

Measurement-Device-Independent Entanglement Detection for Continuous-Variable Systems

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We study the detection of continuous-variable entanglement, for which most of the existing methods designed so far require a full specification of the devices, and we present protocols for entanglement detection in a scenario where the measurement devices are completely uncharacterized. We first generalize, to the continuous variable regime, the seminal results by Buscemi [Phys. Rev. Lett. **108**, 200401 (2012)] and Branciard *et al.* [Phys. Rev. Lett. **110**, 060405 (2013)], showing that all entangled states can be detected in this scenario. Most importantly, we then describe a practical protocol that allows for the measurement-device-independent certification of entanglement of all two-mode entangled Gaussian states. This protocol is feasible with current technology as it makes use only of standard optical setups such as coherent states and homodyne measurements.

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Introduction.—Entanglement is the main resource for a broad range of applications in quantum information science, among which are quantum key distribution [1], quantum computation [2], and quantum metrology [3]. It is therefore crucial to develop methods to detect entanglement that are reliable and practical. The most common method to detect entanglement is given by entanglement witnesses [4]. However, to be reliable this technique requires a perfect implementation of the measurements. Indeed, small calibration errors can lead to false-positive detection of entanglement [5,6], which can be critical when using the wrongly detected entangled state for quantum information purposes. A possible way of circumventing this problem is to move into the so-called device-independent (DI) scenario [7]. In this framework measurements do not need to be characterized, since entanglement is detected through the violation of Bell inequalities, which only use the statistics provided by the experiment, without making any assumptions on the real implementation. The DI scenario is however stringent from an experimental point of view, requiring low levels of noise and high detection efficiencies. This is why other approaches requiring an intermediate level of trust on the devices have been developed. In particular, there exist methods that do not require any characterization of the measurement implemented for entanglement detection, known as measurement device independent (MDI) [8,9].

Here, we consider the problem of entanglement detection in continuous-variable (CV) systems, for which very little is known about methods not requiring a full

characterization of measurement devices. A fully DI approach is complex because of the difficulty of finding useful Bell tests for continuous-variable states. For instance, in the Gaussian regime, which is the most feasible experimentally, DI entanglement detection is impossible because no Bell inequality can be violated [7], hence intermediate approaches are necessary. The main goal of this work is to provide methods for MDI entanglement detection in CV systems. We first demonstrate that, in principle, all entangled states can be detected in this scenario. Then, we describe a MDI protocol that can detect the entanglement of all two-mode Gaussian states. Our protocol only relies on the use of trusted, well-calibrated, sources of coherent states, the easiest to prepare in the lab.

An entanglement detection scenario where two parties, Alice and Bob, do not assume a particular description of their measurement but use trusted sources of states was first introduced by Buscemi [8]. Namely, let us consider that Alice and Bob can produce states $\psi_{A'}^\mu$ and $\psi_{B'}^\nu$ according to some distributions $P_{A'}(\mu)$, $P_{B'}(\nu)$, respectively. Alice and Bob can use these states as inputs to their measurement devices, which return outcomes a and b , respectively. These outcomes occur with probability $P(a, b|\psi_{A'}^\mu, \psi_{B'}^\nu) = \text{Tr}[M_{AA'}^a \otimes N_{BB'}^b(\psi_{A'}^\mu \otimes \rho_{AB} \otimes \psi_{B'}^\nu)]$, where $M_{AA'}^a$ and $N_{BB'}^b$ are unknown measurement operators defining a positive-operator valued measure (POVM). The main goal of Alice and Bob is to determine if ρ_{AB} is entangled based on the knowledge of $\psi_{A'}^\mu$, $\psi_{B'}^\nu$, $P_{A'}(\mu)$, $P_{B'}(\nu)$, and $P(a, b|\psi_{A'}^\mu, \psi_{B'}^\nu)$. Besides the calibration issue discussed before, this scenario is motivated by

cryptographic tasks in which Alice and Bob do not trust the provider of the measurement devices they are using [10–13].

For finite dimensional Hilbert spaces, Buscemi has shown that any entangled state ρ_{AB} can be certified in this scenario, but his proof is not constructive [8]. The authors of Ref. [9] have shown how to construct a MDI entanglement witness from standard entanglement witnesses. A different route was considered in Refs. [14,15], where the question was formulated as a convex optimization problem that can be efficiently solved numerically.

In what follows we first generalize the results of Refs. [8,9] and show that the entanglement of every CV entangled state can in principle be detected in a MDI scenario. We then move to the experimentally relevant case of Gaussian states and operations and show a MDI protocol that is able to certify the entanglement of all two-mode Gaussian entangled states. This protocol is feasible with current technology in that it only requires the production of coherent states and the implementation of homodyne measurements. Moreover, our approach provides an interesting connection between MDI entanglement detection and quantum metrology.

Reduction to process tomography.—In this section, we show that it is possible to detect the entanglement of any entangled state in a MDI scenario where Alice and Bob use coherent states as trusted inputs (the proof is presented for two-mode bipartite states but can be generalized to n modes, see Ref. [16]).

Suppose Alice and Bob are in possession of trusted sources producing coherent states $|\alpha\rangle_{A'}$ and $|\beta\rangle_{B'}$, respectively, according to some distribution. The shared entangled state is ρ_{AB} . The systems AA' and BB' are then projected onto respective two-mode squeezed vacuum (TMSV) states, i.e., the measurement $\{|\Phi^{(r)}\rangle\langle\Phi^{(r)}|, \mathbb{1} - |\Phi^{(r)}\rangle\langle\Phi^{(r)}|\}$ is performed on both AA' and BB' (r is the squeezing parameter). Conditioned on α and β , the probability of both measurements obtaining output “1,” corresponding to the projector $|\Phi^{(r)}\rangle\langle\Phi^{(r)}| \equiv \Phi^{(r)}$, can be expressed as

$$\begin{aligned} P_\rho(1, 1|\alpha, \beta) &= \text{Tr}[(\Phi_{AA'}^{(r)} \otimes \Phi_{BB'}^{(r)})(|\alpha\rangle\langle\alpha|_{A'} \otimes \rho_{AB} \otimes |\beta\rangle\langle\beta|_{B'})] \\ &= \text{Tr}[M_{A'B'}^{(r)}|\alpha\rangle\langle\alpha|_{A'} \otimes |\beta\rangle\langle\beta|_{B'}], \end{aligned} \quad (1)$$

where we have defined

$$M_{A'B'}^{(r)} := \text{Tr}_{AB}[(\Phi_{AA'}^{(r)} \otimes \Phi_{BB'}^{(r)})(\rho_{AB} \otimes \mathbb{1}_{A'B'})], \quad (2)$$

which is a POVM element by construction. Our main observation in this section is that nonseparability of the POVM element defined by Eq. (2) is equivalent to the underlying state being entangled. Namely, we have

Proposition 1. For any $r > 0$, the POVM element $M_{A'B'}^{(r)}$ defined by Eq. (2) is entangled if and only if ρ_{AB} is entangled.

Proof 1.—Let us assume ρ_{AB} is separable, i.e.,

$$\rho_{AB} = \sum_{\mu} p_{\mu} \rho_A^{\mu} \otimes \sigma_B^{\mu}. \quad (3)$$

We can then define

$$\begin{aligned} M_{A'}^{(r)\mu} &:= \text{Tr}_A[\Phi_{AA'}^{(r)}(\rho_A^{\mu} \otimes \mathbb{1}_{A'})], \\ N_{B'}^{(r)\mu} &:= \text{Tr}_B[\Phi_{BB'}^{(r)}(\sigma_B^{\mu} \otimes \mathbb{1}_{B'})], \end{aligned} \quad (4)$$

which are POVM elements by construction. It is easy to see that $M_{A'B'}^{(r)} = \sum_{\mu} p_{\mu} M_{A'}^{(r)\mu} \otimes N_{B'}^{(r)\mu}$, which is separable. It remains to be shown that the POVM element defined by Eq. (2), which can be rewritten as [17]

$$M_{A'B'}^{(r)} = (1 - \lambda^2) \lambda^{\hat{n}_A + \hat{n}_B} \rho_{AB}^T \lambda^{\hat{n}_A + \hat{n}_B} \quad (5)$$

(where $\lambda = \tanh r$ and $\hat{n}_X = a_X^\dagger a_X$ the number operator on mode X), is entangled for all entangled ρ_{AB} . In fact, suppose there exists an entanglement witness W such that $\text{Tr}[\rho W] < 0$ while $\text{Tr}[\rho' W] \geq 0$ for any ρ' separable [18]. From W we can obtain a Hermitian operator \tilde{W} such that $\text{Tr}[M_{A'B'}^{(r)} \tilde{W}] < 0$, whereas for any separable POVM $\text{Tr}[\sum_{\mu} p_{\mu} (M_{A'}^{\mu} \otimes N_{B'}^{\mu}) \tilde{W}] \geq 0$. Consider in fact

$$\tilde{W} = \lambda^{-\hat{n}_A - \hat{n}_B} W^T \lambda^{-\hat{n}_A - \hat{n}_B}. \quad (6)$$

It is then easy to see that

$$\text{Tr}[M_{A'B'}^{(r)} \tilde{W}] = (1 - \lambda^2) \text{Tr}[\rho W] < 0. \quad (7)$$

For separable POVMs, on the other hand, it holds

$$\sum_{\mu} p_{\mu} \text{Tr}[\tilde{W} (M_A^{\mu} \otimes N_B^{\mu})] = \sum_{\mu} p_{\mu} \text{Tr}[W (\tilde{M}_A^{\mu} \otimes \tilde{N}_B^{\mu})], \quad (8)$$

where $\tilde{M}_A^{\mu} = \lambda^{\hat{n}_A} (M_A^{\mu})^T \lambda^{\hat{n}_A}$, $\tilde{N}_B^{\mu} = \lambda^{\hat{n}_B} (N_B^{\mu})^T \lambda^{\hat{n}_B}$. The operators \tilde{M}_A^{μ} and \tilde{N}_B^{μ} are manifestly positive semidefinite, meaning that under a proper renormalization they can be seen as states, thus generating a separable state ρ' such that $\text{Tr}[\rho' W] \geq 0$. This implies

$$\sum_{\mu} p_{\mu} \text{Tr}[W (\tilde{M}_A^{\mu} \otimes \tilde{M}_B^{\mu})] \geq 0, \quad (9)$$

which finishes the proof. \blacksquare

We show in Ref. [16] that the violation of the derived witness Eq. (7) scales as $1/N$, where N is the energy scale (number of photons) defined by the original witness, which

is to be compared with the $1/d$ scaling found in Ref. [9] (d being the Hilbert space dimension of ρ_{AB}).

As a consequence of Proposition 1, Alice and Bob can certify the entanglement of ρ_{AB} in a MDI way, if their output statistics allow them to fully reconstruct the POVM element $M_{A'B'}^{(r)}$. As in the case of discrete variables [14], this can be achieved by means of process tomography [21,22]. The set of all coherent states form a tomographically complete set via the Glauber-Sudarshan P representation [23,24]. Further it has been shown that discrete sets of coherent states can form tomographically complete sets [25,26]. As a special case of process tomography with coherent states, POVMs can be fully reconstructed by their output statistics [27–29]. Once Alice and Bob have reconstructed $M_{A'B'}^{(r)}$, they can determine whether it is nonseparable using an entanglement criterion. We also note that, for a given entangled state, if the witness W is known, all that is necessary is to evaluate $\text{Tr}[M_{A'B'}^{(r)} \tilde{W}]$, which might not require full tomography. In summary, we have the following:

Corollary 1. For every entangled state ρ_{AB} , if $|\alpha\rangle$ and $|\beta\rangle$ are chosen from tomographically complete sets, Alice and Bob can certify the entanglement of ρ_{AB} in a measurement-device-independent way.

The results presented in this section suffer from practical problems in their realization: first, they rely on performing the POVM that projects on the two-mode squeezed states defined in Eq. (1). A typical scheme for such measurement involves photodetection [16], which typically has low efficiency and high cost. Second, the full tomography could be in general experimentally inefficient. Therefore, the previous proof is mostly a proof-of-principle result. Next, we show that feasible schemes for MDI entanglement detection are possible. In fact, we propose an experimentally friendly MDI entanglement detection protocol which is based solely on homodyne measurements and can detect all two-mode Gaussian entangled states.

MDI entanglement witness for all two-mode Gaussian states.—In this section we present a practical method for MDI entanglement certification of Gaussian states that can be implemented using readily available optical components. Our method is inspired by the entanglement witness (EW) introduced in a seminal paper by Duan *et al.* [30] (see also Simon [31]). In that work, it was proven that the inequality

$$\langle \text{EW}_\kappa \rangle \equiv \langle \Delta^2 \hat{u}_\kappa \rangle + \langle \Delta^2 \hat{v}_\kappa \rangle \geq \frac{\kappa^2 + \kappa^{-2}}{2}, \quad (10)$$

where $\langle \Delta^2 \hat{O} \rangle$ is the variance of the operator \hat{O} , and

$$\hat{u}_\kappa = \left(\kappa \hat{x}_A - \frac{\hat{x}_B}{\kappa} \right), \quad \hat{v}_\kappa = \left(\kappa \hat{p}_A + \frac{\hat{p}_B}{\kappa} \right), \quad (11)$$

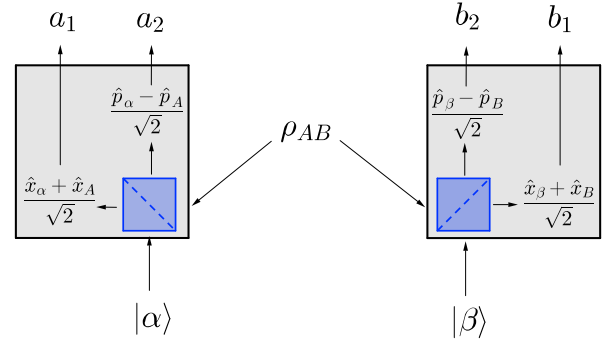


FIG. 1. Experimental setup for the MDI entanglement detection of a two-mode state. Fiduciary coherent states are prepared by the parties and measured together with the corresponding subsystems of the unknown state ρ_{AB} . To compute the bound of the entanglement witness (13), measurements should be seen as uncharacterized black boxes producing the outputs. To obtain a violation, the following specific measurements are implemented: the coherent states are mixed with the respective modes of ρ_{AB} in a 50:50 beam splitter and homodyne measurements of \hat{x} and \hat{p} are performed on the two outputs.

(i) holds for any two-mode separable state, real number κ , where $\kappa > 0$ without loss of generality, and pairs of orthogonal quadratures of the bosonic modes A and B [32], while (ii) for any entangled Gaussian state there exist a value of κ and pairs of quadratures such that Eq. (10) is violated.

Our main result is an experimentally friendly method for MDI entanglement detection inspired by the witness (10) and given by the following proposition:

Proposition 2. Let $|\alpha\rangle$ and $|\beta\rangle$ be coherent states prepared by Alice and Bob according to the Gaussian probability distribution

$$P(\alpha) = \frac{1}{\pi\sigma^2} e^{-|\alpha|^2/\sigma^2} \quad \alpha \equiv \alpha_x + i\alpha_p \quad (12)$$

(for different choices of input distribution, see Ref. [16]). Consider the setup in Fig. 1 in which uncharacterized local measurements are applied jointly on these states and half of an unknown state ρ_{AB} , producing as a result two real numbers (a_1, a_2) for Alice and (b_1, b_2) for Bob. For all local measurements and all separable states one has

$$\langle \text{MDIEW}_\kappa \rangle \equiv \langle U_\kappa^2 \rangle + \langle V_\kappa^2 \rangle \geq \frac{\kappa^2 + \kappa^{-2}}{2} \frac{\sigma^2}{1 + \sigma^2}, \quad (13)$$

where U_κ and V_κ are

$$U_\kappa \equiv \kappa a_1 - \frac{b_1}{\kappa} - \frac{\kappa \alpha_x - \beta_x}{\sqrt{2}},$$

$$V_\kappa \equiv \kappa a_2 + \frac{b_2}{\kappa} - \frac{\kappa \alpha_p + \beta_p}{\sqrt{2}}. \quad (14)$$

For any two-mode entangled Gaussian state, there exist local measurements acting jointly on the state and the input coherent states violating inequality (13).

Looking at its definition, the operational meaning of the witness is clear: Alice and Bob results should be such that their difference and sum, weighted by κ , are as close as possible to the same difference and sum of the quadratures of the coherent states, divided by $\sqrt{2}$. Note also that expectation values in (13) are computed with respect to the quantum state and the distribution of coherent states.

Proof of (13).—To prove the inequality we need to minimize the value of the witness over all separable states $\rho_{AB} = \sum_i p_i \rho_A^{(i)} \otimes \rho_B^{(i)}$. We can restrict the analysis to product states because the witness is linear on the state. It follows that the output probability factorizes

$$p(a_1, a_2, b_1, b_2 | \alpha, \beta) = \text{Tr}[M_{a_1, a_2}^A | \alpha \rangle \langle \alpha |] \text{Tr}[M_{b_1, b_2}^B | \beta \rangle \langle \beta |], \quad (15)$$

and we are left with two independent POVMs on the input states to be optimized. Using $\langle U_\kappa^2 \rangle + \langle V_\kappa^2 \rangle \geq \langle \Delta^2 U_\kappa \rangle + \langle \Delta^2 V_\kappa \rangle$ and the fact that the distribution is completely factorized between the two sides it follows that the value of the witness is lower bounded by the minimum of

$$\frac{1}{2} \left(\kappa^2 + \frac{1}{\kappa^2} \right) (\langle \Delta^2 [\sqrt{2}a_1 - \alpha_x] \rangle + \langle \Delta^2 [\sqrt{2}a_2 - \alpha_p] \rangle). \quad (16)$$

That is, in the absence of correlations, the best Alice and Bob can do is to separately perform the optimal measurements to estimate the input coherent states.

Minimizing the expression in the second parenthesis in (16) looks essentially like a metrology problem. A lower bound, in turn, can be obtained using a multiparameter Bayesian version of the quantum Cramér-Rao bound [34]. The simultaneous estimation of the position and momentum quadratures has been studied thoroughly and is optimized for coherent states by measuring \hat{x} and \hat{p} on two different modes after a 50:50 beam splitter [35,36]. In particular, assuming a Gaussian prior distribution, like in our case, the minimal sum of variances is equal to $\sigma^2/(1 + \sigma^2)$ [35], which proves the bound for the entanglement witness. ■

To prove the violation claimed in Proposition 2, it suffices to show that for any entangled two-mode Gaussian state there exist local measurements and values of (κ, σ) that lead to it. Consider now the optical setup depicted in Fig. 1. Alice and Bob, upon receiving their respective subsystems of ρ_{AB} , first mix them with local coherent states in a balanced beam splitter, and then measure the position and momentum quadratures in the output ports. The output observables are thus

$$\begin{aligned} \hat{A}_1 &= \frac{\hat{x}_\alpha + \hat{x}_A}{\sqrt{2}}, & \hat{A}_2 &= \frac{\hat{p}_\alpha - \hat{p}_A}{\sqrt{2}}, \\ \hat{B}_1 &= \frac{\hat{x}_\beta + \hat{x}_B}{\sqrt{2}}, & \hat{B}_2 &= \frac{\hat{p}_\beta - \hat{p}_B}{\sqrt{2}}. \end{aligned} \quad (17)$$

The quadratures $\hat{x}_A, \hat{p}_A, \hat{x}_B,$ and \hat{p}_B are those used in the standard witness (10). Observables (17) are used to define the measurement outputs needed for the computation of our MDI witness (13). More precisely, consider first the case in which the average values of the state's quadratures are null, $\langle \hat{x}_A \rangle = \langle \hat{p}_A \rangle = \langle \hat{x}_B \rangle = \langle \hat{p}_B \rangle = 0$. Then, by taking (a_1, a_2, b_1, b_2) equal to the statistical output of $(\hat{A}_1, \hat{A}_2, \hat{B}_1, \hat{B}_2)$, respectively, it follows by substituting in (15) [37]

$$\begin{aligned} \langle U_\kappa^2 \rangle &= \left\langle \left(\kappa \frac{\hat{x}_\alpha + \hat{x}_A}{\sqrt{2}} - \frac{1}{\kappa} \frac{\hat{x}_\beta + \hat{x}_B}{\sqrt{2}} - \frac{\kappa \alpha_x - \frac{\beta_x}{\kappa}}{\sqrt{2}} \right)^2 \right\rangle \\ &= \frac{1}{2} \left(\kappa^2 \langle \Delta^2 \hat{x}_\alpha \rangle + \frac{\langle \Delta^2 \hat{x}_\beta \rangle}{\kappa^2} + \langle \Delta^2 \hat{u}_\kappa \rangle \right) \\ &= \frac{1}{2} \left(\frac{\kappa^2 + \kappa^{-2}}{4} + \langle \Delta^2 \hat{u}_\kappa \rangle \right). \end{aligned} \quad (18)$$

Similarly, $\langle V_\kappa^2 \rangle = \frac{1}{2} \{ (\kappa^2 + \kappa^{-2})/4 + \langle \Delta^2 \hat{v}_\kappa \rangle \}$, and consequently we find that in the proposed scheme

$$\langle \text{MDIEW}_\kappa \rangle = \frac{1}{2} \left(\frac{\kappa^2 + \kappa^{-2}}{2} + \langle \text{EW}_\kappa \rangle \right). \quad (19)$$

The generalization to states that have nonzero averages of the quadratures is obtained by simply offsetting the outputs accordingly as follows: a_1 is the output of $\hat{A}_1 - \langle \hat{x}_A \rangle / \sqrt{2}$, a_2 of $\hat{A}_2 + \langle \hat{p}_A \rangle / \sqrt{2}$ and so on.

The violation of the inequality (13) is therefore found for any entangled Gaussian state: indeed, for any such state there exist quadratures and a value of κ such that $\langle \text{EW}_\kappa \rangle < [(\kappa^2 + \kappa^{-2})/2]$, which implies from Eq. (19) that in the proposed scheme $\langle \text{MDIEW}_\kappa \rangle < [(\kappa^2 + \kappa^{-2})/2]$. It is then sufficient to choose σ large enough to violate (13).

Two-mode squeezed state case, with noise tolerance.—At last, to illustrate the feasibility of our scheme, we apply it to the case of ρ_{AB} being a TMSV state. By including noise tolerance, we pave the way for an experimental realization of our MDI entanglement witness.

A TMSV state can be described as the mixing of two single-mode squeezed states (one squeezed in \hat{p} and one in \hat{x}) [33]. In the Heisenberg picture, this results in

$$\begin{aligned} \hat{x}_A &= \frac{e^r \hat{x}_1^{(0)} + e^{-r} \hat{x}_2^{(0)}}{\sqrt{2}}, & \hat{p}_A &= \frac{e^{-r} \hat{p}_1^{(0)} + e^r \hat{p}_2^{(0)}}{\sqrt{2}}, \\ \hat{x}_B &= \frac{e^r \hat{x}_1^{(0)} - e^{-r} \hat{x}_2^{(0)}}{\sqrt{2}}, & \hat{p}_B &= \frac{e^{-r} \hat{p}_1^{(0)} - e^r \hat{p}_2^{(0)}}{\sqrt{2}}, \end{aligned} \quad (20)$$

where the superscript $\{\hat{x}^{(0)}, \hat{p}^{(0)}\}$ represents operators acting on the vacuum. Consider now the two operators $\hat{u}_{\kappa=1} = \hat{x}_A - \hat{x}_B$ and $\hat{v}_{\kappa=1} = \hat{p}_A + \hat{p}_B$. From Eq. (10) we see, by choosing $\kappa = 1$, that these operators satisfy for any separable state $\langle \Delta^2 \hat{u}_{\kappa=1} \rangle + \langle \Delta^2 \hat{v}_{\kappa=1} \rangle \geq 1$. If we compute the above combination for the two-mode squeezed state, we obtain $\hat{u}_{\kappa=1} = \sqrt{2}e^{-r}\hat{x}_2^{(0)}$ and $\hat{v}_{\kappa=1} = \sqrt{2}e^{-r}\hat{p}_1^{(0)}$. Consequently, it holds

$$\langle \text{EW}_{\kappa=1} \rangle_{\text{TMSV}} = e^{-2r} < 1. \quad (21)$$

This is not surprising: as soon as there is squeezing $r > 0$ the state is entangled. Substituting this value into expression (13), we obtain the entangled score for the MDI witness $\langle \text{MDIEW}_{\kappa=1} \rangle = \frac{1}{2}(1 + e^{-2r})$.

To check the noise tolerance, we consider losses in the modes A and B modeled as a beam splitter

$$\begin{aligned} \hat{a}_A(\eta_A) &= \sqrt{1-\eta_A}\hat{a}_A(0) + \sqrt{\eta_A}\hat{a}_{NA}^{(0)}, \\ \hat{a}_B(\eta_B) &= \sqrt{1-\eta_B}\hat{a}_B(0) + \sqrt{\eta_B}\hat{a}_{NB}^{(0)}, \end{aligned} \quad (22)$$

where $\hat{a}_{NX}^{(0)}$ is a vacuum mode acting as a noise on mode X , while $\hat{a}_X(0)$ is the corresponding noiseless mode. We focus on a natural scenario in which the source producing the two-mode squeezed state is between Alice and Bob and losses affect the two modes, not necessarily in a symmetric way. At the same time, the same loss noise [Eq. (22)] applied to the input coherent states would lead only to a renormalization of α and β and can be compensated by increasing the variance σ . Using Eqs. (20) and (22) we can accordingly compute

$$\begin{aligned} \langle \text{EW}_{\kappa} \rangle_{\text{TMSV}, \eta_A, \eta_B} &= \left\langle \Delta^2 \left(\kappa \hat{x}_A(\eta_A) - \frac{\hat{x}_B(\eta_B)}{\kappa} \right) + \Delta^2 \left(\kappa \hat{p}_A(\eta_A) + \frac{\hat{p}_B(\eta_B)}{\kappa} \right) \right\rangle \\ &= \frac{1}{2} \left(\kappa^2 \eta_A + \frac{\eta_B}{\kappa^2} \right) + \frac{e^{2r}}{4} \left(\kappa \sqrt{1-\eta_A} - \frac{\sqrt{1-\eta_B}}{\kappa} \right)^2 + \frac{e^{-2r}}{4} \left(\kappa \sqrt{1-\eta_A} + \frac{\sqrt{1-\eta_B}}{\kappa} \right)^2. \end{aligned} \quad (23)$$

Notice that it is always possible to choose κ such that $\kappa \sqrt{1-\eta_A} - \kappa^{-1} \sqrt{1-\eta_B} = 0$ and r big enough to nullify the last term of Eq. (23), and obtain a score (19)

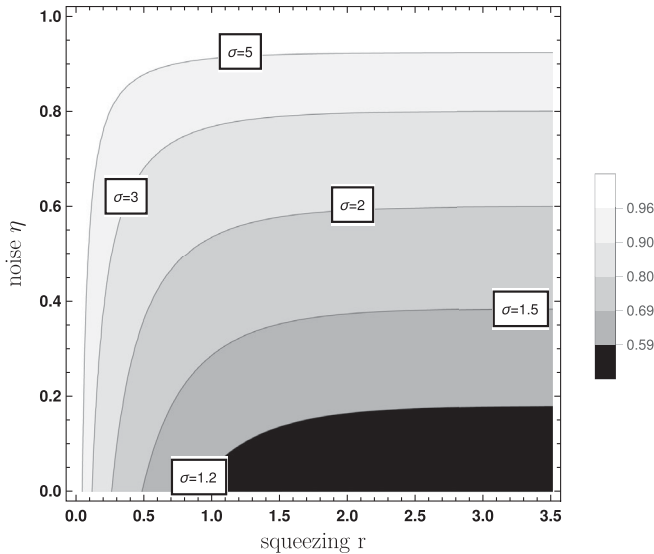


FIG. 2. Contour plot of the obtainable value (19) of $\langle \text{MDIEW}_{\kappa=1} \rangle$, for a two-mode squeezed vacuum state with squeezing parameter r , under the presence of losses with parameter $\eta_A = \eta_B \equiv \eta$ [cf. (23)]. The contours are chosen to match the separable bound (13) for different values of σ , which corresponds to the width of the Gaussian prior used in the experiment. Therefore the area under each σ contour defines the range of parameters for which MDI entanglement is certified.

$\langle \text{MDIEW}_{\kappa} \rangle = \frac{1}{4}(\kappa^2(\eta_A^2 + 1) + (\eta_B^2 + 1)\kappa^{-2})$, which is lower than the separable bound $\frac{1}{2}(\kappa^2 + \kappa^{-2})[\sigma^2/(1 + \sigma^2)]$ for large enough σ . In this sense the entanglement witness we analyzed here is loss resistant. In Fig. 2 we plot the trade-off between noise, entanglement, and variance of the prior distributions σ for the MDI detection of entanglement in the case of symmetric losses ($\eta_A = \eta_B$, $\kappa = 1$).

Discussion.—In this work we have promoted the task of measurement-device-independent entanglement certification to the continuous-variable regime. We first generalized the results of Refs. [8,9] and proved that all continuous-variable entangled states can in principle be detected in this scenario. Then, we showed a simple MDI test able to detect the entanglement of all two-mode Gaussian entangled states. Most importantly, the test only relies on the preparation of coherent states and uses standard experimental setups, thus being readily available with current technology.

Our work also opens up a series of interesting directions. From a general perspective, our work opens the path to the use of CV quantum systems for MDI tasks beyond entanglement detection, such as randomness generation or secure communication. Another possible research direction would be to investigate the possible generality of the connection between entanglement detection and metrology exploited here. In particular, can all MDI entanglement tests be translated into a parameter estimation problem? We are therefore confident that our results will motivate further studies in the field of quantum information with continuous variables.

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- [16] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.126.190502> for technical details and generalizations of the results in the main text.
- [17] Here we used the well-known decomposition of two-mode squeezed states in the Fock basis
- $$|\Phi^{(r)}\rangle = \sqrt{1 - \tanh^2 r} \sum_{i=0}^{\infty} (\tanh r)^i |ii\rangle.$$
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- [32] We use here a notation similar to Ref. [33]. In particular the quadratures are defined as

$$\hat{a} = \hat{x} + i\hat{p}, \quad \hat{a}^\dagger = \hat{x} - i\hat{p},$$

such that

$$[\hat{x}, \hat{p}] = \frac{1}{4i} [\hat{a} + \hat{a}^\dagger, \hat{a} - \hat{a}^\dagger] = \frac{i}{2}.$$

Note for example that the variances of the quadratures will have a factor $1/2$ with respect to the normalization choice of Ref. [30].

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