Where Is the Lightest Charmed Scalar Meson?

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The lightest charmed scalar meson is known as the $D_0^*(2300)$, which is one of the earliest new hadron
congress observed at modern B factories. We show here that the parameters assigned to the lightest scalar D resonances observed at modern B factories. We show here that the parameters assigned to the lightest scalar D meson are in conflict with the precise LHCb data of the decay $B^- \to D^+\pi^-\pi^-$. On the contrary, these data can be well described by an unitarized chiral amplitude containing a much lighter charmed scalar meson, the $D_0^*(2100)$. We also extract the low-energy S-wave $D\pi$ phase of the decay $B^- \to D^+\pi^-\pi^-$ from the data in a model independent wey, and show that its difference from the $D\pi$ septenting phase shift our he traced healt model-independent way, and show that its difference from the $D\pi$ scattering phase shift can be traced back to an intermediate ρ^- exchange. Our work highlights that an analysis of data consistent with chiral symmetry, unitarity, and analyticity is mandatory in order to extract the properties of the ground-state scalar mesons in the singly heavy sector correctly, in analogy to the light scalar mesons $f_0(500)$ and $K_0^*(700)$.

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Introduction.—Since the discovery of the $D_{s0}^*(2317)$ [\[1\]](#page-5-5),
any hadrons were observed beyond the quark model many hadrons were observed beyond the quark model expectations, which have seriously challenged the understanding of the hadron spectrum in terms of the conventional quark model that identifies mesons as $\bar{q}q$ states. The observation that the $D_{s0}^{*}(2317)$ [\[1\]](#page-5-5) and $D_{s1}(2460)$ [\[2\]](#page-5-6) are
significantly lighter than expected by the quark model significantly lighter than expected by the quark model, around 2.48 and 2.55 GeV [3–[5\],](#page-5-7) has driven the development of various models, including $D^{(*)}K$ hadronic mole-cules [6–[13\]](#page-5-8), tetraquark states [\[14,15\]](#page-5-9), and mixtures of $c\bar{q}$ with tetraquarks [\[16\].](#page-5-10) In 2004, two new charm-nonstrange structures, the $D_0^*(2300)$ [\[17,18\],](#page-5-11) called $D_0^*(2400)$ previ-
ously and $D_0(2430)$ [17] were reported as the SU(3) ously, and $D_1(2430)$ [\[17\]](#page-5-11), were reported as the SU(3) partners of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$, respectively.
The observations posed a puzzle: why are the masses of the The observations posed a puzzle: why are the masses of the two nonstrange mesons, $D_0^*(2300)$ and $D_1(2430)$, almost equal to their strange siblings i.e., the $D^*(2317)$ and equal to their strange siblings, i.e., the $D_{s0}^{*}(2317)$ and $D_{s}(2460)$? Thanks to new data from both lattice quantum $D_{s1}(2460)$? Thanks to new data from both lattice quantum chromodynamics (QCD) [\[19](#page-5-12)–24] and the LHCb experiment [\[25\]](#page-5-13), it was recently demonstrated that the various puzzles in the charm meson spectrum can be solved naturally in the framework of unitarized chiral perturbation theory (UChPT) [\[26](#page-5-14)–28] that allows one to calculate the nonperturbative dynamics of Goldstone bosons scattering off the $D_{(s)}^{(*)}$ mesons in a controlled way. The combination of UChPT and lattice QCD not only reproduced the correct $D_{s0}^*(2317)$ mass [\[19\]](#page-5-12), but also predicted its pion mass
dependence [23.20]. The solution provided for the SU(3) dependence [\[23,29\]](#page-5-15). The solution provided for the SU(3) mass hierarchy puzzle mentioned above is that instead of only one heavy state, $D_0^*(2300)$ in the channel
 $(S, I) = (stranomess isospin) - (0, 1/2)$ there are two $(S, I) \equiv$ (strangeness, isospin) = $(0, 1/2)$, there are two states, one lighter and one heavier [\[9,11,13,19,26,30](#page-5-16)– [34\].](#page-5-16) The most recent studies revealed their pole locations to be at $(2105\frac{+6}{8} - i102\frac{+10}{8})$ and $(2451\frac{+35}{8} - i134\frac{+7}{8})$ MeV
[26.27] reprectively The SU(3) partner of the D^* (2317) is [\[26,27\]](#page-5-14), respectively. The SU(3) partner of the $D_{s0}^{*}(2317)$ is
the lighter one, denoted as $D^{*}(2100)$ in the following the lighter one, denoted as $D_0^*(2100)$ in the following,
which restores the expected mass bierarchy. The heavier which restores the expected mass hierarchy. The heavier pole on the other hand is a member of a different multiplet. Support for the presence of two poles comes from an analysis of the high-quality LHCb data on the decays $B^- \rightarrow$ $D^+\pi^-\pi^-$ [\[25\]](#page-5-13), $B_s^0 \to \bar{D}^0 K^-\pi^+$ [\[35\],](#page-5-17) $B^0 \to \bar{D}^0 \pi^- \pi^+$ [\[36\]](#page-5-18), $B^- \to D^+\pi^-K^-$ [\[37\],](#page-5-19) and $B^0 \to \bar{D}^0\pi^-K^+$ [\[38\]](#page-5-20) performed in Refs. [\[27,39\],](#page-5-21) as well as from the fact that their existence is consistent with the lattice energy levels [\[19](#page-5-12)–23] for the relevant two-body scattering [\[26,32,40\].](#page-5-14) This two-pole structure indeed emerges as a more general pattern in the hadron spectrum, see, e.g., Ref. [\[41\].](#page-5-22)

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Despite the phenomenological success of this picture in describing the available lattice and LHCb data, the observation that the lightest D_0^* has a mass around 2.1 GeV has not entered the Review of Particle Physics (RPP) [\[42\]](#page-5-23) yet, which still lists the $D_0^*(2300)$ as the lightest charmed scalar
meson and the $D_1(2430)$ as the corresponding axial-vector meson and the $D_1(2430)$ as the corresponding axial-vector meson. In this Letter, we demonstrate that the $D_0^*(2300)$ as
in the RPP is not consistent with the most precise data for in the RPP is not consistent with the most precise data for $B^{-} \to D^{+}\pi^{-}\pi^{-}$, contrary to the $D_{0}^{*}(2100)$ predicted in
LIChPT and conclude that the positive-parity charm-UChPT, and conclude that the positive-parity charmnonstrange meson spectrum in the RPP needs to be revised.

Dπ S-wave phase of $B^- \to D^+\pi^-\pi^-$.—The decay amplitude in the low-energy region of the $D\pi$ system can be decomposed into S , P , and D waves,

$$
\mathcal{A}_{B^-\to D^+\pi^-\pi^-}(s,z) = \sum_{\ell=0}^2 \sqrt{2\ell+1} \,\mathcal{A}_{\ell}(s) P_{\ell}(z_s), \quad (1)
$$

where $A_{\ell}(s)$ with $\ell = 0, 1, 2$ correspond to the amplitudes with $D^+\pi^-$ in the S, P, and D waves, respectively, s is the c.m. energy squared of the $D^+\pi^-$ system, and $P_{\ell}(z_s)$ are the Legendre polynomials with z_s the cosine of the helicity angle of the $D^+\pi^-$ system, i.e., the angle between the moving directions of the two pions in the $D^+\pi^-$ c.m. frame. The angular moments are determined by weighting the data with the Legendre polynomials $P_{\ell}(z)$ [\[25\].](#page-5-13) They contain contributions from certain partial waves and their interference terms, and thus the corresponding phase variations. The first few moments are given by [\[25,27,39\]](#page-5-13)

$$
\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2,
$$

$$
\langle P_2 \rangle \propto \frac{2}{5} |\mathcal{A}_1|^2 + \frac{2}{7} |\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}} |\mathcal{A}_0| |\mathcal{A}_2| \cos(\delta_2 - \delta_0),
$$

$$
\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |\mathcal{A}_0| |\mathcal{A}_1| \cos(\delta_1 - \delta_0), \quad (2)
$$

with δ_i the phase of A_i , i.e., $A_i = |A_i|e^{i\delta_i}$. As first proposed in Ref. [\[27\],](#page-5-21) we use the linear combination $\langle P_{13} \rangle$ instead of $\langle P_1 \rangle$ and $\langle P_3 \rangle$ individually, since it only depends on the $S - P$ -wave interference up to $\ell = 2$ and is particularly sensitive to the S-wave phase motion.

For $M_{D^+\pi^-}$ < 2.2 GeV, even the D wave can be neglected, since the narrow tensor resonance $D_{s2}(2460)$ is sufficiently far away. This can be verified from the data of the angular moments, i.e., Fig. 3 in Ref. [\[25\].](#page-5-13) Therefore, in this kinematic regime one obtains

$$
\cos(\delta_0 - \delta_1) = \sqrt{\frac{3}{10}} \frac{\langle P_{13} \rangle}{\sqrt{\langle P_2 \rangle} \sqrt{\langle P_0 \rangle - \frac{5}{2} \langle P_2 \rangle}} \tag{3}
$$

for the $S-P$ phase difference. The P wave is dominated by the vector resonance $D^*(2007)^0$ below the $D^+\pi^-$ threshold

FIG. 1. Comparison of the predictions of Eq. [\(5\)](#page-2-1) from UChPT (blue) and a Breit-Wigner parametrization (green) for δ_0 with the phase extracted in Ref. [\[25\]](#page-5-13) (red) and that using Eq. [\(3\)](#page-1-1) (black). The bands correspond to errors propagated from the input UChPT scattering amplitudes and from the Breit-Wigner resonance parameters.

with a width of less than 60 keV [\[43,44\].](#page-5-24) The next vector D^* resonance is far above this energy region. Thus, the phase of the P wave δ_1 can be safely fixed to 180° for the region we are interested in. The S-wave $D\pi$ phase motion below 2.2 GeV can then be extracted, see Fig. [1.](#page-1-0) For comparison, the phase motion of the $D^+\pi^-$ S wave up to 2.4 GeVobtained in the LHCb analysis [\[25\]](#page-5-13) (with the phase at 2.4 GeV fixed to 180°) is also shown, which is fully in line with the phase we extracted from Eq. [\(3\)](#page-1-1) below 2.2 GeV.

For $M_{D^+\pi^-}$ < 2.4 GeV, the effect of the ρ meson could be significant via the coupled channel $B^- \to D^0 \pi^0 \pi^-$; see Fig. [2.](#page-2-0) This follows directly from the large branching ratio $B(B^{-} \rightarrow D^{0} \rho^{-}) = 1.34\%$, which is an order larger than $\mathcal{B}(B^- \to D^+\pi^-\pi^-) = 0.107\%$ [\[42\]](#page-5-23). It is therefore reasonable to assume that the decay $B^-\to D^+\pi^-\pi^-$ is dominated by the process $B^- \to D^0 \rho^- \to D^0 \pi^0 \pi^- \to D^+ \pi^- \pi^-$. By virtue of soft-pion theorems, one has [\[45\]](#page-5-25)

$$
\mathcal{A}(B^- \to D^+\pi^-\pi^-)|_{p_{\pi^-} \to 0} = \frac{1}{F_{\pi}}\mathcal{A}(B^0 \to \bar{D}^0\pi^0),
$$

$$
\mathcal{A}(B^- \to D^0\pi^0\pi^-)|_{p_{\pi^0} \to 0} = -\frac{1}{F_{\pi}}\mathcal{A}(B^- \to D^0\pi^-),
$$
 (4)

where $p_{\pi^-(\pi^0)}$ is the momentum of the $\pi^-(\pi^0)$, and F_π is the pion decay constant (in the chiral limit). From $\mathcal{B}(B^0 \rightarrow$ $\bar{D}^0\pi^0$ = 2.63 × 10⁻⁴ and $\mathcal{B}(B^-\to D^0\pi^-)$ = 4.68 × 10⁻³, one concludes that at low energies for the $D^+\pi^ (D^0\pi^0)$ system, the amplitude of $B^- \to D^0 \pi^0 \pi^-$ is much larger than that of $B^- \to D^+\pi^-\pi^-$. Furthermore, isospin symmetry shows that for the decays $B \to D \pi \pi$ with even relative angular momenta between the pions, the amplitude for $B^- \to D^+\pi^-\pi^-$ is larger than that of $B^- \to D^0\pi^0\pi^-$ by a factor of $2\sqrt{2}$ [\[27,39,46,47\].](#page-5-21) As in addition even angular

FIG. 2. The decay $B^- \to D^+\pi^-\pi^-$ via the coupled channel $B^- \to D^0 \pi^0 \pi^-$. The filled square denotes the $D^0 \pi^0 \to D^+ \pi^- T^$ matrix element.

momenta here imply isospin $I = 2$ and therefore nonresonant partial waves, the relative angular momentum of $\pi^0 \pi^$ in the decay $B^- \to D^0 \pi^0 \pi^-$ is by far dominantly odd in the low-energy regime for $D^0\pi^0$, and the ρ^- plays a crucial role.

If we assume that the decay $B^- \to D^+\pi^-\pi^-$ is dominated by the process in Fig. [2,](#page-2-0) the $D\pi S$ -wave part of the triangle diagram can be estimated by the integral

$$
\mathcal{A}_0^{\text{trig}}(s) = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} ds' \frac{\hat{P}(s')\rho(s')T_{D^0\pi^0 \to D^+\pi^-}(s')}{s'-s}, \quad (5)
$$

where $\hat{P}(s)$ is the production amplitude for $B^- \to D^0 \rho^- \to$ $D^{0}\pi^{0}\pi^{-}$ projected to the $D^{0}\pi^{0}$ s channel, $\rho(s)$ = $\sqrt{\lambda(s,M_D^2,M_{\pi}^2)/(16\pi s)}$ is the $D\pi$ phase space with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ the Källén $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ the Källén function, $T_{D^0\pi^0\to D^+\pi^-}(s)$ the S-wave scattering amplitude for $D^0 \pi^0 \to D^+ \pi^-$, and $s_{\text{th}} = (M_D + M_\pi)^2$. The expression for $\hat{P}(s)$ is the same as $\hat{\mathcal{F}}_0^{1/2}(s)$ in Eq. [\(12\)](#page-3-0) below.
The evaluation of Eq. (5) depends on the asymptot

The evaluation of Eq. [\(5\)](#page-2-1) depends on the asymptotic behavior of the integrand, which is divergent in general. We may estimate Eq. [\(5\)](#page-2-1) using a cutoff at $\sqrt{s_{\text{max}}} = \sqrt{q_{\text{max}}^2 + M_D^2} + \sqrt{q_{\text{max}}^2 + M_Z^2}$, where $q_{\text{max}} \approx$
1 GeV (another way is to introduce a form factor e.g. 1 GeV (another way is to introduce a form factor, e.g., $e^{-(s-s_{th})/s_0}$ with $s_0 = \mathcal{O}(1 \text{ GeV})$ [\[48\]\)](#page-5-26). We evaluate Eq. [\(5\)](#page-2-1) by employing both the $D\pi$ scattering amplitude from UChPT [\[19\]](#page-5-12) and that of a Breit-Wigner (BW) parametrization of the $D_0^*(2300)$ for comparison, despite the defi-
ciencies of the latter discussed in Ref. [39]; see also ciencies of the latter discussed in Ref. [\[39\]](#page-5-27); see also Ref. [\[49\].](#page-5-28)

The results with $q_{\text{max}} = 1$ $q_{\text{max}} = 1$ GeV are shown in Fig. 1, where the solid blue band and the green dashed band correspond to the $D\pi$ scattering amplitudes from UChPT and BW, respectively. The obtained phase describes the data perfectly for the UChPT amplitude, while the BW one fails. We have checked that the obtained phases are insensitive to a variation of the cutoff in a reasonable region, $q_{\text{max}} \in [0.8, 1.2]$ GeV.

Khuri-Treiman formalism.—While Eq. [\(5\)](#page-2-1) provides a reasonable estimation of the S-wave decay amplitude with a clear underlying physical picture, it does not respect three-body unitarity. In order to check if the conclusion formulated above is robust, we cure this deficiency by employing the Khuri-Treiman equations [\[50\]](#page-5-29), which are based on two-body elastic phase shifts and explicitly generate the crossed-channel rescattering between final-state particles. The formulas are constructed from dispersion relations for the related crossed scattering processes and then analytically continued to the decay region, referring to the continuation of the triangle graph [\[51\]](#page-5-30).

We can write amplitudes for $A_{+-}(B^- \to D^+\pi^-\pi^-)$ and $A_{00-}(B^- \to D^0 \pi^0 \pi^-)$ in terms of single-variable functions according to a reconstruction theorem [\[47,52\],](#page-5-31)

$$
\mathcal{A}_{+--}(s,t,u) = \mathcal{F}_0^{1/2}(s) + \frac{\kappa(s)}{4} z_s \mathcal{F}_1^{1/2}(s) \n+ \frac{\kappa(s)^2}{16} (3z_s^2 - 1) \mathcal{F}_2^{1/2}(s) + (t \leftrightarrow s), \n\mathcal{A}_{00-}(s,t,u) = -\frac{1}{\sqrt{2}} \mathcal{F}_0^{1/2}(s) - \frac{\kappa(s)}{4\sqrt{2}} z_s \mathcal{F}_1^{1/2}(s) \n- \frac{\kappa(s)^2}{16\sqrt{2}} (3z_s^2 - 1) \mathcal{F}_2^{1/2}(s) + \frac{\kappa_u(u)}{4} z_u \mathcal{F}_1^1(u),
$$
\n(6)

where the subindex ℓ and superindex I of the singlevariable amplitudes \mathcal{F}_{ℓ}^{I} represent the angular momentum and isospin, respectively, and only the $I < 3/2$ and $\ell < 2$ terms are taken into account. The Mandelstam variables of the B-meson decay $B^{-}(p_B) \to D(p_D) \pi(p_1) \pi^{-}(p_2)$ are $s = (p_B - p_2)^2$, $t = (p_B - p_1)^2$, and $u = (p_B - p_0)^2$. The corresponding angles are given by

$$
z_s \equiv \cos \theta_s = \frac{s(t - u) - \Delta}{\kappa(s)}, \quad z_u \equiv \cos \theta_u = \frac{t - s}{\kappa_u(u)}, \quad (7)
$$

where $\kappa(s) = \lambda^{1/2} (s, M_D^2, M_\pi^2) \lambda^{1/2} (s, M_B^2, M_\pi^2), \quad \kappa_u(u) =$ $\lambda^{1/2}(u, M_B^2, M_D^2)\sqrt{1-4M_{\pi}^2/u}$, and $\Delta = (M_B^2 - M_{\pi}^2)\times$ $(M_D^2 - M_\pi^2)$.
Since we

Since we are interested in the s-channel process, we use the index $A(B)$ to label the two-body channels corresponding to $D^+\pi^-$ and $D^0\pi^0$. The partial-wave decomposition for the decay amplitudes A_A reads

$$
\mathcal{A}_A(s, z_s) = \sum_{I, \ell} b_{I, \ell}^A P_{\ell}(z_s) f_{\ell}^I(s), \tag{8}
$$

with $b_{I,\ell}^A$ denoting Clebsch-Gordan coefficients. By comparing with Eq. [\(1\)](#page-1-2), it is easy to obtain $A_{\ell}(s) =$ $(2\ell + 1)^{-1/2} \sum_{I} b_{I,\ell}^{1} f_{\ell}^{I}(s)$. We have the following partial-
wave unitarity relation for election receptions: wave unitarity relation for elastic rescattering:

$$
\operatorname{disc} f_{\ell}^{I}(s) = 2if_{\ell}^{I}(s) \sin \delta_{\ell}^{I}(s) e^{-i\delta_{\ell}^{I}(s)} \theta(s - s_{\text{th}}), \quad (9)
$$

where $\delta_{\ell}^{l}(s)$ is the elastic final-state scattering phase shift.
The discontinuities of f^{l} and those of the single-variable The discontinuities of f^I_{ℓ} and those of the single-variable

functions $\kappa^{\ell} \mathcal{F}_{\ell}^{I}$ coincide on the right-hand cut by construction. Thus, one has

$$
\text{disc}\mathcal{F}^I_{\ell}(s) = 2i[\mathcal{F}^I_{\ell}(s) + \hat{\mathcal{F}}^I_{\ell}(s)]\sin\delta^I_{\ell}(s)e^{-i\delta^I_{\ell}(s)}\theta(s-s_{\text{th}}),
$$
\n(10)

where the inhomogeneities $\hat{\mathcal{F}}_{\ell}^{I}(s)$ encode the left-hand cut
contributions and are free of discontinuities on the rightcontributions and are free of discontinuities on the righthand cut. This discontinuity relation is solved by

$$
\mathcal{F}^I_{\ell}(s) = \Omega^I_{\ell}(s) \left\{ Q^I_{\ell}(s) + \frac{s^n}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{ds'}{s'^n} \frac{\sin \delta^I_{\ell}(s') \hat{\mathcal{F}}^I_{\ell}(s')}{|\Omega^I_{\ell}(s')|(s'-s)} \right\},\tag{11}
$$

where $\Omega_{\epsilon}^{L}(s) = \exp\{s/\pi \int_{\sin}^{\infty} ds' \delta_{\epsilon}^{L}(s') / [s'(s'-s)]\}$ is the Omnès function [\[53\],](#page-5-32) $Q^I_{\epsilon}(s)$ is a polynomial at least of degree $(n-1)$ (see discussion below) and the number of degree $(n - 1)$ (see discussion below), and the number of subtractions n is chosen to guarantee the convergence of the dispersion integral.

The inhomogeneity $\hat{\mathcal{F}}_{\ell}^{I}$ is determined by the partialwave decomposition of Eq. [\(6\)](#page-2-2) as the projection of the crossed-channel amplitudes onto the considered channel. Around the $D\pi$ threshold in the s channel, $\sqrt{t} \sim 5$ GeV, there is no resonance in the t channel, and thus the interaction is supposed to be very weak. The only possible significant crossed-channel effect is from the ρ meson through $B^- \to D^0 \pi^0 \pi^-$. The resulting inhomogeneity for the S-wave s-channel amplitude is [\[47\]](#page-5-31)

$$
\hat{\mathcal{F}}_0^{1/2}(s) = -\frac{1}{4\sqrt{2}} \int_{-1}^1 dz_s(t-s) \mathcal{F}_1^1(u). \tag{12}
$$

For technical details regarding this integral, see Ref. [\[54\]](#page-5-33) and the Supplemental Material [\[55\].](#page-5-34)

The full solution for the decay amplitudes can be obtained by solving a set of coupled integral equations in terms of a few linearly independent complex subtraction constants contained in $Q_{\ell}^{I}(s)$, which cannot be determined a priori
in the framework of dispersion theory. Since we are only in the framework of dispersion theory. Since we are only interested in the $D\pi$ low-energy regime and especially in its S wave, based on the large branching ratio of $B^- \to D^0 \rho^-$, it is reasonable to approximate $\mathcal{F}_1^1(u)$ in Eq. [\(6\)](#page-2-2) by a BW function for the a meson. In this case, $\mathcal{F}_1^1(u)$ behaves as u^{-1} function for the ρ meson. In this case, $\mathcal{F}_1^1(u)$ behaves as u^{-1}
for $u \to \infty$, thus $\hat{\mathcal{T}}_1^1(x)$ in Eq. (12) approaches a constant so for $u \to \infty$, thus $\hat{\mathcal{F}}_{\ell}^I(s)$ in Eq. [\(12\)](#page-3-0) approaches a constant as $s \to \infty$. The number of the subtractions *n* in Eq. (11) is then $s \to \infty$. The number of the subtractions *n* in Eq. [\(11\)](#page-3-1) is then determined by the asymptotic behavior of the scattering phase $\delta^I_{\ell}(s)$. For the BW phase and that of UChPT taken
from Ref. [19] one single subtraction is sufficient from Ref. [\[19\]](#page-5-12), one single subtraction is sufficient. Moreover, the phase of UChPT, as well as that of the BW, is unreliable at high energies. Thus, the dispersion integral will be evaluated up to a cutoff Λ , and the effect of cutting off the integral may be absorbed into the polynomial $Q^I_{\ell}(s)$. Explicitly, for an integral

$$
g(s) = \int_{s_{\text{th}}}^{\infty} ds' \frac{f(s')}{s' - s} = \int_{s_{\text{th}}}^{\Lambda} ds' \frac{f(s')}{s' - s} + \int_{\Lambda}^{\infty} ds' \frac{f(s')}{s' - s}
$$

$$
\approx g_0 + g_1 s + \int_{s_{\text{th}}}^{\Lambda} ds' \frac{f(s')}{s' - s}.
$$
(13)

For simplicity, we neglect the $I = 3/2$ contribution since it contains no resonances. Therefore, for the S-wave amplitude at low energies, Eq. [\(11\)](#page-3-1) can be written as

$$
\mathcal{F}_0^{1/2}(s) = \Omega_0^{1/2}(s) \left\{ g_0 + g_1 \frac{s - M_D^2}{M_D^2} + \frac{s}{\pi} \int_{s_{\text{th}}}^{\Lambda} \frac{ds'}{s'} \frac{\sin \delta_0^{1/2}(s') \hat{\mathcal{F}}_0^{1/2}(s')} {|\Omega_0^{1/2}(s')|(s'-s)} \right\}.
$$
 (14)

The constants g_0 and g_1 have to be fixed by data.

We fit $\langle P_0 \rangle$, $\langle P_{13} \rangle$, and $\langle P_2 \rangle$ of the decay $B^- \rightarrow$ $D^+\pi^-\pi^-$ up to 2.4 GeV, which is below the D_n and $D_s\bar{K}$ thresholds. To describe the angular moments, one needs the explicit amplitudes for the $D\pi P$ and D waves. The P wave can be safely parametrized as $\delta_1^{1/2}(s) =$
 $\sigma_1^{(2)}(s)$ as discussed below Eq. (2). Consequently $\pi\theta(s - M_{D^{*0}}^2)$ as discussed below Eq. [\(3\).](#page-1-1) Consequently, the dispersion integral (11) for the *B* weven can be the dispersion integral (11) for the *P* wave can be neglected since $\sin \pi = 0$. The *D* wave is dominated by the resonance $D_2^0(2460)$ with a width of 47.5 MeV [\[42\]](#page-5-23),
which is above the region we are interested in Thus, the which is above the region we are interested in. Thus, the D-wave phase is close to 0 below 2.4 GeV, and the corresponding dispersion integral can be neglected as well. Therefore, for the *P*-(*D*-)wave amplitudes $\mathcal{F}_{1,2}^{1/2}$, we can use the same BW forms as those in the LHCb analysis $[25]$, which is equivalent to the corresponding Omnès function multiplied by a polynomial. In the isobar model used in Ref. [\[25\]](#page-5-13), complex factors are introduced for each resonance BW function. Without crossed-channel effects, these factors become real according to Watson's theorem [\[56\].](#page-5-35) For the P and D waves, as discussed above, the Omnès representation should be a good approximation in the energy region we are interested in, and the normalization factor is real. We also consider a complex normalization and find the results unchanged.

For the $D\pi S$ wave, we employ both the scattering phase shifts from UChPT [\[19\],](#page-5-12) which contains the $D_0^*(2100)$, and
the BW for the $D^*(2300)$. The fit results are shown in the BW for the $D_0^*(2300)$. The fit results are shown in
Fig. 3, where the blue and green bands correspond to the Fig. [3](#page-4-0), where the blue and green bands correspond to the best fits from UChPT and the BW, respectively. While UChPT describes the data very well with $\chi^2/\text{d.o.f.} = 1.2$, the BW fails to reproduce the data with $\chi^2/\text{d.o.f.} = 2.0$. The difference of these two values is significant from the statistical point of view: the corresponding p values are 0.1 for the UChPT fit and 3×10^{-5} for the BW fit, respectively. Thus, the former can be accepted as a good description of the data, while the latter is highly disfavored [\[57\].](#page-5-36) The error bands correspond to the 1σ uncertainties propagated from

FIG. 3. Results of UChPT (blue) and the BW description (green), with the best fits $\chi^2/\text{d.o.f.} = 1.2$ and 2.0, respectively.

the input phases. The borders of the band of $\langle P_{13} \rangle$ for the BW are plotted in dotted and dashed curves to make it evident that the data for $\langle P_0 \rangle$ and $\langle P_{13} \rangle$ cannot be described by the BW phase. With the fitted parameters g_0 and g_1 , we obtain the S-wave phase of the decay amplitude for $B^- \rightarrow$ $D^+\pi^-\pi^-$ shown in Fig. [4,](#page-4-1) where the results corresponding to UChPT and the BW are plotted as blue and green bands, respectively. As expected, UChPT describes the S-wave phase extracted using Eq. [\(3\)](#page-1-1) and that obtained in Ref. [\[25\]](#page-5-13) well up to 2.4 GeV. For the BW one, although the error band is broad, either the low-energy or the high-energy region cannot be described.

Conclusion.—The existence of the $D_0^*(2300)$ as given in
B RPP is the starting point of many theoretical analyses the RPP is the starting point of many theoretical analyses (see, e.g., Refs. [58–[61\]\)](#page-5-37). The results obtained in this Letter show that the $D_0^*(2300)$, whose resonance parameters were
obtained using the BW parametrization from the Belle [17] obtained using the BW parametrization from the Belle [\[17\]](#page-5-11) and BABAR [\[62\]](#page-5-38) analyses, is in conflict with the much more precise LHCb data for $B^- \to D^+\pi^-\pi^-$, which, however, can be well reproduced by the UChPT amplitude containing the $D_0^*(2100)$.
We expect the

We expect that the $D_1(2430)$ as given in the RPP [\[42\]](#page-5-23) will also be in conflict with future high-quality data of B^- → $D^{*+}\pi^-\pi^-$ from LHCb [\[63\]](#page-6-0) and Belle-II, and that the

FIG. 4. Phases obtained from Eq. [\(14\)](#page-3-2) with scattering phase shifts from UChPT (blue) and BW (green).

lightest D_1 meson is the $D_1(2250)$ predicted by UChPT [\[26,27\]](#page-5-14).

The D_0^* is analogous to the more famous $f_0(500)$ and (700) whose masses have been significantly shifted $K_0^*(700)$, whose masses have been significantly shifted
from earlier versions of the RPP due to improved data and from earlier versions of the RPP due to improved data and improved theoretical analyses—for recent discussions see Refs. [64–[68\]](#page-6-1) and the review on scalar mesons in the RPP [\[42\]](#page-5-23). We expect a similar change in all systems emerging from the scattering of a pion off an isospin-nonsinglet hadron. The lightest resonance in that case should not be extracted from data using the usual BW form—a parametrization accounting for chiral symmetry and coupled channels is mandatory. The $D\pi S$ wave phase extracted model independently here provides valuable information for further understanding matter-field–Goldstone-boson scattering and the structure of positive-parity heavy hadrons.

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Note added.—Recently, a lattice calculation also concluded that the D_0^* mass should be lower than the RPP value [\[69\]](#page-6-2). The authors found a mass of (2196 ± 64) MeV with a pion mass of 239 MeV, only (77 ± 64) MeV above the $D\pi$ threshold. Thus, our conclusion receives a strong support from lattice QCD calculations.

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