Anomalous Hydrodynamics in a One-Dimensional Electronic Fluid

I. V. Protopopov,[1,2] R. Samanta,[3] A. D. Mirlin,[4,5,6,2] and D. B. Gutman[3]

[1]*Department of Theoretical Physics, University of Geneva, 1211 Geneva, Switzerland*
[2]*Landau Institute for Theoretical Physics, 119334 Moscow, Russia*
[3]*Department of Physics, Bar Ilan University, Ramat Gan 52900, Israel*
[4]*Institute for Quantum Materials and Technologies, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*
[5]*Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76049 Karlsruhe, Germany*
[6]*Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia*

(Received 1 January 2021; accepted 19 May 2021; published 22 June 2021)

We construct multimode viscous hydrodynamics for one-dimensional spinless electrons. Depending on the scale, the fluid has six (shortest lengths), four (intermediate, exponentially broad regime), or three (asymptotically long scales) hydrodynamic modes. Interaction between hydrodynamic modes leads to anomalous scaling of physical observables and waves propagating in the fluid. In the four-mode regime, all modes are ballistic and acquire Kardar-Parisi-Zhang (KPZ)-like broadening with asymmetric power-law tails. "Heads" and "tails" of the waves contribute equally to thermal conductivity, leading to $\omega^{-1/3}$ scaling of its real part. In the three-mode regime, the system is in the universality class of a classical viscous fluid [O. Narayan and S. Ramaswamy, Anomalous Heat Conduction in One-Dimensional Momentum-Conserving Systems, Phys. Rev. Lett. **89**, 200601 (2002)., H. Spohn, Nonlinear fluctuating hydrodynamics for anharmonic chains, J. Stat. Phys. **154**, 1191 (2014).]. Self-interaction of the sound modes results in a KPZ-like shape, while the interaction with the heat mode results in asymmetric tails. The heat mode is governed by Levy flight distribution, whose power-law tails give rise to $\omega^{-1/3}$ scaling of heat conductivity.

DOI: 10.1103/PhysRevLett.126.256801

Understanding properties of interacting electronic systems is fundamentally important across various branches of physics. The problem is extremely nontrivial and multifaceted due to the impact of quantum coherence and strong interactions as well as other important ingredients, including the underlying crystal lattice and/or disorder. Progress has been achieved by constructing effective theories for long-living modes of electronic systems. Such theories are universal, i.e., insensitive to microscopic details and mostly determined by qualitative aspects such as dimensionality, symmetries, and topology. Paradigmatic examples of effective descriptions are Landau's Fermi-liquid theory [1], the theory of superfluid liquids [2], and the theory of diffusive modes in disordered conductors [3].

Recent advances in experimental techniques have made available several systems [4–9] realizing (in a certain temperature range) the hydrodynamic regime of electron transport. In this regime, the dynamics is dominated by electron-electron collisions (rather than by impurity or electron-phonon scattering) and can be described by a set of equations of hydrodynamic type governing the evolution of conserved densities (a charge, momentum, energy, etc.). Two aspects make such systems spectacular. First, they exhibit electron transport that is profoundly different from that observed in conventional Drude conductors. It is manifested in the Gurzhi effect [10], spatial nonlocality [11], and unconventional magnetoresistance [12,13], see Refs. [14,15] for a recent review. Second, topologically induced qualitative diversity of underlying electronic spectra gives rise to unconventional hydrodynamic regimes, such as relativistic hydrodynamics in graphene [16].

The hydrodynamics of one-dimensional (1D) interacting electrons is of special interest. It often involves an extended (in integrable systems even infinite) number of conserved hydrodynamic charges [17–23]. Furthermore, the reduced dimensionality of the system greatly promotes hydrodynamic fluctuations [24], which can invalidate the mean-field hydrodynamic description at sufficiently long scales and drive the system into a fluctuation-dominated regime characterized by nontrivial scaling of physical observables [25–28]. The relevance of fluctuational hydrodynamics and in particular of the celebrated Kardar-Parisi-Zhang (KPZ) model in the context of 1D electronic fluids was discussed recently in Refs. [29,30].

In this Letter, we explore the full multimode fluctuational hydrodynamics of 1D spinless fermions with short-range interaction. Our focus is on real-time dynamics and on thermal transport that was probed recently in several closely related experimental setups [31–39]. We confirm the frequency scaling of the thermal conductivity, $\kappa \propto \omega^{-1/3}$, advocated recently based on a self-consistent kinetic theory of bosonic excitations (see Ref. [30] and references therein). We find, however, that the earlier kinetic treatment fails to predict the correct dependence of the prefactor in this scaling on temperature and other parameters of the system.

To construct the nonlinear hydrodynamic description of the system, we employ the bosonization technique [40–43] taking into account the curvature of the electronic spectrum (i.e., finite fermion mass m) [44–47]. Relaxation processes in such a "nonlinear Luttinger liquid" were analyzed in several works, see Ref. [48] for a review. It was shown in Ref. [49] (see also Refs. [50–54]) that at temperatures below Fermi-Bose duality temperature $T_{\mathrm{FB}} = 1/ml^2$ [$T < T_{\mathrm{FB}} < \epsilon_F$, where ϵ_F is the Fermi energy and l is the range of the electron-electron interaction], thermal excitations in a nonlinear Luttinger liquid are "composite" fermions with renormalized Fermi velocity $u \sim v_F$, an effective mass $m_* \sim m$, and weak interactions vanishing in the zero-momentum limit [55]. The composite fermions are characterized by a long lifetime τ_F,

$$\tau_F^{-1} \sim l^4 T^7 / m_*^2 u^8. \tag{1}$$

Focusing on this low-temperature regime, we employ the Hamiltonian of the composite fermions and derive the corresponding kinetic equation [56]

$$\frac{\partial N_F(p)}{\partial t} + v_p^F \frac{\partial N_F(p)}{\partial x} = \hat{I}_p[N_F]. \tag{2}$$

Here N_F is a distribution function, v_p^F is the momentum-dependent velocity of fermionic quasiparticles, and $\hat{I}$ is the collision integral. The hydrodynamic equations arise after the projection of the kinetic equation on the zero modes of the collision integral and are valid at scales larger than the fermionic mean free path $u\tau_F$. The collision integral in (2) is nullified by Fermi-Dirac function $n_F[(\epsilon_p - vp - \mu)/T]$ with chemical potential μ, temperature T, and the boost velocity v. These three parameters of the equilibrium distribution correspond to the three exactly conserved densities of the model: particle number, energy, and momentum. Peculiarities of the 1D kinematics give rise, however, to other quasiconserved quantities (soft modes of the collision integral). First, equilibration of the particle number between the left and right movers requires processes involving a deep hole near the bottom of the band. In the bosonic description of the Luttinger liquid, such processes correspond to the umklapp scattering and manifest themselves only at exponentially long length scale [66–68]

$$L_u \sim u T^{-3/2} \epsilon_F^{1/2} e^{\epsilon_F/T}. \tag{3}$$

Thus, at scales shorter than L_u the system possesses four conserved quantities (total energy, total momentum, and two chiral number densities). Second, a detailed analysis of collision processes leading to Eq. (1) shows that in such a collision the energy and momentum exchange between the chiral sectors is parametrically suppressed (compared to the thermal energy or momentum) by a factor $(T/\epsilon_F)^2 \ll 1$ [56]. Correspondingly, at scales shorter than

$$L_4 \sim (\epsilon_F/T)^2 u \tau_F \tag{4}$$

the chiral sectors are effectively decoupled and six hydrodynamic modes exist in the system.

In the six-mode regime the particle densities, momentum, and energies of each chiral sector are separately conserved and we combine them into two chiral vectors $\mathbf{q}_\eta^T = (\rho_\eta, \pi_\eta, \epsilon_\eta)$, $\eta = R, L$. We denote by $\boldsymbol{\phi}_\eta^T = T_\eta^{-1}(\mu_\eta, v_\eta, -1)$ the vector of the corresponding conjugate thermodynamic variables. The conserved quantities obey the continuity equations

$$\partial_t q_\eta^i + \partial_x J_\eta^i = 0, \tag{5}$$

with index i specifying the conserved charge and the corresponding flux, $\mathbf{J}_\eta = (J_\eta^\rho, J_\eta^\pi, J_\eta^\epsilon)$.

On the linear level, one relates

$$\mathbf{q}_\eta(\omega, k) = \chi_\eta^{\mathrm{ret}}(\omega, k) \boldsymbol{\phi}_\eta(\omega, k), \tag{6}$$

via the polarization operator $\chi_{i,j;\eta}^{\mathrm{ret}}(x,t) = -i\theta(t) \times \langle [\hat{\mathbf{q}}_{i,\eta}(x,t), \hat{\mathbf{q}}_{j,\eta}(0,0)] \rangle$. Similarly, currents can be represented in terms of current response function M,

$$\mathbf{J}_\eta(\omega, k) = M_\eta(\omega, k) \boldsymbol{\phi}_\eta(\omega, k)/ik. \tag{7}$$

In the $\omega = 0$, small-k limit, the matrix $M_\eta(k) = (ikA + k^2 D)\chi_\eta$ is built out of matrices of velocities (A), diffusion coefficients (D), and static susceptibilities $\chi_\eta \equiv \chi_\eta^{\mathrm{ret}}(\omega = 0, k \to 0)$. The velocity matrix A and the matrix of static susceptibilities are thermodynamic quantities and can be computed straightforwardly in the approximation neglecting the composite-fermion interaction. The matrix of diffusion coefficients D requires more work; it can be obtained from the linearized kinetic equation (2). See Supplemental Material [56] for explicit expressions for χ and M.

To incorporate nonlinear effects into the hydrodynamic description, we extend the expressions for hydrodynamic currents by terms of second order in the conserved densities:

$$\mathbf{J}_\eta = (M_\eta/ik)\chi_\eta^{-1}\mathbf{q}_\eta + \frac{1}{2}\sum_{i,j} \mathbf{H}_{\eta;i,j} q_\eta^i q_\eta^j. \tag{8}$$

Here, we have taken the static limit $\omega = 0$ and the (vector-valued) coefficients $\mathbf{H}_{\eta;i,j}$ can be computed neglecting the interaction of composite fermions [56].

Equations (5), (6), and (8) describe the six-mode hydrodynamics that exist at short length scales, $L < L_4$.

At longer scales, the collisions equilibrate the temperatures and the boost velocities in the two chiral sectors. The hydrodynamic theory of the four-mode regime can be obtained through the reduction of the six-mode equations by setting $T_L = T_R = T$, $v_L = v_R = v$ and working with the total energy and momentum densities, $\epsilon = \epsilon_R + \epsilon_L$ and $\pi = \pi_R + \pi_L$.

At still larger length scales, $L > L_u$, the system reaches equilibrium with respect to particle exchange between the chiral sectors. The corresponding three-mode hydrodynamics can be obtained through the reduction of the four-mode theory by setting $\mu_L = \mu_R = \mu$.

In the linear hydrodynamic approximation, the continuity equations dictate that

$$\chi_\eta^{\rm ret}(\omega, k) = M_\eta(i\omega\chi_\eta - M_\eta)^{-1}\chi_\eta. \tag{9}$$

The information encoded in the polarization operator enables one to compute the full set of kinetic coefficients, accessible via linear-response measurements. At first glance, the nonlinear terms in hydrodynamic equations are irrelevant for the discussion of such linear-response quantities. This conclusion is, however, invalidated by hydrodynamic fluctuations arising due to finite temperature that were so far neglected. Once the fluctuations are taken into account, the nonlinear hydrodynamic couplings induce strong renormalizations of bare kinetic coefficients, totally modifying the linear-response characteristics of the system.

To account for fluctuations we promote the hydrodynamic equations (5) to the Keldysh action (of Martin-Siggia-Rose type) [56]. Since at hydrodynamic scales the system is locally at equilibrium, the fluctuation-dissipation theorem holds. Therefore, the retarded part of the polarization operator $\chi^{\rm ret}$ determines also the Keldysh components and thus the entire action at the Gaussian level. The quadratic terms in the hydrodynamic currents (8) correspond to cubic vertices in the action.

Following Ref. [27], we analyze the resulting Keldysh action of fluctuational hydrodynamics within the mode-coupling approximation [69]. To perform the calculation it is convenient to pass to the eigenmodes of the linearized hydrodynamic theory. We define a new basis $\mathbf{\Psi} = R\mathbf{q}$, where R diagonalizes the velocity matrix A, $RAR^{-1} = \mathrm{diag}(v_1, \ldots, v_N)$. Because of the mode separation caused by different mode velocities, only diagonal correlations survive in the long-time limit, and the Keldysh pair-correlation functions of the eigenmodes

$$f_j(x,t) = \langle \Psi_j(x,t)\Psi_j(0,0)\rangle \tag{10}$$

satisfy the self-consistent Dyson equations [56]

$$(\partial_t + v_j\partial_x - \tilde{D}_j\partial_x^2)f_j(x,t) = \int_{-\infty}^{\infty} dy \int_0^t ds \times f_j(x-y, t-s)\partial_y^2 R_j(y,s). \tag{11}$$

Here

$$R_j(y,s) = \frac{1}{T^5}\sum_{l,m=1}^{N} \lambda_{jlm}^2 f_l(y,s) f_m(y,s), \tag{12}$$

$\tilde{D}_j$ are diagonal elements of the effective diffusion matrix $\tilde{D}$ describing broadening of eigenmodes, and coupling constants λ_{jlm} account for the mode interaction. These constants are computed from microscopic parameters of the original fermionic model [56].

We now employ this theory to study pulse propagation in an electronic fluid as well as its linear-response properties.

We consider the time evolution of a generic disturbance created in a limited region of the fluid. Because of energy relaxation for times longer than fermionic energy relaxation time τ_F, any disturbance is fully projected onto eigenmodes of the collision integral. At times shorter than L_4/u, this yields six hydrodynamic modes Ψ_j. The degree to which the modes are excited depends on the overlap of the disturbance with Ψ_j. These modes give rise to six ballistic pulses propagating through the fluid. Because of differences between the mode velocities, $\Delta u_{ij} \equiv u_i - u_j$, the separation between the peaks grows linearly with time, $L_{ij} = \Delta u_{ij} t$. The width of each peak is broadened, within the linear hydrodynamics, by the corresponding diffusion process as $(\tilde{D}_j t)^{1/2}$. The nonlinear couplings further broaden the shape of the pulses and modify their shape. At $L \sim L_4$, the number of hydrodynamic modes is reduced to four and the pulses are reshaped into four peaks. In the four-mode regime all nonlinear couplings are of the same order, $\lambda_{ijk} \sim \lambda \equiv T u^{3/2}$. However, only the interaction between modes propagating in the same direction is significant. Thus, Eq. (11) splits into two sets of chiral equations. Comparing the linear and nonlinear terms in Eq. (11) one finds that nonlinear broadening dominates over the normal diffusion at scales beyond $L_* = u^{13}/l^{12}T^{13} \gg L_4$. The same conclusion is reached by analyzing the RG flow [56]. Essentially, at this stage, one can drop the bare diffusion terms in Eq. (11).

Near the maximum of any given mode, the coupling to other modes is parametrically small and to the first approximation can be neglected. Equation (11) describes the long time limit of the pair velocity correlation function in the stochastic Burgers equation and the corresponding KPZ problem [70]. Thus, near the maximum

$$f_i(x,t) \sim \frac{T^2}{(\lambda t)^{2/3}} f_{\rm KPZ}\left(\frac{T(x-u_i t)}{(\lambda t)^{2/3}}\right). \tag{13}$$

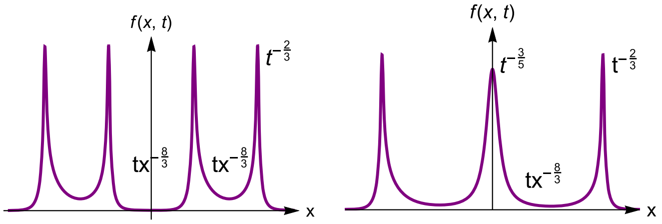

FIG. 1. Schematic shape of pulse evolution through four and three mode regimes. The scaling of the heads and tails of the peaks with time is depicted, see text for details.

Here $f_{\mathrm{KPZ}}(x)$ is the universal dimensionless KPZ function, with $f_{\mathrm{KPZ}}(x) \sim 1$ for $|x| \leq 1$ and $f_{\mathrm{KPZ}}(x) \sim e^{-0.3|x|^3}$ for $|x| \gg 1$ [71,72]. Away from the maximum the interaction between modes plays a role and creates nonsymmetric power-law tails [56], see Fig. 1. The fast modes develop power-law rear tails, while slow modes develop power-law front and rear tails:

$$f_i(x,t) \sim \sum_j \theta[(x-u_i t)\mathrm{sgn}(\Delta u_{ji})]u^2\left(\frac{T}{|\Delta u_{ji}|}\right)^{1/3} \times t|x-u_i t|^{-8/3} \quad \text{for } |x-u_i t| \gg \frac{u t^{2/3}}{T^{1/3}}. \tag{14}$$

One may interpret this as a propagation of one degree of freedom away from its light cone via the interaction with a faster or slower degree of freedom. In Eqs. (13), (14) and below we omit numerical coefficients of order unity, as emphasized by the sign $\sim$ replacing the equality sign.

At distances larger than L_u, the fluid is described by three hydrodynamic modes. This is a universal regime representing the ultimate infrared fixed point of any nonintegrable system. It is characterized by two ballistic sound modes (index $j = 2, 3$) and one static (i.e., zero-velocity) heat mode ($j = 1$). The pulse propagation in such a regime was analyzed in the context of classical fluids in Refs. [26,27,73]. The sound mode acquires the KPZ shape, Eq. (13). For the corresponding self-coupling constant we find $\lambda \equiv \lambda_{222} \sim T^4/m^3u^{9/2}$. Because of the time-reversal symmetry, the self-coupling of the heat mode is identically zero. Therefore, in the absence of the intermode coupling, the spread of the heat mode would be diffusive. The nonlinear interaction between the heat and sound mode, which is characterized by a coupling $\lambda_{122} \sim T^3/m^2u^{5/2}$, leads to the formation of power-law tails for the heat and sound modes [56]. It transforms the heat mode into symmetric Levy-flight distribution with $\alpha = 5/3$,

$$f_1(x,t) \sim \frac{T}{\delta x(t)} f_{\mathrm{Levy},\alpha=5/3}\left(\frac{x}{\delta x(t)}\right). \tag{15}$$

The heat mode has a maximum at $x = 0$ and the width $\delta x(t) \sim t^{3/5}T^{4/5}m^{-6/5}u^{-7/5}$. The $t^{3/5}$ scaling of the width was also obtained in the context of classical anharmonic chains [74,75]. The value at the maximum is $f_1(0,t) \sim T/\delta x$. Away from the maximum (for $x \gg \delta x$) the heat mode has power law tails [76] that scale as $f_1(x,t) = (T^{7/3}/m^2u^{7/3})tx^{-8/3}$, implying anomalous heat diffusion.

We now consider the linear response properties of the electronic fluid. Generally speaking, an N-component liquid has $N(N-1)/2$ independent linear response coefficients, that can be computed via the Kubo formula [56]. The anomalous scaling observed in the pulse propagation problem manifests itself through the linear response coefficients as well. To be specific, we focus on thermal conductivity, a quantity that describes the rate of irreversible heat propagation. To compute the thermal conductivity, one needs first to define the heat current. In interacting many-body problems, expressions for heat currents are in general rather complicated and spatially nonlocal. Luckily, the operator of heat current J_T for fluids is local and can be computed by subtracting an advective contribution from the energy current J_E [77],

$$J_T = J_E - \bar{w}J_\rho, \tag{16}$$

where $\bar{w}$ is the enthalpy of the fluid per one electron and J_ρ the particle current. The Kubo formula for thermal conductivity reads [78]

$$\sigma_T(\omega,k) = \frac{1}{-i\omega T}[K_{\mathrm{TT}}(\omega,k) - K_{\mathrm{TT}}(0,0)], \tag{17}$$

where $K_{\mathrm{TT}}(\omega,k) = -i\langle[\hat{J}_T(x,t),\hat{J}_T(0,0)]\rangle^{\mathrm{ret}}(\omega,k)$. Employing Eq. (17) and setting $k = 0$, we find [56] that in the six-mode regime

$$\sigma_T(\omega) = -\frac{\pi^2 uT}{3i\omega} + \frac{\pi^2 u}{6m^2l^4T^2}. \tag{18}$$

The Drude peak corresponds to the ballistic propagation of heat [79,80], while the real part of conductivity is due to heat diffusion. As the system enters the four-mode regime, the propagation of all modes remains ballistic. Hence, the imaginary part of the heat conductivity is unchanged, $\mathrm{Im}\sigma_T(\omega) = \pi^2uT/3\omega$. The real part of the heat conductivity, on the other hand, is renormalized. The effects of the renormalization are associated with an anomalous broadening of the pulses, with two contributions coming from the head and the tails of the peak. Both happen to be of the same order and lead to

$$\mathrm{Re}\sigma_T(\omega) \sim uT^{1/3}\omega^{-1/3}. \tag{19}$$

Finally, we discuss the three-mode regime, where the heat conductivity is determined solely by the static mode. Therefore, the ballistic contribution is suppressed, giving rise to an exponentially large constant: $i/\omega \mapsto \tau_U$. The real part of the thermal conductivity thus scales as

$$\mathrm{Re}\sigma_T(\omega) \sim uT\tau_U + \frac{T^{7/3}}{m^2u^3}\omega^{-1/3}. \qquad (20)$$

To assess the experimental relevance of the above-studied physics, we estimate characteristic scales for a single-wall carbon nanotube. For the Coulomb interaction [81] and $T \sim 10$–100 K, the electron-electron scattering length where the hydrodynamic regime starts is $u\tau_F = l_{ee} \sim 10^{-6} - 10^{-7}$ m. The electron-phonon energy relaxation length observed experimentally at room temperature [82,83] is $l_{\text{e-ph}} \simeq 10^{-5}$ m; lowering T down to $T \sim 10$–100 K will lead to its further strong increase, thus allowing for electronic fluid to form. For the gated device with $\epsilon_F = 200$ K, the transition to the four-mode regime should occur at $L_4 \sim 10^{-4} - 10^{-7}$ m.

While the physics studied here is very generic, the model considered above is a minimal one. In a real experiment, the physics may be enriched by additional degrees of freedom. In particular, inclusion of spin will double the number of approximately conserved hydrodynamic charges, with early stages of evolution where spin and charge are decoupled. Similarly, fabrication based on narrow channel devices [84] will yield modes with different transversal quantization.

To summarize, we have developed a multimode hydrodynamic approach for the electronic fluid. Depending on the number of conserved charges, the fluid has six, four, or three hydrodynamic modes. Though the three-mode regime is an ultimate long-distance fixed point, it is only reached at exponentially long distances, leaving room for an exponentially long four-mode viscous hydrodynamic regime.

The interaction between the hydrodynamic modes leads to the renormalization of transport coefficients, giving rise to universal scaling behavior, and shapes of the pulses propagating through the fluid. In the six- and four-mode regimes, all pulses propagate ballistically. The "head" of every pulse is controlled by self-interaction, resulting in a KPZ scaling of pulse width ($t^{2/3}$) and amplitude ($t^{-2/3}$) with time. Interaction between the modes propagating with different velocities results in power-law tails scaling as $x^{-8/3}$ with distance x from the mode center and directed towards another mode. As the system reaches a three-mode regime, the pulses redistribute, and a static heat and two ballistic sound peaks are formed. The width of the ballistic modes has KPZ scaling with time. The interaction between the sound waves and the heat mode gives rise to power-law tails for all peaks. Each sound mode acquires a rear tail. The static heat mode is a Levy flight function with $\alpha = 5/3$, with symmetric tails. The anomaly in peak shapes leads to anomalous kinetic coefficients, in particular, the thermal conductivity.

We conclude by comparing the results of the present analysis for σ_T with earlier calculations performed within the self-consistent kinetic approach [30]. Reassuringly, both approaches yield two regimes of anomalous scaling of σ_T separated by the scale L_U. Further, the kinetic approach yields for these regimes results analogous to Eqs. (20) and (19), with the same $\omega^{-1/3}$ scaling. Such agreement in scaling resulting from self-consistent kinetic [24,29,85] and classical renormalization-group [25] approaches has been known for a long time. This agreement is highly nontrivial and perhaps even puzzling. Indeed, although our starting point here is a transport coefficient computed within fermionic kinetic theory [30,86], the subsequent analysis in this work and Ref. [30] is very different. In the kinetic framework of Ref. [30], the $\omega^{-1/3}$ scaling results from subthermal bosons with wave vectors $k \ll T/u$ that can propagate anomalously large distances without scattering. At the same time, in the present framework, this enhancement of thermal conductivity results from the interaction between bosonic (hydrodynamic) modes leading to anomalous hydrodynamics. Importantly, while the frequency scaling agrees in the two approaches, the prefactors (in particular, the temperature scaling) are essentially different. Effects of renormalization controlling results of the present work turn out to be dominant for σ_T, in both four-mode and three-mode regimes.

D. G. was supported by ISF-China 3119/19 and ISF 1355/20.

Note added.—Recently, we learned about a paper [87] that studies the linear hydrodynamics in the six-mode regime.

[1] L. D. Landau, Two-fluid model of liquid helium II, J. Phys. (USSR) **5**, 71 (1941); Teorya sverkhtekuchesti geliya-II, Sov. Phys. JETP **11**, 592 (1941).
[2] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2* (Elsevier, Oxford, 1980).
[3] *50 Years of Anderson Localization*, edited by E. Abrahams (World Scientific, Singapore, 2010).
[4] L. W. Molenkam and M. J. M. de Jong, Observation of Knudsen and Gurzhi transport regimes in a two dimensional wire, Solid State Electron. **37**, 551 (1994).
[5] M. J. M. deJong and L. W. Molenkamp, Hydrodynamic electron flow in high-mobility wires, Phys. Rev. B **51**, 13389 (1995).
[6] D. A. Bandurin, I. Torre, R. K. Kumar, M. B. Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Negative local resistance caused by viscous electron backow in graphene, Science **351**, 1055 (2016).
[7] P. Moll, P. Kushwaha, N. Nandi, B. Schmidt, and A. P. Mackenzie, Evidence for hydrodynamic electron flow in $PdCoO_2$, Science **351**, 1061 (2016).
[8] J. Gooth, F. Menges, N. Kumar, V. Süß, C. Shekhar, Y. Sun, U. Drechsler, R. Zierold, C. Felser, and B. Gotsmann, Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide, Nat. Commun. **9**, 4093 (2018).

[9] D. A. Bandurin, A. V. Shytov, L. S. Levitov, R. K. Kumar, A. I. Berdyugin, M. Ben Shalom, I. V. Grigorieva, A. K. Geim, and G. Falkovich, Fluidity onset in graphene, Nat. Commun. **9**, 4533 (2018).

[10] R. N. Gurzhi, Hydrodynamic effects in solids at low temperature, Sov. Phys. Usp. **11**, 255 (1968).

[11] L. Levitov and G. Falkovich, Electron viscosity, current vortices and negative nonlocal resistance in graphene, Nat. Phys. **12**, 672 (2016).

[12] A. V. Andreev, S. A. Kivelson, and B. Spivak, Hydrodynamic Description of Transport in Strongly Correlated Electron systems, Phys. Rev. Lett. **106**, 256804 (2011).

[13] P. S. Alekseev, Negative Magnetoresistance in Viscous Flow of Two-Dimensional Electrons, Phys. Rev. Lett. **117**, 166601 (2016).

[14] B. N. Narozhny, I. V. Gornyi, A. D. Mirlin, and J. Schmalian, Hydrodynamic approach to electronic transport in graphene, Ann. Phys. (Amsterdam) **529**, 1700043 (2017).

[15] A. Lucas and K. C. Fong, Hydrodynamics of electrons in graphene, J. Phys. Condens. Matter **30**, 053001 (2018).

[16] S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Theory of the Nernst effect near quantum phase transitions in condensed matter and in dyonic black holes, Phys. Rev. B **76**, 144502 (2007).

[17] K. A. Matveev and A. V. Andreev, Hybrid Sound Modes in One-Dimensional Quantum Liquids, Phys. Rev. Lett. **121**, 026803 (2018).

[18] K. A. Matveev and A. V. Andreev, Two-fluid dynamics of one-dimensional quantum liquids in the absence of Galilean invariance, Phys. Rev. B **100**, 035418 (2019).

[19] B. Bertini, M. Collura, J. De Nardis, and M. Fagotti, Transport in Out-of-Equilibrium XXZ Chains: Exact Profiles of Charges and Currents, Phys. Rev. Lett. **117**, 207201 (2016).

[20] O. A. Castro-Alvaredo, B. Doyon, and T. Yoshimura, Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium, Phys. Rev. X **6**, 041065 (2016).

[21] V. B. Bulchandani, R. Vasseur, C. Karrasch, and J. E. Moore, Bethe-Boltzmann hydrodynamics and spin transport in the XXZ chain, Phys. Rev. B **97**, 045407 (2018).

[22] B. Doyon, Lecture notes on generalised hydrodynamics, SciPost Phys. Lect. Notes **18** (2020).

[23] F. Weiner, P. Schmitteckert, S. Bera, and F. Evers, High-temperature spin dynamics in the Heisenberg chain: Magnon propagation and emerging Kardar-Parisi-Zhang scaling in the zero-magnetization limit, Phys. Rev. B **101**, 045115 (2020).

[24] A. F. Andreev, The hydrodynamics of two and one dimensional liquids, Sov. Phys. JETP **51**, 1038 (1980).

[25] O. Narayan and S. Ramaswamy, Anomalous Heat Conduction in One-Dimensional Momentum-Conserving Systems, Phys. Rev. Lett. **89**, 200601 (2002).

[26] H. Spohn, Nonlinear fluctuating hydrodynamics for anharmonic chains, J. Stat. Phys. **154**, 1191 (2014).

[27] H. Spohn, Fluctuating hydrodynamics approach to equilibrium time correlations for anharmonic chains, edited by S. Lepri, in *Thermal Transport in Low Dimensions: From Statistical Physics to Nanoscale Heat Transfer*, Lecture Notes in Physics (Springer, New York, 2016), p. 107.

[28] M. Kardar, G. Parisi, and Y.-C. Zhang, Dynamic Scaling of Growing Interfaces, Phys. Rev. Lett. **56**, 889 (1986).

[29] M. Arzamasovs, F. Bovo, and D. M. Gangardt, Kinetics of Mobile Impurities and Correlation Functions in One-Dimensional Superfluids at Finite Temperature, Phys. Rev. Lett. **112**, 170602 (2014).

[30] R. Samanta, I. V. Protopopov, A. D. Mirlin, and D. B. Gutman, Thermal Transport in One Dimensional Electronic Fluid, Phys. Rev. Lett. **122**, 206801 (2019).

[31] K. Schwab, E. Henriksen, J. Worlock, and M. Roukes, Measurement of the quantum of thermal conductance, Nature (London) **404**, 974 (2000).

[32] M. Meschke, W. Guichard, and J. P. Pekola, Singlemode heat conduction by photons, Nature (London) **444**, 187 (2006).

[33] S. Jezouin, F. D. Parmentier, A. Anthore, U. Gennser, A. Cavanna, Y. Jin, and F. Pierre, Quantum limit of heat flow across a single electronic channel, Science **342**, 601 (2013).

[34] L. Cui, W. Jeong, S. Hur, M. Matt, J. C. Klöckner, F. Pauly, P. Nielaba, J. C. Cuevas, E. Meyhofer, and P. Reddy, Quantized thermal transport in single-atom junctions, Science **355**, 1192 (2017).

[35] E. Sivre, A. Anthore, F. D. Parmentier, A. Cavanna, U. Gennser, A. Ouerghi, Y. Jin, and F. Pierre, Heat Coulomb blockade of one ballistic channel, Nature (London) **14**, 145 (2018).

[36] C. Altimiras, H. le Sueur, U. Gennser, A. Anthore, A. Cavanna, D. Mailly, and F. Pierre, Energy Relaxation in the Integer Quantum Hall Regime, Phys. Rev. Lett. **109**, 026803 (2012).

[37] V. Venkatachalam, S. Hart, L. Pfeiffer, K. West, and A. Yacoby, Local thermometry of neutral modes on the quantum Hall edge, Nat. Phys. **8**, 676 (2012).

[38] H. Inoue, A. Grivnin, Y. Ronen, M. Heiblum, V. Umansky, and D. Mahalu, Proliferation of neutral modes in fractional quantum Hall states, Nat. Commun. **5**, 4067 (2014).

[39] Y. Cohen, Y. Ronen, W. Yang, D. Banitt, J. Park, M. Heiblum, A. D. Mirlin, Y. Gefen, and V. Umansky, Synthesizing a fractional $\nu = 2/3$ state from particle and hole states, Nat. Commun. **10**, 1920 (2019).

[40] M. Stone, *Bosonization* (World Scientific, Singapore, 1994).

[41] J. von Delft and H. Schoeller, Bosonization for beginners-refermionization for experts, Ann. Phys. (N.Y.) **7**, 225 (1998).

[42] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization in strongly correlated systems* (University Press, Cambridge, 1998).

[43] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press Oxford, Oxford, 2004).

[44] M. Schick, Flux quantization in a one-dimensional model, Phys. Rev. **166**, 404 (1968).

[45] B. Sakita, *Quantum Theory of Many-variable Systems and Fields* (Wolrd Scientific, Singapore, 1985).

[46] A. Jevicki and B. Sakita, The quantum collective field method and its application to the planar limit, Nucl. Phys. **B165**, 511 (1980).

[47] F. D. M Haldane, Luttinger liquid theory of one-dimensional quantum fluids, J. Phys. C **14**, 2585 (1981).

[48] A. Imambekov, T. L. Schmidt, and L. I. Glazman, One-dimensional quantum liquids: Beyond the Luttinger liquid paradigm, Rev. Mod. Phys. **84**, 1253 (2012).

[49] I. V. Protopopov, D. B. Gutman, and A. D. Mirlin, Relaxation in Luttinger liquids: Bose-Fermi duality, Phys. Rev. B **90**, 125113 (2014).

[50] A. V. Rozhkov, Density-density propagator for one-dimensional interacting spinless fermions with nonlinear

dispersion and calculation of the Coulomb drag resistivity, Phys. Rev. B **77**, 125109 (2008); Class of exactly soluble models of one-dimensional spinless fermions and its application to the Tomonaga-Luttinger Hamiltonian with nonlinear dispersion, Phys. Rev. B **74**, 245123 (2006); Fermionic quasiparticle representation of Tomonaga-Luttinger Hamiltonian, Eur. Phys. J. C **47**, 193 (2005).

[51] A. Imambekov and L. I. Glazman, Universal theory of nonlinear luttinger liquids, Science **323**, 228 (2009); Phenomenology of One-Dimensional Quantum Liquids Beyond the Low-Energy Limit, Phys. Rev. Lett. **102**, 126405 (2009).

[52] I. V. Protopopov, D. B. Gutman, M. Oldenburg, and A. D. Mirlin, Dissipationless kinetics of one-dimensional interacting fermions, Phys. Rev. B **89**, 161104(R) (2014).

[53] M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, Fermi-Luttinger liquid: Spectral function of interacting one-dimensional fermions, Phys. Rev. B **76**, 155402 (2007).

[54] A. M. Lunde, K. Flensberg, and L. I. Glazman, Three-particle collisions in quantum wires: Corrections to thermopower and conductance, Phys. Rev. B **75**, 245418 (2007).

[55] The fermionic quasiparticles introduced in this way are advantageous over the original electrons of the model because their interaction vanishes in the low-energy limit. In particular, they have flat density of states at the Fermi surface.

[56] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.126.256801 for the details, which includes Refs. [57–65].

[57] F. D. M. Haldane, Effective Harmonic-Fluid Approach to Low-Energy Properties of One-Dimensional Quantum Fluids, Phys. Rev. Lett. **47**, 1840 (1981).

[58] I. V. Protopopov, D. B. Gutman, P. Schmitteckert, and A. D. Mirlin, Dynamics of waves in 1D electron systems: Density oscillations driven by population inversion, Phys. Rev. B **87**, 045112 (2013).

[59] E. M. Lifshits and L. P. Pitaevskii, *Physical Kinetics: Volume 10* (Pergamon Press, Oxford, 1989).

[60] A. Kamenev, *Field Theory of Non Equilibrium Systems* (Cambridge University Press, Cambridge, England, 2011).

[61] J. Rammer and H. Smith, Quantum field-theoretical methods in transport theory of metals, Rev. Mod. Phys. **58**, 323 (1986).

[62] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Elsevier, Oxford, 1990).

[63] J. Lin, K. A. Matveev, and M. Pustilnik, Thermalization of Acoustic Excitations in a Strongly Interacting One-Dimensional Quantum Liquid, Phys. Rev. Lett. **110**, 016401 (2013).

[64] L. Kadanoff and P. Martin, Hydrodynamic equations and correlation functions, Ann. Phys. (N.Y.) **24**, 419 (1963).

[65] P. Kovtun, Lectures on hydrodynamic fluctuations in relativistic theories, J. Phys. A **45**, 473001 (2012).

[66] K. A. Matveev and A. V. Andreev, Equilibration of a spinless Luttinger liquid, Phys. Rev. B **85**, 041102(R) (2012).

[67] T. Micklitz, J. Rech, and K. A. Matveev, Transport properties of partially equilibrated quantum wires, Phys. Rev B **81**, 115313 (2010).

[68] K. A. Matveev and A. V. Andreev, Scattering of hole excitations in a one-dimensional spinless quantum liquid, Phys. Rev B **86**, 045136 (2012).

[69] While it is difficult to give a rigorous justification to the mode-coupling treatment of the 1D fluctuation hydrodynamics, this is known to produce the results that are in excellent agreement with numerical simulations and exact solution in the case of the paradigmatic KPZ theory.

[70] H. van Beijeren, Exact Results for Anomalous Transport in One-Dimensional Hamiltonian Systems, Phys. Rev. Lett. **108**, 180601 (2012).

[71] M. Prahofer, Exact scaling functions for one-dimensional stationary KPZ growth. http://www-m5.ma.tum.de/KPZ.

[72] M. Prahofer and H. Spohn, Exact scaling functions for one-dimensional stationary KPZ growth, J. Stat. Phys. **115**, 255 (2004).

[73] T. Mai, A. Dhar, and O. Narayan, Equilibration and Universal Heat Conduction in Fermi-Pasta-Ulam Chains, Phys. Rev. Lett. **98**, 184301 (2007).

[74] A. Pereverzev, Fermi-Pasta-Ulam β lattice: Peierls equation and anomalous heat conductivity, Phys. Rev. E **68**, 056124 (2003).

[75] H. Spohn and J. Lukkarinen, Anomalous energy transport in the FPU-β chain, Commun. Pure Appl. Math. **61**, 1753 (2008).

[76] V. V. Uchaikin and V. M. Zolotarev, *Chance and Stability, Stable Distributions and their Applications* (W. de Gruyter, Berlin, 1999).

[77] J. M. Luttinger, Theory of thermal transport coefficients, Phys. Rev. **135**, A150 (1964).

[78] C. Castellani, C. Di Castro, G. Kotliar, P. A. Lee, and G. Strinati, Thermal Conductivity in Disordered Interacting-Electron Systems, Phys. Rev. Lett. **59**, 477 (1987).

[79] C. L. Kane and M. P. A. Fisher, Thermal Transport in a Luttinger Liquid, Phys. Rev. Lett. **76**, 3192 (1996).

[80] A. Levchenko, T. Micklitz, J. Rech, and K. A. Matveev, Transport in partially equilibrated inhomogeneous quantum wires, Phys. Rev. B **82**, 115413 (2010).

[81] T. Karzig, L. I. Glazman, and F. von Oppen, Energy Relaxation and Thermalization of Hot Electrons in Quantum Wires, Phys. Rev. Lett. **105**, 226407 (2010).

[82] T. Hertel and G. Moos, Electron-Phonon Interaction in Single-Wall Carbon Nanotube: A Time-Domain Study, Phys. Rev. Lett. **84**, 5002 (2000).

[83] S. Reich, M. Dvorzhak, A. Hoffmann, C. Thomsen, and M. Strano, Excited-state carrier life time in single-wall carbon nanotube, Phys. Rev. B **71**, 033402 (2005).

[84] J. A. Sulpizio, L. Ella, A. Rozen, J. Birkbeck, D. J. Perello, D. Dutta, M. Ben-Shalom, T. Taniguchi, K. Watanabe, T. Holder, R. Queiroz, A. Principi, A. Stern, T.Scaffidi, A. K. Geim, and Sh. Ilani, Visualizing Poiseuille flow of hydrodynamic electrons, Nature (London) **576**, 75 (2019).

[85] K. Samokhin, Lifetime of excitations in a clean Luttinger liquid, J. Phys. Condens. Matter **10**, L533 (1998).

[86] K. A. Matveev and Z. Ristivojevic, Thermal conductivity of the degenerate one-dimensional Fermi gas, Phys. Rev. B **99**, 155428 (2019).

[87] K. A. Matveev, Sound in a system of chiral one-dimensional fermions, Phys. Rev. B **102**, 155401 (2020).