}} = \langle\Phi^+|\rho^{\mathrm{distill}}_{\mathrm{pol}}|\Phi^+\rangle_{\mathrm{pol}}$ of the distilled state $\rho^{\mathrm{distill}}_{\mathrm{pol}}$ to the Φ^+_{pol} state determines whether the distillation was successful. Since the single-copy scheme inherently consists of two independent error channels, we make use of the gain $G = F^{\mathrm{distill}}_{\mathrm{pol}} - \mathrm{Max}(F^{\mathrm{noisy}}_{\mathrm{pol}}, F^{\mathrm{noisy}}_{e-t})$ as our figure of merit. Experimentally, the fidelity F to the Φ^+ state is obtained by measuring the interference visibility $V_{ii} = \langle\sigma_i \otimes \sigma_i\rangle$ in three mutually unbiased bases $\{\sigma_x, \sigma_y, \sigma_z\}$, as $F = (1 + V_{xx} - V_{yy} + V_{zz})/4$.

We first introduce a bit-flip error ($\rho^{\mathrm{err}}_{\mathrm{pol}} = |\Psi^+\rangle\langle\Psi^+|_{\mathrm{pol}}$) to the polarization domain and compare the gain of the experimental data [Fig. 3(a)] with the theory prediction [Fig. 3(b)]. Our experimentally obtained fine-grained error map in Fig. 3 allows us to analyse the performance of the distillation protocol in great detail. First, the region of positive gain ($G > 0$) is approximately symmetrical about the diagonal line defined by $F^{\mathrm{noisy}}_{\mathrm{pol}} \simeq F^{\mathrm{noisy}}_{e-t}$. All distilled states within this region have a higher fidelity to the maximally entangled Φ^+_{pol} state than both noisy states before the distillation. The nondistillable states outside of this area ($G < 0$) are highly asymmetric in their error contribution in polarization and energy-time, while the highest measured gains, up to $G = 0.101$ (13.8% relative gain), can be observed for approximately symmetric error contributions ($F^{\mathrm{noisy}}_{\mathrm{pol}} = 0.734$, $F^{\mathrm{noisy}}_{e-t} = 0.732$). A cross section along the diagonal [Fig. 3(c)] reveals the gain curve known from a bit-flip channel in two-copy entanglement distillation [14] and indicates, in addition to cross sections at constant energy-time fidelity [Fig. 3(d)], a good agreement between our experimental data and the theory.

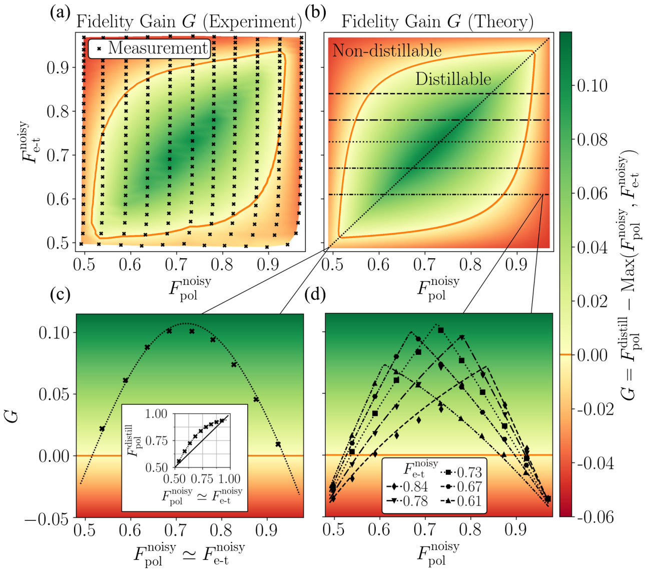

FIG. 3. Fidelity gain after single-copy entanglement distillation in polarization and energy-time. (a) The gain of the experimental data points are triangulated to form the heat map. Starting from an initial fidelity to the Φ^+ state of $F_{\text{pol}}^{\text{init}} = 97.1\%$ and $F_{e-t}^{\text{init}} = 96.8\%$ (top right measurement point) we gradually increase the bit-flip (bit and bit-phase-flip) error in polarization (energy-time) down to a fidelity of ~50%. (b) The heat map corresponds to the model, with the initial fidelities and the imperfect CNOT unitary as the only model parameters. (c) A cut through the heat map at $F_{\text{pol}}^{\text{noisy}} \simeq F_{e-t}^{\text{noisy}}$ reveals the characteristic behavior of two-copy distillation [14]. (d) Cuts through the heat map at constant F_{e-t}^{noisy}. Both in (c) and (d) lines correspond to the model, while markers correspond to measurement points with a standard deviation smaller than the marker size. The orange line separates the region of distillable ($G > 0$, green) from the region of nondistillable states ($G < 0$, red).

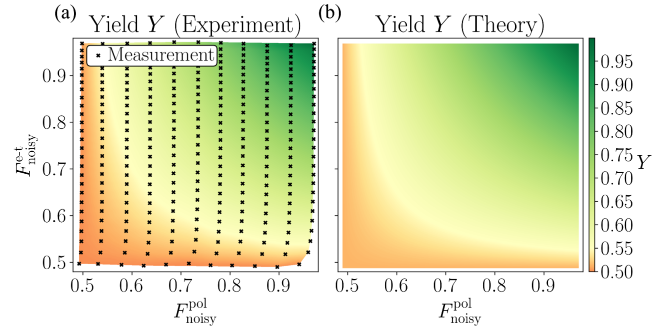

FIG. 4. Yield of single-copy entanglement distillation in polarization and energy-time. (a) The yield Y corresponds to the measurement in Fig. 3. The heat maps are triangulations of (a) the measurement points and (b) the model data points.

Another important figure of merit for distillation protocols is the yield. For a single step of the protocol, we define the yield Y as the number of distilled photon pairs divided by the total number of coincident photon pairs. In our experiment, the yield peaks at $Y = 98.6\%$ for states close to the ideal source state in Eq. (1), while it does not fall below $Y = 50\%$ for any noisy state (Fig. 4). This is an improvement by a factor of 2 compared to the two-copy distillation protocol, which by default has to sacrifice the target photon pair to herald a successful distillation.

In order to demonstrate the universality of our scheme, we examine the bit-phase-flip error ($\rho_{\text{pol}}^{\text{err}} = |\Psi^-\rangle\langle\Psi^-|_{\text{pol}}$) as an additional error type. We experimentally observe that this error shows the same gain and yield characteristics as the bit-flip error, and we again obtain a maximal gain of $G = 0.094$ (12.76% relative gain) at approximately symmetric error contributions ($F_{\text{pol}}^{\text{noisy}} = 0.733, F_{e-t}^{\text{noisy}} = 0.733$). Thus, our results show that single-copy distillation can deal with various noise scenarios.

We experimentally demonstrated an entanglement distillation scheme based on interference between the polarization and the energy-time domain of a single photon pair. Our work constitutes the first experimental realization of entanglement distillation based on CNOT gates, as initially devised in Refs. [14,26], between these DOF. By using degrees of freedom which have been widely tested in long-distance experiments, the scheme we introduced can directly be implemented over existing fiber and free-space links. Our thorough analysis of different error types and strengths allowed us to characterize the domain of distillable states and quantify the fidelity gain. These results are of vital importance for future implementations in which, in general, different DOF are faced with different errors during transmission.

Apart from the experimental feasibility of our approach, the major advantage rests upon its high efficiency as compared to two-copy distillation [23]. For typical experimental parameters and link losses of 20 dB in both channels, the single-copy distillation rate outperforms the tow-copy scheme by 8 orders of magnitude (see Supplemental Material [48] for a detailed rate comparison). This illustrates that both the creation and the distribution of multiple photon pairs is costly, while a single hyperentangled photon pair can equivalently be utilized for entanglement distillation using significantly less resources. Additionally, hyperentanglement is naturally produced in SPDC [27] and does not necessarily increase the complexity of photon pair sources, which is essential for the establishment of entanglement distribution infrastructure such as satellite-based entanglement sources [9]. Furthermore, our implementation using the energy-time DOF is more robust in out-of-the-laboratory implementations as compared to realizations using the path DOF [32], which additionally requires modifications of the SPDC source.

As opposed to the two-copy scheme, our experiment neither depends on temporal synchronization between Alice and Bob at the order of the photon's correlation time [11], nor on the storage in a quantum memory [50]. For quantum cryptography in the polarization domain, our scheme offers the added advantage of path multiplexing, potentially enhancing the secure key rates significantly [51]. While in its first conception [14,15], entanglement distillation was thought of as an asymptotic procedure converging on perfectly entangled states, single-copy distillation schemes are of course inherently limited to a finite number of distillation steps given by the number of accessible DOF (in our case two). Although the observed characteristic of the fidelity gain is evidence of its suitability for recurrence protocols [52], practical considerations make clear that for all known applications in quantum information processing, perfect Bell pairs are not required and usually, crossing a certain noise threshold is sufficient, e.g., for yielding nonzero key rates in quantum key distribution. As our experiment shows, this is where single-copy distillation shows its true power. In low-noise settings, the fidelity gains are moderate, while large gains are obtained in a regime where it really matters—that of high noise. This enables a significant increase in state quality in noise-dominated scenarios, recovering the potential for quantum communication applications in regimes that would otherwise be unattainable.

Photonic hyperentanglement in polarization and energy-time has already been successfully demonstrated over a free-space link [38], where environmental noise is one of the limiting factors. Our experiment realistically simulates local noise on the polarization degree of freedom, such as noise from stress-induced polarization changes in fibers [53]. Additionally, embedding the polarization state in a higher-dimensional state space of energy-time or spatial modes can lead to the dilution of noise induced by background light or accidental coincidences (isotropic noise) [54]. Thus, combining the noise advantages of higher-dimensional systems with our implemented distillation procedure constitutes a promising path forward.

The presented scheme is, in principle, not limited to photonic implementations, and might be extended to other systems featuring hyperentanglement such as atoms [55] or ions [56]. A natural extension of distillation protocols in two DOF will be the exploitation of more photonic DOF, enabling multiple distillation steps on a single photon pair. The only prerequisite for including additional entangled DOF is the existence of corresponding CNOT gates [30,32,57]. Another viable path is the employment of generalized CNOT gates in high-dimensional entanglement distillation [58].

Distributing entanglement between remote parties in the face of noise is an essential task in quantum information processing. Here, we tackled the problem of inefficient entanglement distillation by exploiting hyperentanglement of a single copy instead of exploiting two copies of a photon pair. Our single-copy entanglement distillation approach enables the distribution of high-fidelity entangled states at practical rates and can thus become a vital building block of a future quantum internet.

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