Combined Test of the Gravitational Inverse-Square Law at the Centimeter Range

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(Received 6 January 2021; accepted 4 May 2021; published 26 May 2021)

Experiments measuring the Newtonian gravitational constant G can offer uniquely sensitive probes of the test of the gravitational inverse-square law. An analysis of the non-Newtonian effect in two independent experiments measuring G is presented, which permits a test of the $1/r^2$ law at the centimeter range. This work establishes the strongest bound on the magnitude α of Yukawa-type deviations from Newtonian gravity in the range of 5–500 mm and improves the previous bounds by up to a factor of 7 at the length range of 60–100 mm.

DOI: 10.1103/PhysRevLett.126.211101

General relativity and the standard model provide an impressive description of four fundamental forces in nature. As the attempts to quantize gravity have been plagued with difficulties, these two theories appear to be essentially incompatible. For the purpose of unifying gravity with the rest of standard model physics, theoretical physicists have proposed many new quantum theories of gravity [1–8]. However, these speculations predict a deviation of the gravitational inverse-square law (ISL). Any experimental test of ISL therefore has the potential to offer insight about the new physics. The standard parameterization of non-Newtonian physics is usually described by the Yukawa potential, which can be written as

$$V(r) = -G_N \frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}), \tag{1}$$

where r is the separation between two masses; G_N is the Newtonian gravitational constant independent of r; and α and λ are the strength and length scale, respectively, of any new interaction. Up to now, numerous experiments have been performed searching for ISL deviations [9–31]. In a short range ($\lambda \leq 1$ cm), inspired by the predictions of many new non-Newtonian theories, many groups performed experiments and the constraints for $\alpha - \lambda$ have been substantially improved [9–24]. For larger ranges ($\lambda \geq 10$ m), the constraints on α are dependent on the geophysical and astronomical data, which may be improved significantly in the next few years by using the lunar-laser–ranging (LLR) technology [25–27]. However, for a range of $\lambda = 0.01$–10 m, the current constraints on α have remained essentially unchanged [28–32] since the publication of Ref. [30]. To improve the sensitivity of α down to 10^{-5}, Boynton *et al.* have proposed their experimental design, but they have not yet reported their result [31]. Therefore, the current situations motivate us to conduct further analysis of ISL deviation at the centimeter range.

Among all the experimental methods, one interesting option for testing ISL is offered by experiments measuring the Newtonian gravitational constant G [28,33]. The basic principle is that the results of measuring G in various benchtop experiments may have a distance dependence due to non-Newtonian gravity. In particular, for the form of the Yukawa potential in Eq. (1), G can be written as

$$G(r) = G_N[1 + \alpha(1 + r/\lambda)e^{-r/\lambda}], \tag{2}$$

where $G(r)$ is the value obtained from the experiment. That is to say, one can obtain the information about ISL deviation by comparing the value of $G(r)$ with G_N. However, G_N is still the poorest known among all fundamental physical constants. To weaken the dependence on the G_N value, it is more appropriate to compare the values of $G(r)$ obtained from two different mass separations. For example, in Ref. [28], Long conducted an experiment to test ISL in this way.

The experiments measuring G at Huazhong University of Science and Technology (HUST) have obtained the smallest uncertainties by using two independent methods [34]. Because the distances between masses in these two experiments are different, the discrepancy between the obtained values of G may exist in the presence of the Yukawa effect. Based on this prediction, we present a direct analysis of the Yukawa effect in these two experiments and obtain a new test result of ISL at the centimeter range.

The two methods in Ref. [34] are the time-of-swing (TOS) method and the angular acceleration feedback (AAF) method, respectively. As the principles of these methods are different, we will analyze the Yukawa effect separately. The TOS method is based on detecting the change of the angular oscillation frequencies for the source masses at two configurations ("near" position and "far" position). The basic design and the operation of this method are described in Ref. [34]. In this method, an Al-coated block pendulum is suspended by a thin fused silica fiber and two stainless-steel spheres are used as the source masses. The "near" and "far" positions are switched by a turntable. Considering the Yukawa potential from interaction between the pendulum and the source masses, the equation of the pendulum motion can be expressed as [35]

$$I\ddot{\theta} + b\dot{\theta} + k\theta = \tau_{Ng}(\theta) + \tau_{Yg}(\theta), \tag{3}$$

where I is the moment of inertia of the pendulum; k is the torsional spring constant of the fiber; b is the damping coefficient; θ is the torsion angle; and $\tau_{Ng}(\theta)$ and $\tau_{Yg}(\theta)$ are the Newtonian torque and the Yukawa torque, respectively. The Yukawa torque (similarly the Newtonian torque) may be expanded as

$$\tau_{Yg}(\theta) = -\alpha K_{1Yg}\theta - \alpha K_{3Yg}\theta^3 + o(\theta^5), \tag{4}$$

where $K_{1Yg} = \{[\partial^2 V_{Yg}(\theta)]/(\partial\theta^2)\}$ and $K_{3Yg} = \frac{1}{6}\{[\partial^4 V_{Yg}(\theta)]/(\partial\theta^4)\}$ are evaluated at $\theta = 0$ with $V_{Yg} = -G_N[(m_1 m_2)/r]e^{-r/\lambda}$ being the Yukawa potential energy between the pendulum and the source masses. K_{3Yg} and higher terms represent the nonlinearity effect of the Yukawa gravity. If the nonlinearity effect is neglected, we may treat K_{1Yg} as the effective Yukawa gravitational torsion constants [35] and we can write K_{1Yg} as $G_N C_{Yg}$, where C_{Yg} is determined by the mass distributions of the pendulum and source masses. In the laboratory coordinate system (X, Y, Z), C_{Yg} can be written as

$$C_{Yg} = -m\frac{\partial^2}{\partial\theta^2}\int \frac{\rho_p e^{-\sqrt{(X_1-x)^2+(Y_1-y)^2+(Z_1-z)^2}/\lambda}dxdydz}{\sqrt{(X_1-x)^2+(Y_1-y)^2+(Z_1-z)^2}}\bigg|_{\theta=0}, \tag{5}$$

where m is the mass of source mass, ρ_p is the density of the pendulum, (X_1, Y_1, Z_1) is the center of the sphere, and (x, y, z) is the coordinate of the point mass in the pendulum. We can obtain the corresponding form of C_{Ng} for the Newtonian torque by replacing the Yukawa term in Eq. (5) with the Newtonian term [36]. Hence, the difference between the frequency squared of small oscillations of the pendulum at two configurations is

$$\Delta\omega^2 = \frac{\Delta k + G_N\Delta C_{Ng} + G_N\alpha\Delta C_{Yg}}{I}, \tag{6}$$

where Δk denotes the possible change of spring constant of the fiber at the "near" and "far" positions, which is caused by the anelasticity of the fiber [35], $\Delta C_{Ng} = C_{Ngn} - C_{Ngf}$ and $\Delta C_{Yg} = C_{Ygn} - C_{Ygf}$, where the subscripts n and f represent the "near" and "far" source mass positions. Then, G can be determined by

$$G_{\mathrm{TOS}} = \frac{I\Delta\omega^2 - \Delta k}{\Delta C_{Ng}} = G_N\left(1 + \alpha\frac{\Delta C_{Yg}}{\Delta C_{Ng}}\right), \tag{7}$$

where G_{TOS} is the value obtained from the TOS method and is polluted by the possible Yukawa effect.

The main geometric parameters in the TOS method are given in Ref. [34]. The coefficients ΔC_{Yg} and ΔC_{Ng} can be calculated via an integral over the geometry of the pendulum and the source masses. The corresponding uncertainties $\delta\Delta C_{Yg}$ ($\delta\Delta C_{Ng}$) are estimated by calculating the deviations of ΔC_{Yg} (ΔC_{Ng}) caused by the geometrical metrology errors. After considering all the errors, the value of ΔC_{Ng} is calculated to be $1.1884\ \mathrm{kg}^2\cdot\mathrm{m}^{-1}$ and the relative error $\delta\Delta C_{Ng}/\Delta C_{Ng}$ is 12 parts per million (ppm). For the Yukawa coefficient ΔC_{Yg}, we estimate the values with different λ. The third column of Table I shows the uncertainties caused by the main error sources with $\lambda = 0.05$ m. The calculated values of ΔC_{Yg} and $\delta\Delta C_{Yg}$ are $0.9547\ \mathrm{kg}^2\cdot\mathrm{m}^{-1}$ and $1.0786\times10^{-5}\ \mathrm{kg}^2\cdot\mathrm{m}^{-1}$, respectively (see Supplemental Material [37]). The results show that the ratios of $\delta\Delta C_{Yg}$ to ΔC_{Yg} are $\leq 0.58‰$ with $0.001 \leq \lambda \leq 10$ m, based on which the Yukawa violation parameters are constrained.

Furthermore, we make an evaluation to the nonlinearity effect of Yukawa gravity (K_{3Yg} and higher terms). Using the Krylov-Bogoliubov-Mitropolsky (KBM) method of nonlinear oscillations [38], we can obtain the approximate solution of Eq. (3). Similar to the calculations of Eqs. (6) and (7), the influence of the nonlinearity effect on G_{TOS} can be determined. The results show that the uncertainties due to this effect are no more than 14 ppm, which can be also combined to give the constraint on the Yukawa violation parameter.

The basic design and the operation of AAF method are described in Ref. [34]. In the AAF method, two turntables are used to rotate the torsion pendulum coaxially and the source masses individually. The twist angle of the fiber is reduced to about zero by means of a high-gain feedback control system. Then the G value can be obtained from the angular acceleration of the pendulum. The motion equation of the torsion pendulum in the rotating frame can be written as [39]

$$I\ddot{\theta} + b\dot{\theta} + k\theta = (\tau_{Ng0} + \tau_{Yg0})\sin(\omega_s t) - I\zeta(t), \tag{8}$$

TABLE I. Main error contributions of the Yukawa coefficient ΔC_{Yg} (with 1σ, $\lambda = 0.05$ m).

Main error sources	Measured values	$\delta\Delta C_{Yg}(\mathrm{kg}^2 \cdot \mathrm{m}^{-1})$
Pendulum:		
Mass	68.099 37 (22) g	3.3515×10^{-10}
Length	91.005 75 (11) mm	1.7747×10^{-6}
Width	11.086 88 (9) mm	9.2529×10^{-7}
Height	30.668 46 (12) mm	4.6625×10^{-7}
Coating layer:		
Mass	4.22 (12) mg	1.9837×10^{-6}
Ratio of side face layer thickness to end face	0.824 (100)	1.6645×10^{-6}
Source masses:		
Mass of sphere 2	778.1629 (6) g	3.6673×10^{-7}
Mass of sphere 4	777.9647 (6) g	3.6946×10^{-7}
Distance of GCs	157.193 92 (33) mm	9.4856×10^{-6}
Relative positions:		
Centric height of pendulum	46.699 (17) mm	3.6764×10^{-7}
Centric height of sphere 2	46.700 (11) mm	8.8786×10^{-7}
Centric height of sphere 4	46.712 (16) mm	1.4769×10^{-6}
Position of fiber in X axis	19(4) μm	1.0179×10^{-6}
Position of fiber in Y axis	11(4) μm	7.6469×10^{-8}
Position of turntable in X axis	0 ± 12 μm	2.6965×10^{-6}
Position of turntable in Y axis	0 ± 8 μm	3.0945×10^{-7}
Total:		
ΔC_{Yg}	0.9547 $\mathrm{kg}^2 \cdot \mathrm{m}^{-1}$	1.0786×10^{-5}

where I, k, b, and θ have the same expressions as in Eq. (3); $\zeta(t)$ is the angular acceleration of the inner turntable; ω_s is the signal frequency; and τ_{Yg0} and τ_{Ng0} are the Yukawa and Newtonian gravitational torques due to the source masses, respectively. Because the twist angle of the fiber is reduced to about zero, the angular acceleration of the pendulum is equal to the gravitational angular acceleration generated by the source masses,

$$(\tau_{Ng0} + \tau_{Yg0}) \sin(\omega_s t) = I\zeta(t), \tag{9}$$

where we may write $\tau_{Ng0} + \tau_{Yg0}$ as $G_N I(P_{Ng} + \alpha P_{Yg})$. By extracting the amplitude of $\zeta(t)$ at the signal frequency ω_s, the angular acceleration is obtained as

$$\zeta(\omega_s) = G_N(P_{Ng} + \alpha P_{Yg}), \tag{10}$$

Then, G can be determined by

$$G_{\mathrm{AAF}} = \frac{\zeta(\omega_s)}{P_{Ng}} = G_N\left(1 + \alpha\frac{P_{Yg}}{P_{Ng}}\right), \tag{11}$$

where G_{AAF} is the value obtained from the AAF method and is polluted by the possible Yukawa effect. For the calculations of P_{Ng} and P_{Yg}, we expand the Newtonian gravitational torque in spherical multipole moments

$$P_{Ng} = -\frac{8\pi}{I}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\frac{1}{2l+1} m q_{l,m} Q_{l,m}, \tag{12}$$

where $q_{l,m}$ and $Q_{l,m}$ are respectively the Newtonian multipole moments of the pendulum and the Newtonian multipole fields of the source masses

$$q_{l,m} = \int \rho_p(r_p) Y_l^{m*}(\theta_p, \phi_p) r_p^l d^3 r_p \tag{13}$$

and

$$Q_{l,m} = \int \rho(r_s) Y_l^m(\theta_s, \phi_s) \frac{1}{r_s^{l+1}} d^3 r_s, \tag{14}$$

where $\rho_p(r_p)$ and $\rho(r_s)$ are determined by the mass distributions of the torsion pendulum and the source masses, respectively. For the Yukawa potential, we can obtain the same expression by using Green's function [40]

$$P_{Yg} = -\frac{8\pi}{I}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\frac{1}{2l+1} m q_{l,m}^Y Q_{l,m}^Y, \tag{15}$$

with

$$q_{l,m}^Y = \int \rho_p(r_p)\lambda^l (2l+1)!! i_l\left(\frac{r_p}{\lambda}\right) Y_l^{m*}(\theta_p, \phi_p) d^3 r_p \tag{16}$$

TABLE II. Main error contributions of the Yukawa coefficient P_{Y_g} (with 1σ, $\lambda = 0.05$ m).

Main error sources	Measured values	$\delta P_{Y_g}(\mathrm{kg \cdot m^{-3}})$
Pendulum:		
Mass	40.0379 (3) g	1.3642×10^{-6}
Width	91.052 43 (29) mm	4.1550×10^{-4}
Thickness	4.002 40 (8) mm	2.5269×10^{-4}
Height	49.924 41 (24) mm	9.5018×10^{-5}
Coating layer:		
Mass	57.9 (4) mg	3.2922×10^{-5}
Ratio of side face layer thickness to end face	0.75 (17)	3.3906×10^{-3}
Source masses:		
Mass of sphere 09	8541.4183 (52) g	2.5303×10^{-4}
Mass of sphere 07	8543.5812 (53) g	2.5791×10^{-4}
Mass of sphere 10	8540.5288 (52) g	2.5300×10^{-4}
Mass of sphere 12	8541.5593 (52) g	2.5297×10^{-4}
Horizontal distance of GCs 07-09	342.2874 (19) mm	1.9620×10^{-2}
Horizontal distance of GCs 10-12	342.3074 (19) mm	1.9610×10^{-2}
Vertical distance of GCs 09-12	139.7997 (15) mm	9.2870×10^{-3}
Vertical distance of GCs 07-10	139.7822 (17) mm	1.0526×10^{-2}
Relative positions:		
Offset of two turntables	0 ± 56 μm	6.4853×10^{-4}
Centric height of sphere 09	0 ± 31 μm	2.2558×10^{-4}
Position of sphere in pendulum axis	0 ± 0.121 mm	1.4392×10^{-3}
Total		
P_{Y_g}	$1655.74\ \mathrm{kg \cdot m^{-3}}$	3.1320×10^{-2}

being the Yukawa multipole moment of the pendulum, and

$$Q_{l,m}^{Y} = \int \rho(r_s) \frac{1}{\lambda^{l+1}(2l-1)!!} k_l\left(\frac{r_s}{\lambda}\right) Y_l^m(\theta_s, \phi_s) d^3 r_s \quad (17)$$

being the Yukawa multipole moment of the source masses, calculated in the lab frame [40]; $i_l(r/\lambda)$ and $k_l(r/\lambda)$ are the spherical modified Bessel functions. Therefore, the above parameters (P_{Ng} and P_{Yg}) can be calculated by using Eqs. (12) and (15).

The main geometric parameters in the AAF method can be obtained from Ref. [34]. The calculation methods for δP_{Ng} and δP_{Yg} are similar to those for $\delta \Delta C_{Ng}$ and $\delta \Delta C_{Yg}$. After considering all the errors, the value of P_{Ng} is calculated to be $6926.41\ \mathrm{kg \cdot m^{-3}}$ and the relative error $\delta \Delta C_{Ng}/\Delta C_{Ng}$ is 11 ppm. We estimate the values of the Yukawa coefficient P_{Yg} with different λ. The third column of Table II shows the uncertainties caused by the main error sources with $\lambda = 0.05$ m. The calculated values of P_{Yg} and δP_{Yg} are $1655.74\ \mathrm{kg \cdot m^{-3}}$ and $3.1320 \times 10^{-2}\ \mathrm{kg \cdot m^{-3}}$, respectively (see Supplemental Material [37]). The ratios of δP_{Yg} to P_{Yg} are ≤ 1.80‰ with $0.001 \leq \lambda \leq 10$ m, which have been combined in the constraint on the Yukawa violation.

For a better perspective of the Yukawa effect in these two experiments, we plot the relative coefficients ($\Delta C_{Yg}/\Delta C_{Ng}$ and P_{Yg}/P_{Ng}), and the difference between them, as a function of λ. As shown in Fig. 1, the difference between these two coefficients increases rapidly while λ ranges from 0.004 m to 0.4 m and decreases rapidly if λ is in the 0.4 m to 1 m range, which can be interpreted as the distance effect. The separation between the pendulum and the source mass is about 130 mm in the TOS method and about 40 mm in the AAF method. Therefore, the Yukawa strength in the

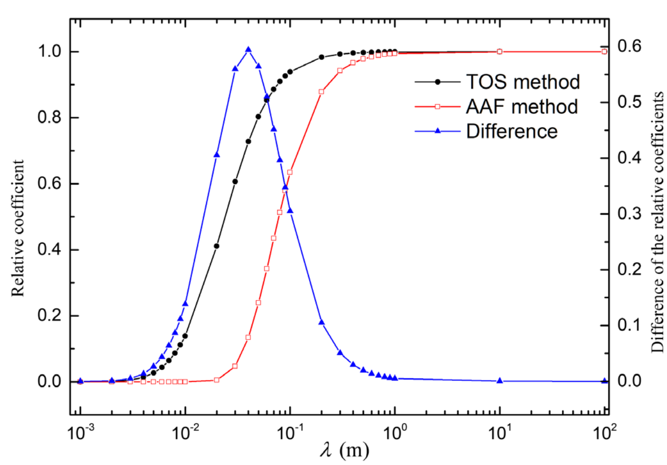

FIG. 1. The relative coefficients of the TOS method ($\Delta C_{Yg}/\Delta C_{Ng}$, black and dot) and the AAF method (P_{Yg}/P_{Ng}, red and square) and the difference of the relative coefficients between these two methods (blue and triangle) for different values of λ.

TOS method is greater than that in AAF method, which corresponds well with our calculation. Thus, we can obtain information about ISL deviation through the comparison of the values of G in the above methods.

Combining Eqs. (7) and (11), the difference between these two methods can be written as

$$\frac{\Delta G}{G_N} = \frac{G_{\mathrm{TOS}} - G_{\mathrm{AAF}}}{G_N} = \alpha\left(\frac{\Delta C_{Yg}}{\Delta C_{Ng}} - \frac{P_{Yg}}{P_{Ng}}\right). \tag{18}$$

In Ref. [34], the results of G_{TOS} and G_{AAF} are $6.674\,184(78)G_0$ and $6.674\,484(78)G_0$, respectively, where $G_0 = 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$. Combining the sensitivities in previous experiments at the centimeter range [19,28–30], we assume G_N as $6.67\times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ and make a conservative estimation to the constraint. Then, the difference between two values of G is

$$\frac{\Delta G}{G_N} = \frac{\sqrt{3^2 + (2\times 0.78)^2 + (2\times 0.78)^2}\times 10^{-15}}{G_N} = 5.58\times 10^{-5}, \tag{19}$$

where $3\times 10^{-15}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the fixed difference between these two methods, $0.78\times 10^{-15}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the uncertainty in each method, and we calculate the constraint at the 2σ level. Thus, the constraint on α can be obtained through Eq. (18), as shown in Fig. 2. The curves labeled HUST and Maryland are taken directly from Refs. [19] and [30]. The original results in Ref. [29] are determined at the 1σ level. For a better comparison, we convert these into the data at 2σ level. Our work sets the strongest bound on α in the range of 5–500 mm. At the length range 60–100 mm, we improve the previous bounds by up to a factor of 7.

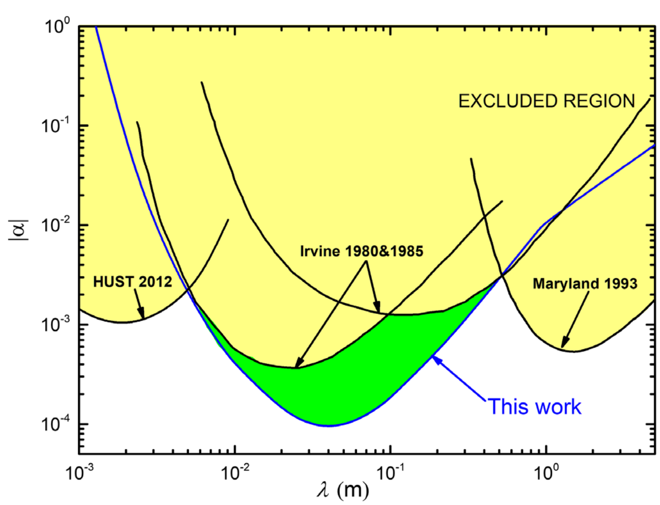

FIG. 2. Constraints on Yukawa violation of the Newtonian $1/r^2$ law. The heavy lines labeled HUST [19], This work, Irvine [29], and Maryland [30] show the experimental constraints, respectively (all plot with 2σ limits).

The current constraint on α can be further improved once the discovery of new error sources can reduce the difference of G in Ref. [34]. It is concluded that the continuous improvement of the accuracy of measuring G will contribute to the test of ISL. In summary, the increase in sensitivity in the constraint on α has the potential to motivate the development of the area of searching new physics and can draw attention to the problem of the substantial discrepancies between the results of G values.

This work is supported by National Key R &D Program of China (Grant No. 2020YFC2200500), the National Natural Science Foundation of China (Grants No. 11575160, No. 11805074, and No. 11925503), Guangdong Major Project of Basic and Applied Basic Research (Grant No. 2019B030302001), and the Fundamental Research Funds for the Central Universities, HUST: 2172019kfyRCPY029.

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