Observing Dynamical Quantum Phase Transitions through Quasilocal String Operators

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(Received 24 November 2020; revised 17 March 2021; accepted 20 April 2021; published 17 May 2021)

We analyze signatures of the dynamical quantum phase transitions in physical observables. In particular, we show that both the expectation value and various out of time order correlation functions of the finite length product or string operators develop cusp singularities following quench protocols, which become sharper and sharper as the string length increases. We illustrated our ideas analyzing both integrable and nonintegrable one-dimensional Ising models showing that these transitions are robust both to the details of the model and to the choice of the initial state.

DOI: 10.1103/PhysRevLett.126.200602

Understanding out-of-equilibrium dynamics of quantum many body systems is an exciting field of recent research both from theoretical and experimental viewpoints [1–6]. In this regard, dynamical and equilibrium quantum phase transitions (DQPTs), manifested as real-time singularities in time-evolving integrable and nonintegrable quantum systems, are indeed an emerging and intriguing phenomena [7–11]. To probe DQPTs, a quantum many body system is prepared initially in the ground state $|\psi(0)\rangle$ of some Hamiltonian. At time $t = 0$ a parameter λ of the Hamiltonian in suddenly changed to say $\lambda = \lambda_f$ and the subsequent temporal evolution of the system generated by the time-independent final Hamiltonian $H(\lambda_f)$ is tracked. DQPTs occur at those instants of time t when the evolved state $|\psi(t)\rangle = \exp[-iH(\lambda_f)t]|\psi(0)\rangle$, becomes orthogonal to the initial state $|\psi(0)\rangle$, i.e., the so-called Loschmidt overlap (LO), $\mathcal{L}(t) = |\langle\psi(0)|\psi(t)\rangle|^2$ vanishes. At those critical instants, the so-called dynamical free energy density (or the rate function of the return probability) defined as $\mathcal{F} = -(1/N)\log|\mathcal{L}(t)|$, N being the system size, develops nonanalytic singularities (cusps in 1D systems) in the thermodynamic limit [7].

Following the initial proposal [7], there have been a plethora of studies investigating intricacies of DQPTs in several integrable and nonintegrable, one-dimensional (as well as two-dimensional) quantum systems occurring subsequent to both sudden [12–45], and smooth [46–49] ramping protocols. The notion of a DQPT has also been generalized for *mixed* initial states [50–53] and finally also in open quantum systems [54]. Analogous to equilibrium phase transitions, it has been established that one expects universal scaling of the dynamical free energy density near the critical instants with identifiable critical exponents. (For reviews on various aspects of DQPTs, we refer to Refs. [55–57].) Remarkably, these nonanalyticities have been detected experimentally subsequent to a rapid quench from a topologically trivial system into a Haldane-like system [58].

Recently, DQPTs were also experimentally [59] detected in trapped-ion setups simulating a long-range interacting transverse field Ising model (TFIM). Starting from a degenerate ground state manifold, it was established that following a quench in an interacting chain of $^{40}Ca^{+}$ ions, the dynamical free energy density develops cusplike singularities at critical instants signalling DQPTs. However, a thorough understanding of the phenomena in harmony with the now well-understood notion of equilibrium quantum phase transitions is far from being complete. Although DQPTs may be characterized by a topological dynamical order parameter [20,30] indicating the emergence of momentum space vortices at critical instants, there is an ongoing search for spatially local observables which are able to capture these nonequilibrium quantum phase transitions [10,11,60]. There has been many attempts aiming at finding real-time observable effects of the dynamical transitions on many-body observables such as work distributions and the growth of entanglement in quenched systems [43,61]. A very interesting perspective on DQPTs was put forward in another recent trapped ion quantum simulator [62], where the authors experimentally studied singularities in the domain wall statistics following a sudden quench of the transverse magnetic field in the system with long range Ising type interactions.

One obvious drawback of DQPTs is that they are manifested in the overlap of the wave functions, which is difficult to observe experimentally. However, the experiment of J. Zhang *et al.* (Ref. [62]) showed that DQPTs can also be manifested in the behavior of nonlocal, stringlike observables. But the precise mathematical connection between DQPTs and this experiment remains unclear. Currently, the broad questions which remain under scrutiny are: (i) Can the singular transitions at DQPTs be captured in the real-time behavior of strong observable quantities? (ii) What can one infer about the spatiotemporal locality of the observables required to detect the DQPTs? (iii) Is it

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possible to obtain the critical exponents associated with the dynamical transitions through a measurement of time-evolving observables [38]? In this work, we approach these issues by defining finite length string operators. As the length of these operators increases they effectively play the role of the projection operators to polarized spin states. Using an exact diagonalization scheme [63,64], we show that these observables are able to capture the critical singularities in their time evolving expectations and temporal correlators. We observe that early time rate function of the out-of-time order correlator (OTOC), which is an important quantity to study scrambling of information in chaotic systems [65] and quantum phase transitions [42,66], quickly becomes nonanalytic at the critical instants with the operator length and show universal critical scaling near DQPTs.

To exemplify, we consider a ferromagnetic TFIM with nearest and next nearest neighbor interactions ($J > 0$ and $J_2 > 0$) and a noncommuting external field h, having N spins,

$$H = -J\sum_{i=1}^{N}\sigma_z^i\sigma_z^{i+1} - J_2\sum_{i=1}^{N}\sigma_z^i\sigma_z^{i+2} - h\sum_{i=1}^{N}\sigma_x^i \qquad (1)$$

under periodic boundary conditions where σ's are the Pauli Matrices satisfying standard SU(2) commutation relations. The presence of both the nearest neighbor and the next nearest neighbour interactions renders the model non-integrable with an integrable point at $J_2 = 0$.

For concreteness we start from a fully spin-polarized state $|\uparrow\uparrow\uparrow\ldots\rangle$, which is a ground state corresponding to a zero transverse field. However, this assumption can be lifted without affecting the results of this work as we will observe similar nonanalyticities in the infinite temperature OTOCs (also see Supplemental Material Ref. [67] for a discussion on generic initial states). As an observable we consider the translationally invariant Pauli string operator,

$$P_n(0) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2^n}\prod_{i}^{i+n}(\mathbb{I}_i + \sigma_i^z), \qquad (2)$$

having a finite string size n of a system of size N and probe its time evolution following a sudden quench of the transverse field h at $t = 0$. When the strings in Eq. (2) span the whole system (i.e., when $n = N$), the quantity $P_n(0)$ simply reduces to the projector over the complete initial state $|\psi(0)\rangle$.

Originally DQPTs are defined through emergent non-analyticities of the rate function [7,12] defined as

$$f(t) = -\lim_{N\to\infty}\frac{1}{N}\log\mathcal{L}(t), \quad \text{where } \mathcal{L}(t) = |\langle\psi(0)|\psi(t)\rangle|^2. \qquad (3)$$

These nonanalyticities develop at critical instants of time ($t = t_c$). It is easy to see that formally the rate function can be understood through the time-dependent expectation value of the ground state projection operator $P = |\psi(0)\rangle\langle\psi(0)|$ in the Heisenberg representation: $P(t) = \exp[iHt]P(0)\exp[-iHt]$, such that,

$$f(t) = -\lim_{N\to\infty}\frac{1}{N}\log\langle P(t)\rangle, \qquad (4)$$

where, $\langle\cdots\rangle = \langle\psi(0)|\ldots|\psi(0)\rangle$. In the one-dimensional Ising models, close to the critical point $|f(t) - f(t_c)| \sim |t - t_c|^\alpha$ with the universal critical exponent $\alpha = 1$ [7,12,19]. For the initially polarized state $|\uparrow\uparrow\uparrow\ldots\rangle$, clearly $P \equiv P_N(0)$. The idea of this work is to look into the expectation value of the operator $P_n(t)$ ($n \leq N$) instead of the full projector through the observable

$$\mathcal{O}_n(t) = -\frac{1}{n}\log\langle P_n(t)\rangle. \qquad (5)$$

We find (see Fig. 1) that the observable $\mathcal{O}_n(t)$ develops emergent cusp singularities at the critical instants in both integrable and integrability-broken systems, thus establishing that DQPTs can be detected using normal physical observables. As it is evident from the plot the singularities quickly develop with increasing n, becoming very sharp for $n \gtrsim 6$ for the parameters used to generate this plot. The singularities were also seen to develop with increasing string length in the thermodynamic limit ($N \to \infty$)

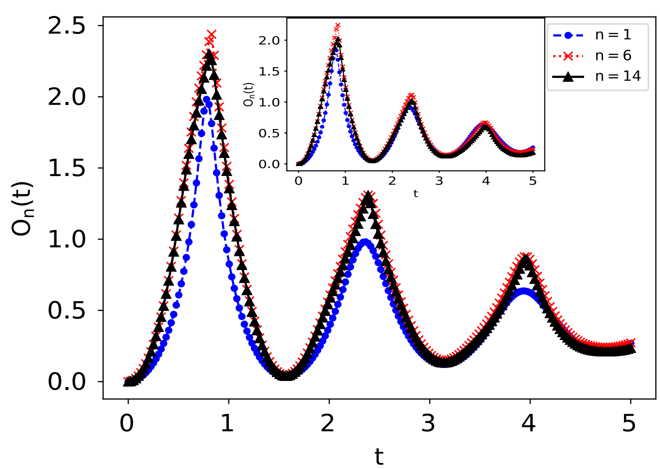

FIG. 1. Emergent cusp singularities in the observable $\mathcal{O}_n(t)$ [see Eq. (5)] for finite string lengths n following an integrable sudden quench in the transverse field (with $4J = 1.0$, $2J_2 = 0.0$), from the completely polarized ferromagnetic ground state ($2h = 0$) to a paramagnetic phase ($2h = 4.0$). The emergence of the singularities is shown for various string lengths comparing the cusps with increasing string length n. (Inset) The same observable following a quench in the nonintegrable Ising chain ($4J = 1.0$, $4J_2 = 0.5$). The simulations have been performed in a chain containing $N = 16$ spins using exact diagonalization.

following an exact calculation for the integrable situation as the expectation $\langle P_n(t_c)\rangle$ vanishes at the critical instants exponentially fast in n [67].

These results can be generalized for quenches starting from an arbitrary initial state [67], like a ground state corresponding to a finite transverse field within the paramagnetic phase or a mixed initial density matrix, for example, corresponding to a finite temperature ensemble. To see how it works, let us observe that for any initial density matrix $\rho(0)$ we have

$$C(t) \equiv \mathrm{Tr}[\rho(t)P(0)P(-t)P(0)] = p(t)\mathcal{L}(t), \qquad (6)$$

where, $p(t) = \langle\psi(0)|\rho(t)|\psi(0)\rangle$. It is easy to check that,

$$\mathrm{Tr}[\rho(t)P(0)P(-t)P(0)] = \mathrm{Tr}[\rho(0)P(t)P(0)P(t)],$$

such that,

$$C(t) = p(t)\mathcal{L}(t) = \mathrm{Tr}[\rho(0)P(t)P(0)P(t)]. \qquad (7)$$

The function $p(t)$ is a projection of the time dependent density matrix to the ground state $|\psi(0)\rangle$. In some situations, like when the initial state is the ground state of the initial Hamiltonian, $p(t) = \mathcal{L}(t)$ such that $C(t) \propto \mathcal{L}^2(t)$. If the initial state, on the other hand, is stationary with respect to the final Hamiltonian $H(\lambda_f)$ then $p(t) = \mathrm{const}(t)$ and $C(t) \propto \mathcal{L}(t)$. In both cases $-(1/N)\log[C(t)]$ is expected to show the exact same singularities as the rate function and scale similar to $f(t)$ with the same exponent α near the critical point.

The function $C(t)$ is nothing but the OTOC of the projection operator, which can be measured through, for example, quantum echo protocols [22,68–71].

Like before instead of the full $C(t)$ we define the quasilocal truncated OTOC

$$\mathcal{C}_n(t) = -\frac{1}{n}\log\langle P_n(t)P_n(0)P_n(t)\rangle, \qquad (8)$$

where the average now is over the initial density matrix $\rho(0)$, which we first take to be the same as before $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$ and later show that the results qualitatively do not change if we start from general mixed ensembles.

Similar to the expectation $\mathcal{O}_n(t)$ the postquench rate function $\mathcal{C}_n(t)$ of the OTOC develops nonanalytic cusp singularities [see Figs. 2(a) and 2(b)] even for finite length string operators $P_n(0)$. We find that the observable $\mathcal{C}_n(t)$ apart from being singular at critical times, also shows a collapse to an universal scaling for sufficiently long strings near the critical point $t = t_c$, having critical exponent $\alpha \sim 1$ for quenches in both the integrable and nonintegrable chains. Consequently, the growth rate of the OTOC in its early time dynamics, shows singular discontinuous jumps at the critical instants of DQPTs. In Fig. 3 we show

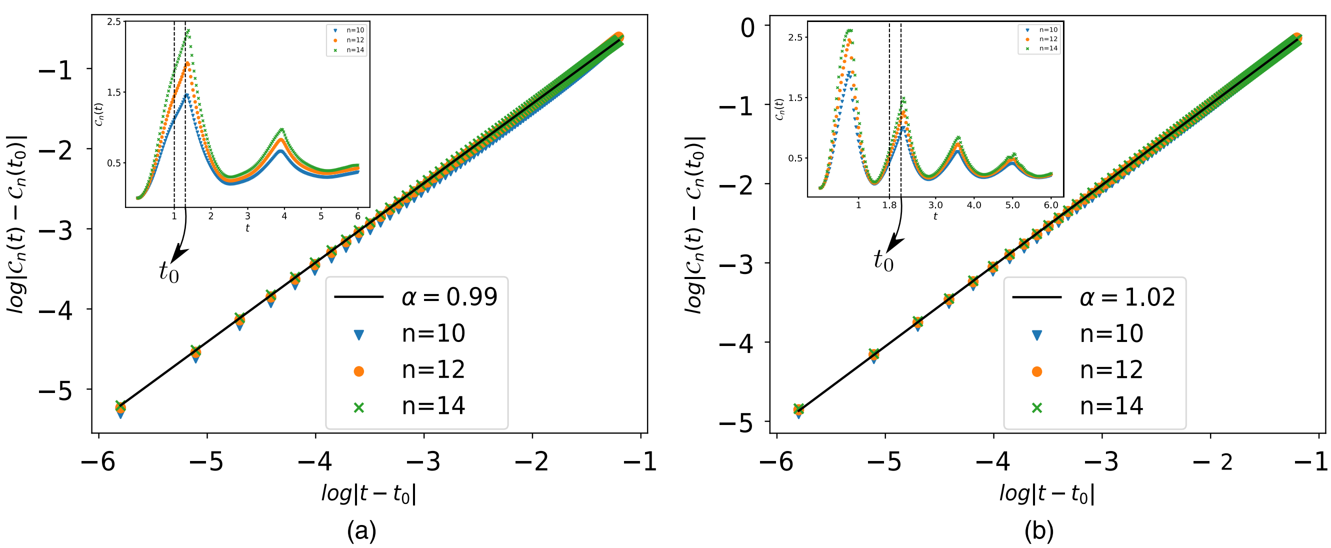

FIG. 2. (a) The universal scaling of the OTOCs for an integrable chain near the critical point. The deviation $\log|\mathcal{C}_n(t) - \mathcal{C}_n(t_0)|$ vs $\log|t - t_0|$ (in the region shown within the vertical dashed lines in inset) exhibits a scaling collapse for different string lengths n (see also Ref. [72]); here we have chosen an instant $t_0 = 1.3$ to be a point near the critical point such that it captures the universal linear collapse region. (Inset) Corresponding emergent cusp singularities in the rate function of the OTOC $\mathcal{C}_n$ [see Eq. (8)] for finite string lengths following a sudden quench in the transverse field (with $4J = 1.5$, $4J_2 = 0$ for $N = 16$) from the completely polarized ferromagnetic ground state ($2h = 0$) to a paramagnetic phase ($2h = 2.5$). (b) The linear scaling of $\mathcal{C}_n$ (in the region shown within the vertical dashed lines in inset) and the corresponding cusps (inset) at critical times in a nonintegrable chain (with $4J = 1.0$, $4J_2 = 1.0$ for $N = 16$) subsequent to a quench from $2h = 0.0$ to $2h = 4.5$ and choosing a $t_0 = 2.1$ near the critical point. Both the integrable and the nonintegrable situations exhibit a universal critical exponent of $\alpha \sim 1$ as seen by the linear fits depicted by black solid lines. The linear scaling in both (a) and (b) has been shown for $t < t_c$ (left of the critical instant). Similar scaling has also been checked to hold for $t > t_c$.

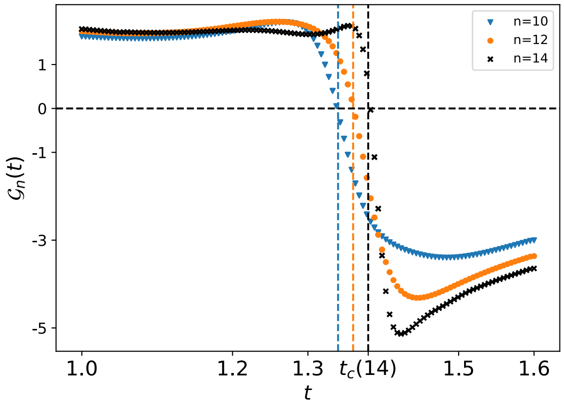

FIG. 3. Emergent jump discontinuities in the OTOC growth rate $\mathcal{G}_n$ [see Eq. (9)] following an integrable sudden quench in the transverse field with the quench parameters being the same as in Fig. 2(a). The quantity $\mathcal{G}_n(t)$ vs t, is shown for various n comparing the sharp jumps at the critical instant $t = t_c(n)$ with increasing string length. The critical instants $t_c(n)$ corresponding to the peaks in the rate function of the OTOC, for a string length n can be determined by the emergent jump singularities in the OTOC growth rate (shown by vertical dashed lines in the figure where the rate crosses zero, with $t_c(14)$ explicitly marked). Similar jump discontinuities were also observed in the non-integrable situations.

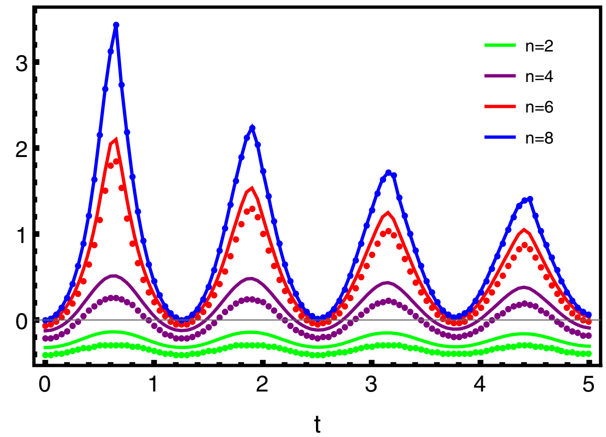

FIG. 4. (Solid lines) The logarithm of the infinite temperature OTOC $\tilde{\mathcal{C}}_n(t)$ [see Eq. (10)] at the critical instants show sharper singularities with increasing peak heights as the string size n increases. (dotted lines) The corresponding postquench autocorrelation functions $\tilde{\mathcal{O}}_n(t)$. The system is initially chosen to be in a completely polarized ground state of an integrable TFIM having a transverse field $h = 0.0$, $J = 0.1$ and suddenly quenched to a final field in the paramagnetic phase $h = 2.5$ at $t = 0$. The simulations have been performed for a chain of $N = 8$ spins while considering finite strings of lengths n. The singularities were seen to become sharper for higher string sizes.

that the jump singularities in the early-time OTOC growth rate at critical instants,

$$\mathcal{G}_n(t) = \frac{d\mathcal{C}_n(t)}{dt}, \tag{9}$$

for finite string operators $P_n(0)$ emerge with increasing sharpness for increasing string lengths.

Although we have demonstrated the development of emergent singularities through a quenched TFIM, the phenomena is not explicitly model dependent as has been previously established in literature. We also stress that the information about the complete initial ground state is not a necessary requirement to study the nonequilibrium phase transitions which can also be detected in temporal correlators of the strings when summed over *arbitrary* complete basis states. To elaborate, consider the infinite temperature autocorrelator and OTOCs,

$$\tilde{\mathcal{O}}_n(t) = -\frac{1}{n}\log \mathrm{Tr}[P_n(t)P_n(0)],$$
$$\tilde{\mathcal{C}}_n(t) = -\frac{1}{n}\log \mathrm{Tr}[P_n(t)P_n(0)P_n(t)], \tag{10}$$

respectively. Remarkably, we find that though the traced correlations are independent of the full initial state $|\psi(0)\rangle$, they develop cusplike singularities at the critical instants for finite but sufficiently long strings and are therefore sufficient to observe the nonequilibrium transitions (see Fig. 4). This also establishes that the emergent critical behavior stays robust in the observables even when the dynamics starts from *arbitrary* excited initial states.

In conclusion, we demonstrated that DQPTs can be observed experimentally as postquench singularities developing in time for stringlike observables. These singularities become sharper with increasing string length. We showed that for the initial ground state these singularities emerge in the expectation values of string operators, while for generic, even infinite temperature, initial states they appear in the OTOC of such operators and can be detected through echo-type protocols. It is interesting that similar signatures of postquench singularities in two-spin observables were found in Refs. [10,60]. The precise relation of the results of that work to the present one regarding DQPTs is yet to be understood.

One can also check (see Ref. [67]) that the coefficient of variation of the observable $P_n(t_c)$ at a critical point remains finite and nonzero for local strings (having a finite length n) even in the thermodynamic limit, unlike that of the full projector $P_N(t_c)$ which diverges exponentially in system size. This suggests a further experimental advantage of the proposed observables as unlike the full Loschmidt overlap, it takes only a finite number of measurements to accurately determine the expectations of finite projectors even for infinitely large system sizes.

S. B. acknowledges support from a PMRF fellowship, MHRD, India. A. D. and A. P. acknowledge financial support from SPARC program, MHRD, India. A. P. was supported by NSF DMR-1813499 and AFOSR FA9550-16-1-0334.

We acknowledge Sourav Bhattacharjee and Somnath Maity for comments. We acknowledge QuSpin and HPC-2010, IIT Kanpur for computational facilities.

Note added.—Recently, we came across a similar study [72] which also exhibits the efficacy of local operators in capturing DQPTs.

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