Quantum Noise in Fibers with Arbitrary Nonlinear Profiles

J. Bonetti * and D. F. Grosz
Depto. de Ingeniería en Telecomunicaciones, Centro Atómico Bariloche, Comisión Nacional de Energía Atómica, Río Negro 8400, Argentina and Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), San Carlos de Bariloche 8400, Argentina

S. M. Hernandez
Instituto Balseiro, Universidad Nacional de Cuyo, Bariloche, Río Negro 8400, Argentina

(Received 16 March 2021; accepted 29 April 2021; published 26 May 2021)

In this Letter we introduce a novel equation addressing the effect of quantum noise in optical fibers with arbitrary frequency-dependent nonlinear profiles. To the best of our knowledge, such an endeavor has not been undertaken before despite the growing relevance of fiber optics in the design of new quantum devices. We show that the stochastic generalized nonlinear Schrödinger equation, derived from a quantum theory of optical fibers, leads to unphysical results such as a negative photon number and the appearance of a dominant anti-Stokes sideband when applied to this kind of waveguides. Starting from a recently introduced master-equation approach to propagation in fibers, we derive a novel stochastic photon-conserving nonlinear Schrödinger equation suitable for modeling arbitrary nonlinear profiles, thus greatly enhancing the study of fiber-based quantum devices.

DOI: 10.1103/PhysRevLett.126.213602

Nonlinear waveguides are becoming a fundamental tool in the design of novel quantum-technology devices. A clear example of this is found in the interface with quantum memories [1], performed by the frequency conversion of single photons in nanowires [2–4], as also in the generation of entangled photon pairs in nonlinear fibers for quantum key distribution systems [5–8]. Further examples can be found in several kinds of nonclassical states of light attainable by means of nonlinear fibers, such as squeezed and entangled light [9–11], or in the quantum-state engineering with an array of nonlinear waveguides recently proposed in Ref. [12]. These applications have triggered a remarkable interest in highly nonlinear waveguides (HNLWs) manufactured from emerging materials such as graphene [13–15], nanoparticle-doped glasses [16], silicon photonic crystals [17], calchogenide glasses [18,19], and several metamaterials [20], allowing for an enhanced and tailored nonlinear coefficient. All these incipient technologies are already paving the way towards commercial integrated quantum devices [21,22].

Interestingly, the large nonlinearity in HNLWs is commonly associated with a strong frequency dependence of the nonlinear profile [16,23,24], leading to very unusual and intriguing phenomena in nonlinear optics such as the existence of solitons and modulation instability in the normal dispersion regime, or to a controllable self-steepening parameter [25,26], among others. Additionally, HNLWs with a frequency-dependent nonlinearity are also relevant in the area of solitons in fiber lasers [27,28]. However, as we have recently shown, this frequency dependence may lead to unphysical results from the well-established equations used to model propagation in fibers [29]. In particular, the generalized nonlinear Schrödinger equation (GNLSE) was shown to produce unphysical results in fibers with arbitrary nonlinear profiles (i.e., whenever the fiber nonlinear coefficient is an arbitrary real function of frequency), such as the nonconservation of the mean photon number or the pulse energy even in lossless media. Based on this observation, we derived a new propagation equation suitable for modeling fibers with a frequency-dependent nonlinearity. The photon-conserving generalized nonlinear Schrödinger equation (pcGNLSE) [30], which can be solved with the same efficient algorithms used for the GNLSE, produces consistent results even for arbitrary nonlinear profiles, allowing for the analysis of new kinds of waveguides, such as silver-nanoparticle-doped fibers, silicon nanowires, and graphene-decorated waveguides. Note, however, that neither the GNLSE nor the pcGNLSE are suitable for the analysis of quantum devices as they deal with classical fields. Quantum effects in fibers are usually modeled by the stochastic GNLSE (stoGNLSE), derived from a quantum theory of fibers, in which the different sources of quantum noise are introduced by means of stochastic processes [31]. Therefore, the simulation of HNLWs in quantum devices should be performed by resorting to the stoGNLSE rather than the classical GNLSE. However, given the proven inadequacy of the GNLSE for modeling a frequency-dependent nonlinearity, it is only natural to wonder whether the stoGNLSE may also fail when applied to such waveguides. The main goal of this work is to derive a new stochastic equation, with a computational complexity similar to that of the stoGNLSE, but capable to tackle propagation in HNLWs with an arbitrary frequency-dependent nonlinearity.

We revisit the analysis of photon-pair generation in nonlinear waveguides, in order to highlight a shortcoming of the stoGNLSE in this scenario. We emphasize that only the positive P representation of the stoGNLSE [stoGNLSE$(+P)$] produces physical results in highly nonclassical experiments. In addition, we show that the stoGNLSE$(+P)$ is no longer adequate if an arbitrary frequency-dependent nonlinearity is considered, as stated in our main hypothesis. Starting from the recently introduced master-equation approach to propagation in nonlinear fibers [32], we derive a novel stochastic equation suitable for such nonlinear profiles. This equation is reminiscent of a stochastic version of the previously developed pcGNLSE, resembling a photon-conserving analogue of the stoGNLSE. We compare simulation results from both the new equation and the stoGNLSE$(+P)$ in order to show that a physically consistent modeling of HNLWs does require the use of the novel stochastic equation here derived.

Photon-pair generation via a spontaneous four-wave mixing (FWM) process is perhaps the most easily observable quantum phenomenon occurring in nonlinear waveguides. The basic setup consists of a high-intensity continuous-wave (CW) laser, the *pump*, launched into a nonlinear fiber. As the wave propagates into the fiber, two photons are created by spontaneous FWM at both sides of the pump following the simultaneous annihilation of two photons from the CW pump. A quantum picture of this parametric process can be expressed as $2\hbar\omega_p \to \hbar\omega_s + \hbar\omega_i$, where ω_p is the pump frequency, and ω_s and ω_i are the *signal* and *idler* frequencies, respectively, satisfying $\omega_s + \omega_i = 2\omega_p$. As a consequence of this process, the spectrum at the output end of the fiber is no longer a simple CW, but a CW with correlated noisy sidebands located at frequencies for which the process is phase matched. Note that this simple experiment cannot be reproduced by simulation with the GNLSE, as a CW input produces a steady-state CW solution but no sidebands due to the generated photon pairs.

A first approach to spontaneous FWM in fibers can be performed by resorting to the stoGNLSE [33], a truncated Wigner representation of the quantum theory of nonlinear fibers that reads [31]

$$\partial_z A_t = \left(i\hat{\beta} - \frac{\hat{\alpha}}{2}\right)A_t + i\hat{\gamma}\left[\int_0^\infty R(\tau)|A_{t-\tau}|^2 d\tau + \sigma_t\right]A_t, \tag{1}$$

where z is the propagation axis, A is the normalized complex envelope of the electric field, $|A|^2$ is the optical power, $\hat{\alpha}$ and $\hat{\beta}$ are linear operators defined by the eigenvalue-equations $\hat{\alpha}e^{-i\omega t} = \alpha_\omega e^{-i\omega t}$, $\hat{\beta}e^{-i\omega t} = \beta_\omega e^{-i\omega t}$, α_ω, and β_ω are the frequency profiles of loss and dispersion, respectively; $\hat{\gamma} = \gamma_0(1 + i/\omega_0 \partial_t)$ is the nonlinear operator, γ_0 is the waveguide nonlinear coefficient, ω_0 is the envelope central frequency, and $R(t)$ is the nonlinear response function including both the instantaneous (electronic) and the delayed Raman response. Quantum noise is included by the stochastic process σ_t, whose frequency-domain correlations are $\langle \tilde{\sigma}^*_\mu \tilde{\sigma}_{\mu'}\rangle = -\hbar\omega_0 g_{-\mu}\delta_{\mu\mu'}/T\gamma_0$, where T is the envelope period, δ is the Dirac delta, $g_\mu = 2\mathrm{Im}[\tilde{R}_{|\mu|}](H(\mu) + n_\mu)$, $\tilde{R}_\omega = \int_{-\infty}^{\infty} R(t)e^{i\omega t}dt$, $H(\mu)$ is the Heaviside step function, and $n_\mu = [\exp(\hbar|\mu|/k_B T_w) - 1]^{-1}$; k_B is the Boltzmann constant and T_w the temperature of the waveguide. It must be noted that Eq. (1) also includes effects that are highly detrimental to the generation of photon pairs, such as fiber loss and Raman scattering. The initial condition for the simulation of photon-pair generation is $A_t(0) = \sqrt{P_0} + \eta_t$, where P_0 is the pump power and η_t is the stochastic process representing the phenomenological inclusion of quantum-limited shot noise [33] with correlation $\langle \tilde{\eta}_\mu \tilde{\eta}^*_{\mu'}\rangle = \hbar(\omega_0 + \mu)\delta_{\mu\mu'}/2T$. However, we must emphasize that this usual approach is not adequate for the analysis of nonclassical experiments, as in the case of spontaneous FWM. As a simple example, Fig. 1 displays results of a simulation for a particular case in which the stochastic equation fails (simulation details can be found in the Supplemental Material [34]). In particular, it predicts an unphysical negative mean photon number in a certain frequency range. Opposite to the stoGNLSE, the stochastic

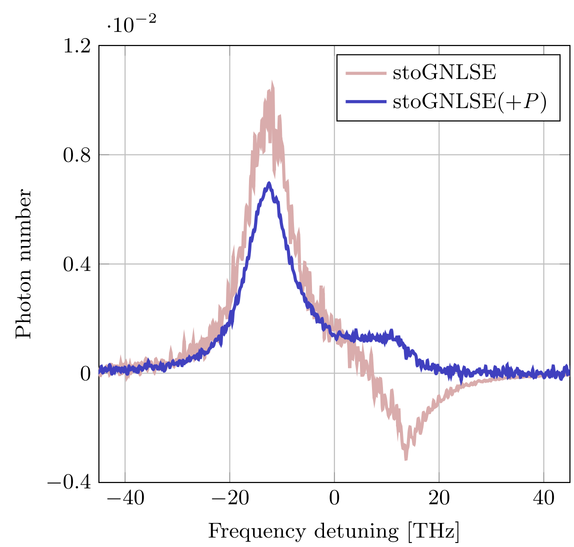

FIG. 1. Numerical results of the stoGNLSE and its positive P representation for a 1-cm standard single-mode fiber pumped by a high-power CW laser at 1550 nm. Curves were obtained by averaging 10 000 noise realizations (the pump laser is not shown for the sake of clarity.) The $+P$ representation does not display an unphysical negative photon number as predicted by the stoGNLSE.

equation derived as the evolution of the mean value of field operators from the master equation presented in Ref. [32], which is the same as Eq. (1) but with frequency-domain correlations given by

$$\begin{cases} \langle \tilde{\sigma}_\mu \tilde{\sigma}_{-\mu'} \rangle = -i \frac{\hbar\omega_0}{T\gamma_0} (\tilde{R}_\mu + i g_{-\mu}) \delta_{\mu\mu'} \\ \langle \tilde{\sigma}^*_\mu \tilde{\sigma}_{\mu'} \rangle = -\frac{\hbar\omega_0}{T\gamma_0} g_{-\mu} \delta_{\mu\mu'}, \end{cases} \tag{2}$$

produces a physically sound result. Moreover, the initial conditions for this equation do not require the *ad hoc* addition of shot noise to the pump. It must be noted that the stochastic terms given by Eq. (2) agree with the positive P representation of the quantum theory of fibers put forth by P. D. Drummond and J. F. Corney [31], where the difference between the two representations is pointed out. Here we simply remark about the overlooked fact that whenever quantum noise plays a significant role, as in the case of quantum devices, the usual form of the stoGNLSE is rendered inadequate and the positive P representation must be used.

As we have recently shown in Refs. [29,30], the GNLSE [i.e., Eq. (1) without the stochastic term] may produce unphysical results when applied to fibers with a frequency-dependent nonlinear profile. This fact suggests that, likewise, the stoGNLSE may not be suitable for modeling such waveguides, as it will be demonstrated. Indeed, in Ref. [31] the fiber nonlinear coefficient was assumed constant. In order to find an adequate stochastic equation for the case of an arbitrary nonlinearity, we start from the quantum theory of nonlinear fibers recently introduced in Ref. [32], where the evolution of the reduced density matrix ρ of the complex envelope is proposed to follow the master equation

$$\partial_z \rho = i\frac{T}{\hbar}[\hat{P}, \rho] + \frac{T}{\hbar}\sum_{m=1}^{2}\sum_{\mu} \hat{L}_\mu^{(m)} \rho \hat{L}_\mu^{\dagger(m)} - \frac{1}{2}\{\rho, \hat{L}_\mu^{\dagger(m)} \hat{L}_\mu^{(m)}\}, \tag{3}$$

where $\hat{P}$ represents processes of dispersion and four-wave mixing [35], and $\hat{L}_\mu^{(1)}$ and $\hat{L}_\mu^{(2)}$ represent the waveguide linear loss and the Raman scattering process [36], respectively. Unlike in Ref. [32] these operators are expressed in the more convenient form [30],

$$\begin{aligned} \hat{P} = &\sum_\omega \frac{\beta_\omega \hat{A}^\dagger_\omega \hat{A}_\omega}{\omega_0 + \omega} \\ &+ \frac{1}{4}\sum_{\omega_1,\omega_2,\mu} (\hat{C}^\dagger_{\omega_1} \hat{C}^\dagger_{\omega_2} \hat{B}_{\omega_1-\mu} \hat{B}_{\omega_2+\mu} + \hat{B}^\dagger_{\omega_1} \hat{B}^\dagger_{\omega_2} \hat{C}_{\omega_1-\mu} \hat{C}_{\omega_2+\mu}) \\ &+ \frac{1}{2}\sum_{\omega_1,\omega_2,\mu} (\mathrm{Re}[\tilde{R}_\mu] - 1) \hat{B}^\dagger_{\omega_1} \hat{B}^\dagger_{\omega_2} \hat{B}_{\omega_1-\mu} \hat{B}_{\omega_2+\mu}, \end{aligned} \tag{4}$$

$$\hat{L}_\mu^{(1)} = \sqrt{\frac{\alpha_\mu}{\omega_0 + \mu}} \hat{A}_\mu, \tag{5}$$

and

$$\hat{L}_\mu^{(2)} = \sqrt{g_\mu} \sum_\omega \hat{B}^\dagger_{\omega-\mu} \hat{B}_\omega, \tag{6}$$

where we introduced the nonlinearity-dependent field operators $\hat{B}_\omega = r_\omega \hat{A}_\omega$ and $\hat{C}_\omega = r^*_\omega \hat{A}_\omega$, being $r_\omega = \sqrt[4]{\gamma_\omega/(\omega_0 + \omega)}$. It is easy to prove that these operators are equivalent to those proposed in Ref. [32] when a conventional nonlinear profile, $\gamma_\omega = \gamma_0 \omega$ with $\gamma_0 > 0$, is assumed. However, this formulation allows for a straightforward inclusion of arbitrary nonlinear profiles, while preserving the correct modeling of the various quantum processes in terms of the creation and the annihilation of photons. As explained in Refs. [27,28], the operators in Eqs. (4) and (6) allow for a consistent quantum modeling of a frequency-dependent Kerr coefficient by means of a straightforward generalization of Miller's rule. In a similar fashion to that in Ref [32], it can be shown that the evolution of any normally ordered quantum operator can be calculated as

$$\langle \hat{A}^\dagger_{\omega_1} \hat{A}^\dagger_{\omega_2} \ldots \hat{A}_{\omega_3} \hat{A}_{\omega_4} \ldots \rangle = \langle A^*_{\omega_1} A^*_{\omega_2} \ldots A_{\omega_3} A_{\omega_4} \ldots \rangle, \tag{7}$$

where A_ω are c numbers whose evolution follows the stochastic equation

$$\begin{aligned} \partial_z A_\omega = &\left(i\beta_\omega - \frac{\alpha_\omega}{2}\right) A_\omega \\ &+ i\frac{\tilde{\gamma}_\omega}{2} \mathcal{F}[(C^*_t B_t + \sigma_t^{(1)}) B_t] + i\frac{\tilde{\gamma}^*_\omega}{2} \mathcal{F}[(B^*_t C_t + \sigma_t^{(2)}) C_t] \\ &+ i\tilde{\gamma}^*_\omega \mathcal{F}\left[\left(\int_0^\infty [R(\tau) - \delta(\tau)] |B_{t-\tau}|^2 d\tau + \sigma_t^{(3)}\right) B_t\right], \end{aligned} \tag{8}$$

where $\tilde{\gamma}_\omega = (\omega_0 + \omega) r_\omega$, B_t and A_t are the time-domain version of the fields $B_\omega = r_\omega A_\omega$ and $C_\omega = r^*_\omega A_\omega$, respectively, and $\sigma_t^{(i)}$ are stochastic processes with frequency-domain correlations given by

$$\begin{cases} \langle \tilde{\sigma}^{(1)}_\mu \tilde{\sigma}^{(1)}_{-\mu'} \rangle = \langle \tilde{\sigma}^{(2)}_\mu \tilde{\sigma}^{(2)}_{-\mu'} \rangle = -2i \frac{\hbar}{T} \delta_{\mu\mu'} \\ \langle \tilde{\sigma}^{(3)}_\mu \tilde{\sigma}^{(3)}_{-\mu'} \rangle = -i \frac{\hbar}{T} (\tilde{R}_\mu - 1 + i g_{-\mu}) \delta_{\mu\mu'} \\ \langle \tilde{\sigma}^{(3)*}_\mu \tilde{\sigma}^{(3)}_{\mu'} \rangle = -\frac{\hbar}{T} g_{-\mu} \delta_{\mu\mu'}. \end{cases} \tag{9}$$

We refer the interested reader to the Supplemental Material for details of these calculations. While $\sigma_t^{(1)}$ and $\sigma_t^{(2)}$ model spontaneous FWM in the context of a frequency-dependent Kerr coefficient, $\sigma_t^{(3)}$ introduces a consistent model of Raman scattering for arbitrary nonlinear profiles. If stochastic processes were not considered, Eq. (8) is equivalent to the pcGNLSE proposed in Ref. [30], an equation that allows for the correct modeling of classical-light propagation in nonlinear fibers with an arbitrary frequency-dependent nonlinearity. As a consequence, the strict conservation of the photon number is ensured by Eq. (8), referred to as the stochastic photon-conserving generalized nonlinear Schrödinger equation (sto-pcGNLSE). It is easy to prove that for the particular case of a conventional nonlinear profile, $\gamma_\omega = \gamma_0 \omega$, with $\gamma_0 > 0$, the sto-pcGNLSE reduces to the stoGNLSE($+P$) [Eq. (1)]. In addition, Eq. (8) can be solved by means of the same efficient numerical algorithms used for solving the stoGNLSE($+P$) and allows for the consistent modeling of arbitrary higher-order nonlinear terms in novel and relevant schemes [37].

Figure 2 portrays a comparison between numerical results obtained with the stoGNLSE($+P$) and with the sto-pcGNLSE, and for a waveguide with the nonlinear profile shown in the top panel (see the Supplemental Material for technical details about the simulation and waveguide parameters). This particular nonlinear profile combines negative and positive values displaying a zero-nonlinearity wavelength, a feature commonly observed in silver-nanoparticle-doped photonic crystal fibers and other metamaterials [16]. As it can be observed in Fig. 2, the standard stoGNLSE($+P$) produces an unphysical outcome (i.e., a negative photon number) despite considering the positive P representation. On the other hand, results obtained with the proposed sto-pcGNLSE are consistently positive all across the spectrum, resembling the comparison shown in Fig. 1. This fact suggests that, in the same way that the positive P representation is crucial for a correct modeling of highly nonclassical experiments, the sto-pcGNLSE is instrumental for a suitable simulation of waveguides possessing arbitrary frequency-dependent nonlinear profiles. In addition, we compare these two equations by way of a relevant example: a HNLW with a negative nonlinear coefficient, a feature found, for instance, in graphene-decorated nanowires [15]. Figure 3 shows the output spectra obtained by numerically solving Eqs. (1) and (8) (see the Supplemental Material for details.) In this case, and as suggested by the spectrum asymmetry with respect to the pump frequency, most of the energy in the spectral sidebands is produced by Raman scattering instead by four-wave mixing. Even though this scenario is not a suitable setup for the generation of photon pairs, it clearly illustrates another problematic aspect of applying the stoGNLSE($+P$) to model arbitrary nonlinear profiles, as it predicts an unphysical prevalence of the anti-Stokes over the Stokes sideband. The novel sto-pcGNLSE, however, predicts a reasonably higher Stokes sideband even in the presence of a negative fiber nonlinear coefficient.

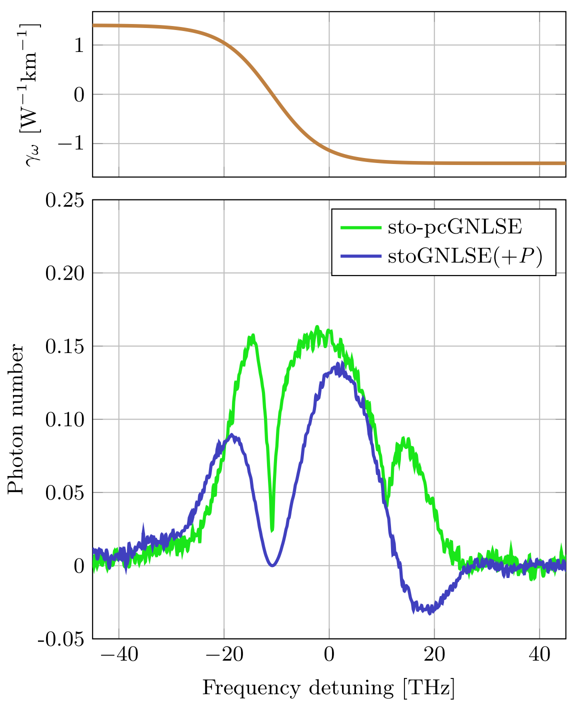

FIG. 2. (top panel) Frequency-dependent nonlinear profile. (bottom panel) Numerical results of the positive P stoGNLSE and the proposed sto-pcGNLSE. Curves correspond to the average of 10 000 noise realizations (the CW is not shown for the sake of clarity.) The sto-pcGNLSE avoids the unphysical negative photon number as predicted by the stoGNLSE($+P$).

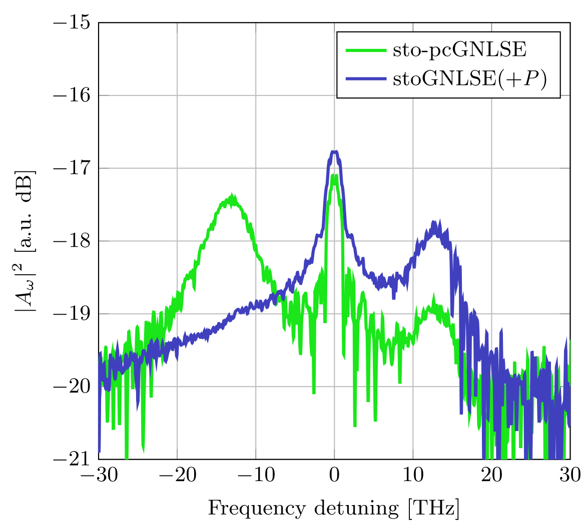

FIG. 3. Numerical results of the positive P stoGNLSE and the sto-pcGNLSE for a HNLW with a negative nonlinear coefficient. Curves were obtained as an average of 1000 noise realizations. Unlike the sto-pcGNLSE, the stoGNLSE($+P$) predicts an unphysical growth of the anti-Stokes sideband.

In conclusion, we investigated predictions of well-established stochastic propagation equations when applied to waveguides with arbitrary frequency-dependent nonlinear profiles. Numerical simulation of relevant cases strongly supports our main hypothesis: The stoGNLSE$(+P)$ is not adequate to correctly modeling this kind of waveguides as it may produce unphysical results. Consequently, we derived a novel propagation equation, the sto-pcGNLSE, starting from a master-equation approach to nonlinear optical fibers. This original proposal, whose numerical simulation does not involve a higher degree of computational complexity, was shown to reduce to the usual stoGNLSE when a standard nonlinearity is considered, and to reduce to the recently introduced pcGNLSE when the effect of quantum noise is neglected, ensuring strict conservation of the photon number. In addition, the sto-pcGNLSE produces physically sound results in cases where the stoGNLSE$(+P)$ fails. Finally, we believe to have put forth a powerful tool for the modeling and design of novel quantum devices based on highly nonlinear waveguides.

[*]juan.bonetti@ib.edu.ar

[1] N. Maring, P. Farrera, K. Kutluer, M. Mazzera, G. Heinze, and H. de Riedmatten, Photonic quantum state transfer between a cold atomic gas and a crystal, Nature (London) **551**, 485 (2017).

[2] H. J. McGuinness, M. G. Raymer, C. J. McKinstrie, and S. Radic, Quantum Frequency Translation of Single-Photon States in a Photonic Crystal Fiber, Phys. Rev. Lett. **105**, 093604 (2010).

[3] Q. Li, M. Davanço, and K. Srinivasan, Efficient and low-noise single-photon-level frequency conversion interfaces using silicon nanophotonics, Nat. Photonics **10**, 406 (2016).

[4] C. Joshi, A. Farsi, S. Clemmen, S. Ramelow, and A. L. Gaeta, Frequency multiplexing for quasi-deterministic heralded single-photon sources, Nat. Commun. **9**, 1 (2018).

[5] A. A. Shukhin, J. Keloth, K. Hakuta, and A. A. Kalachev, Heralded single-photon and correlated-photon-pair generation via spontaneous four-wave mixing in tapered optical fibers, Phys. Rev. A **101**, 053822 (2020).

[6] B. J. Smith, P. Mahou, O. Cohen, J. S. Lundeen, and I. A. Walmsley, Photon pair generation in birefringent optical fibers, Opt. Express **17**, 23589 (2009).

[7] O. Cohen, J. S. Lundeen, B. J. Smith, G. Puentes, P. J. Mosley, and I. A. Walmsley, Tailored Photon-Pair Generation in Optical Fibers, Phys. Rev. Lett. **102**, 123603 (2009).

[8] R. J. Francis-Jones, R. A. Hoggarth, and P. J. Mosley, All-fiber multiplexed source of high-purity single photons, Optica **3**, 1270 (2016).

[9] M. Stefszky, R. Ricken, C. Eigner, V. Quiring, H. Herrmann, and C. Silberhorn, Waveguide Cavity Resonator as a Source of Optical Squeezing, Phys. Rev. Applied **7**, 044026 (2017).

[10] F. Kaiser, B. Fedrici, A. Zavatta, V. d'Auria, and S. Tanzilli, A fully guided-wave squeezing experiment for fiber quantum networks, Optica **3**, 362 (2016).

[11] K. Thyagarajan, J. Lugani, S. Ghosh, K. Sinha, A. Martin, D. B. Ostrowsky, O. Alibart, and S. Tanzilli, Generation of polarization-entangled photons using type-II doubly periodically poled lithium niobate waveguides, Phys. Rev. A **80**, 052321 (2009).

[12] D. Barral, M. Walschaers, K. Bencheikh, V. Parigi, J. A. Levenson, N. Treps, and N. Belabas, Quantum state engineering in arrays of nonlinear waveguides, Phys. Rev. A **102**, 043706 (2020).

[13] A. Ishizawa, R. Kou, T. Goto, T. Tsuchizawa, N. Matsuda, K. Hitachi, T. Nishikawa, K. Yamada, T. Sogawa, and H. Gotoh, Optical nonlinearity enhancement with graphene-decorated silicon waveguides, Sci. Rep. **7**, 45520 (2017).

[14] H. Zhang, S. Virally, Q. Bao, L. K. Ping, S. Massar, N. Godbout, and P. Kockaert, Z-scan measurement of the nonlinear refractive index of graphene, Opt. Lett. **37**, 1856 (2012).

[15] N. Vermeulen, D. Castelló-Lurbe, M. Khoder, I. Pasternak, A. Krajewska, T. Ciuk, W. Strupinski, J. Cheng, H. Thienpont, and J. Van Erps, Graphene's nonlinear-optical physics revealed through exponentially growing self-phase modulation, Nat. Commun. **9**, 2675 (2018).

[16] S. Bose, R. Chattopadhyay, S. Roy, and S. K. Bhadra, Study of nonlinear dynamics in silver-nanoparticle-doped photonic crystal fiber, J. Opt. Soc. Am. B **33**, 1014 (2016).

[17] A. Blanco-Redondo, C. Husko, D. Eades, Y. Zhang, J. Li, T. F. Krauss, and B. J. Eggleton, Observation of soliton compression in silicon photonic crystals, Nat. Commun. **5**, 3160 (2014).

[18] J. Fatome, C. Fortier, T. N. Nguyen, T. Chartier, F. Smektala, K. Messaad, B. Kibler, S. Pitois, G. Gadret, C. Finot *et al.*, Linear and nonlinear characterizations of chalcogenide photonic crystal fibers, J. Lightwave Technol. **27**, 1707 (2009).

[19] E. C. Mägi, L. B. Fu, H. C. Nguyen, M. R. E. Lamont, D. I. Yeom, and B. J. Eggleton, Enhanced kerr nonlinearity in sub-wavelength diameter as2se3 chalcogenide fiber tapers, Opt. Express **15**, 10324 (2007).

[20] H. Zhu, X. Yin, L. Chen, Z. Zhu, and X. Li, Manipulating light polarizations with a hyperbolic metamaterial waveguide, Opt. Lett. **40**, 4595 (2015).

[21] J. Leuthold, C. Koos, and W. Freude, Nonlinear silicon photonics, Nat. Photonics **4**, 535 (2010).

[22] J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, Integrated photonic quantum technologies, Nat. Photonics **14**, 273 (2020).

[23] N. C. Panoiu, X. Liu, J. R. Osgood, and M. Richard, Self-steepening of ultrashort pulses in silicon photonic nanowires, Opt. Lett. **34**, 947 (2009).

[24] P. Ferrari, S. Upadhyay, M. V. Shestakov, J. Vanbuel, B. De Roo, Y. Kuang, M. Di Vece, V. V. Moshchalkov, J. P. Locquet, P. Lievens *et al.*, Wavelength-dependent nonlinear optical properties of Ag nanoparticles dispersed in a glass host, J. Phys. Chem. C **121**, 27580 (2017).

[25] C. Husko and P. Colman, Giant anomalous self-steepening in photonic crystal waveguides, Phys. Rev. A **92**, 013816 (2015).

[26] F. R. Arteaga-Sierra, A. Antikainen, and G. P. Agrawal, Soliton dynamics in photonic-crystal fibers with frequency-dependent Kerr nonlinearity, Phys. Rev. A **98**, 013830 (2018).

[27] H. Zhang, D. Y. Tang, L. M. Zhao, Q. Bao, and K. P. Loh, Vector dissipative solitons in graphene mode locked fiber lasers, Opt. Commun. **283**, 3334 (2010).

[28] Y. Song, X. Shi, C. Wu, D. Y. Tang, and H. Zhang, Recent progress of study on optical solitons in fiber lasers, Appl. Phys. Rev. **6**, 021313 (2019).

[29] J. Bonetti, N. Linale, A. Sánchez, S. Hernandez, P. Fierens, and D. Grosz, Modified nonlinear schrödinger equation for frequency-dependent nonlinear profiles of arbitrary sign, J. Opt. Soc. Am. B **36**, 3139 (2019).

[30] J. Bonetti, N. Linale, A. Sánchez, S. Hernandez, P. Fierens, and D. Grosz, Photon-conserving generalized nonlinear Schrödinger equation for frequency-dependent nonlinearities, J. Opt. Soc. Am. B **37**, 445 (2020).

[31] P. D. Drummond and J. F. Corney, Quantum noise in optical fibers. I. Stochastic equations, J. Opt. Soc. Am. B **18**, 139 (2001).

[32] J. Bonetti, S. M. Hernandez, and D. F. Grosz, Master equation approach to propagation in nonlinear fibers, Opt. Lett. **46**, 665 (2021).

[33] K. L. Corwin, N. R. Newbury, J. M. Dudley, S. Coen, S. A. Diddams, K. Weber, and R. S. Windeler, Fundamental Noise Limitations to Supercontinuum Generation in Microstructure Fiber, Phys. Rev. Lett. **90**, 113904 (2003).

[34] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.126.213602 for full derivation of the sto-pcGNLSE and technical details of its numerical simulation.

[35] Y. Lai and H. A. Haus, Quantum theory of solitons in optical fibers. I. Time-dependent Hartree approximation, Phys. Rev. A **40**, 844 (1989).

[36] G. P. Agrawal, *Nonlinear Fiber Optics*, 6th ed. (Academic Press, New York, 2019).

[37] D. Y. Tang, H. Zhang, L. M. Zhao, and X. Wu, Observation of High-Order Polarization-Locked Vector Solitons in a Fiber Laser, Phys. Rev. Lett. **101**, 153904 (2008).