Non-Fermi Liquids in Conducting Two-Dimensional Networks

Jongjun M. Lee,[1,5,*] Masaki Oshikawa,[2,3,4,†] and Gil Young Cho[1,5,‡]
[1]Department of Physics, Pohang University of Science and Technology (POSTECH), Pohang 37673, Republic of Korea
[2]Institute for Solid State Physics, The University of Tokyo, Kashiwa 277-8581, Japan
[3]Kavli Institute for the Physics and Mathematics of the Universe, Kashiwa 277-8583, Japan
[4]Trans-scale Quantum Science Institute, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan
[5]Center for Artificial Low Dimensional Electronic Systems, Institute for Basic Science (IBS), Pohang 37673, Korea

(Received 1 October 2020; accepted 1 April 2021; published 6 May 2021)

We explore the physics of novel fermion liquids emerging from conducting networks, where 1D metallic wires form a periodic 2D superstructure. Such structure naturally appears in marginally twisted bilayer graphenes, moire transition metal dichalcogenides, and also in some charge-density wave materials. For these network systems, we theoretically show that a remarkably wide variety of new non-Fermi liquids emerge and that these non-Fermi liquids can be *classified* by the characteristics of the junctions in networks. Using this, we calculate the electric conductivity of the non-Fermi liquids as a function of temperature, which show markedly different scaling behaviors than a regular 2D Fermi liquid.

DOI: 10.1103/PhysRevLett.126.186601

Introduction.—The ubiquity of Fermi liquids [1] makes it particularly interesting to find and elucidate systems in which the Fermi liquid theory breaks down, namely, "non-Fermi liquid" (NFL) behavior. Experimentally, one of the signatures of NFL is the resistivity exponent α defined as the resistivity

$$\rho_{xx}(T) = \rho_0 + cT^{\alpha}, \tag{1}$$

as a function of the temperature T. The Fermi liquid is characterized by $\alpha = 2$. On the other hand, various values of the exponent $\alpha \neq 2$ have been observed experimentally in strongly correlated electron systems, indicating NFL. However, a controlled theoretical description of NFLs in $d \geq 2$ dimensions remains a challenging problem [2–4]. In this Letter, we propose a simple theoretical model of NFL in $d \geq 2$ dimensions at finite temperatures, including a "strange insulator" with a negative exponent α [5–7], in terms of a network made of 1D conducting segments.

While our theory is specific to network superstructures, such systems appear in a surprisingly wide variety of materials. To name only a few, marginally twisted bilayer graphene [8–10], moire transition metal dichalcogenides [11–13], helium atoms absorbed on graphene [14], and certain charge-density wave materials [15–17] show such superstructures. The possibility of engineering a network in ultracold gas experiments is discussed in the Supplemental Material [18]. A series of intriguing many-body phenomena have been observed in these systems, including superconductivity [17,19–21,36–38] and metal-insulator transition [38,39]. Indeed, the NFL behavior with the resistivity exponent varying with pressure or gate voltage was observed in $1T$-$TiSe_2$ [17,21].

Motivated by these observations, we will study the electric conduction through the network superstructure. The electronic properties of conducting networks have been studied theoretically in various contexts. For instance, our previous works [15,40] have shown that $1T$-TaS_2 in nearly commensurate charge-density wave states hosts a conducting honeycomb network via STM [15] and that the network supports a cascade of *anomalously stable* flat bands, which can explain unusual enhancement of the superconductivity [40] and higher-order topology [40,41]. Also the network systems have received some attention in connection with the phenomenology of the magic-angle graphene and Chalker-Coddington physics [42–51]. However, systematic investigation of electric conduction through the network in the presence of electron interaction has been largely lacking (see also phenomenological discussions in [22,51]). In this Letter, we take a first step toward elucidating universal NFL behaviors in networks.

First, generalizing the Landauer-Büttiker approach, we can naturally derive the macroscopic Pouillet's law so that the conduction of the entire network is characterized by the conductivity of a single junction. Furthermore, by including the effects of the electron-electron interactions, we find a remarkably broad set of NFL behaviors emerging naturally in the conducting network systems. This originates from the Tomonaga-Luttinger liquid (TLL) nature of the 1D segments of interacting electrons. We will explain when and why NFL behaviors are expected. Furthermore, we will argue that these NFLs can be *one-to-one* matched with the characteristics of the junctions, i.e., the "boundary conditions" for electrons at the junctions. As a consequence, the resistivity exponent α of the network is determined by the Luttinger parameter K, which describes

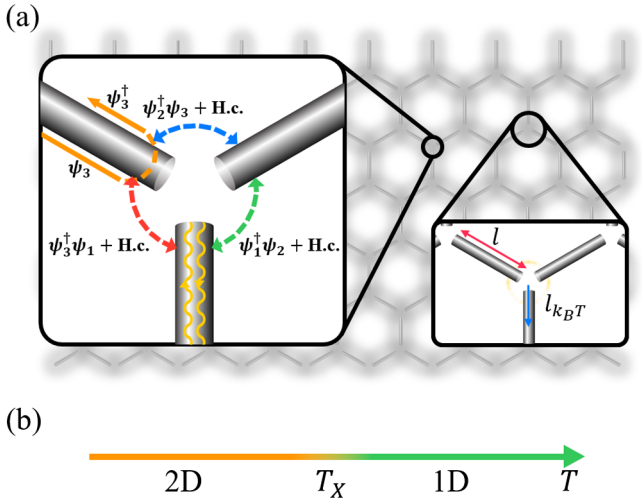

FIG. 1. (a) Schematic picture of honeycomb network. Each link is a TLL [Eq. (2)]. Left: shows a possible $H_{Y;(a,b,c)}$ around a single junction. Right: represents the scales of the problem in our RG process. Here l_{k_BT} is the thermal coherence length at temperature T and $l_{k_BT} < l$ for $T > T_X$, where T_X is the crossover temperature. (b) Phase diagram of the network in temperature.

each 1D segment, potentially explaining the intriguing variation of the resistivity exponent observed in the experiments [17,21].

Model.—In this Letter, as an example, we will mainly consider the minimal honeycomb network model, which consists of 1D segments of interacting spinless electrons as in Fig. 1(a). The network is analyzed in terms of the renormalization group (RG), starting from the microscopic energy scale (such as the bandwidth W of the 1D wire). The RG transformation should be terminated at the energy scale given by the temperature T. At sufficiently low temperatures $T \ll W$, the interacting electrons in the 1D segment of length l can be described as a TLL characterized by a Luttinger parameter K and the (renormalized) velocity v_F,

$$H_{\mathrm{TLL};a} = \frac{v_F}{2}\int_0^l dx\left[K(\partial_x\phi_a)^2 + \frac{1}{K}(\partial_x\theta_a)^2\right], \quad (2)$$

where a is the index labeling each 1D segment. Here v_F and K are different from those of the bare, free electrons, due to interactions between them (see Supplemental Material [18] for our bosonization convention). Here we assume that the electron filling per wire is incommensurate to avoid unnecessary complications and that there is no disorder. The effective Hamiltonian of the whole network reads

$$H = \sum_a H_{\mathrm{TLL};a} + \sum_{\langle a,b,c\rangle} H_{Y;(a,b,c)}, \quad (3)$$

where $H_{Y;(a,b,c)}$ is the *local* interactions between the three neighboring wires around the Y junctions, such as the hopping between the wires [see Fig. 1(a)]. Although the precise form of $H_{Y;(a,b,c)}$ depends on microscopic details, we will show that the essential NFL behaviors are independent of them and thus universal. In addition, we will assume coupling of electrons with external environment (typically phonons). However, the result is again independent of its precise form, as we will explain later.

Strictly speaking, our model is based on the standard TLL theory, which applies to systems with only short-range interactions. Nevertheless, the long-range Coulomb interaction between electrons is often screened. Indeed, the TLL behaviors are rather ubiquitous in actual quasi-1D materials, e.g., [52–56]. As for the candidate materials hosting network superstructures, the layered quasi-2D systems, TaS_2 and $TiSe_2$, are metallic [17,19], and thus the screening is expected. For 2D materials such as twisted bilayer graphene, substrates [57] can provide screening of long-range Coulomb interactions. Therefore, our model will describe a wide variety of material realizations of electrons on networks.

Dimensional crossover.—The energy scale ϵ can be translated to the length scale $l_\epsilon \sim \hbar v_F/\epsilon$. Thus, we can introduce the crossover temperature

$$T_X \sim \frac{\hbar v_F}{k_B l}, \quad (4)$$

which is the temperature where the thermal coherence length touches the wire length l [58]. For $T \lesssim T_X$, the electrons can "feel" the finiteness of the wire length and recognize the network as the 2D system. The physics of this regime is essentially two dimensional. On the other hand, above the crossover temperature, $T \gtrsim T_X$, we are probing the system at the length scale shorter than the segment length l. Thus the system does not "know" that the wires form a 2D network, and the physics is largely governed by the properties of 1D wires and 0D junctions. Based on these observations, we can draw a "phase diagram" in T [Fig. 1(b)]. In real materials, we roughly estimate $T_X \gtrsim \mathcal{O}(90)$ K in 1T-TaS_2 and $T_X \gtrsim O(50)$ K in twisted bilayer graphenes (for $l = 140$ nm, though it is tunable). For 1T-$TiSe_2$, we estimate $T_X \gtrsim \mathcal{O}(6)$ K with some assumptions. For the details and possible implications in existing experimental data, see the Supplemental Material [18].

We compare our crossover temperature [Eq. (4)] of the network with that of coupled wire systems or sling LLs [59–69]. There, the wires are aligned along the same directions, and we assume only the short-ranged terms. Then the crossover temperature toward a regular 2D FL from the 1D TLL limit depends on both the intra- and interwire terms [59–65]

$$T_X^{\text{coupled wires}} \sim t_\perp \cdot \left(\frac{t_\perp}{W}\right)^{\frac{1}{2-\Delta(K)}}, \quad (5)$$

where $t_\perp$ is the strength of the interwire hopping, and $\Delta(K)$ is its scaling dimension.

In this Letter, we focus on the electric conduction in the network in the "1D TLL physics" regime

$$T_X \lesssim T \ll W, \tag{6}$$

which has not been much explored. Since each segment is described by a scale-invariant TLL, the temperature dependence of the transport can be associated with the properties of the junctions. Hence, to study the network in this regime, we can utilize the RG analysis of junctions of TLLs [23,24,58,70,71]. Indeed, we find that the emergent NFLs in the network can be characterized by the RG fixed points of a single junction of TLLs. We will show that the RG fixed points of the junctions determine not only the leading dc conductivities but also leading scaling corrections, which are power law in temperature T. Namely, the NFL behavior is universal in the 1D regime (6) where the junction is described by a RG fixed point, which pins down the scaling dimensions of all the possible perturbations. Below we will explicitly illustrate this for the two simplest fixed points [24,71], namely, "disconnected fixed point" ($K > 1$) and "connected fixed points." As our terminology suggests, the connected fixed point gives rise to the maximum conductances between the wires [24]. Some properties and the stability of these fixed points are reviewed in the Supplemental Material [18].

Conduction through the network.—Here we show that the resistivity of the network is, in general, given by the power law (1) where the exponent α is determined by the Luttinger parameter of the 1D TLL segment and the boundary conditions at the junctions. More precisely, $\alpha = 2\Delta(K) - 2$ where $\Delta(K)$ is the scaling dimension of the leading irrelevant operators at the junctions. The numerical coefficient c depends on microscopic details. For example, for the disconnected fixed point, $\rho_0 = 0$ and

$$\alpha(K) = 2K - 2. \tag{7}$$

To establish this, we first show that the 0D electric *conductance* at a single junction determines the 2D *conductivity* of the whole network. For this, we imagine a perfect 2D network made out of the identical Y junctions [Fig. 1(a)]. We apply a uniform voltage drop across the y direction and then calculate the electric current flowing through the network (Fig. 2). In materials, electrons interact with the external environment, such as phonons. In this Letter, as we consider the fairly high temperatures $T \gtrsim T_X$, we assume that the electron-phonon coherence length is shorter than the segment length l. Then the electrons in the segment equilibrate, so that each segment has a well-defined voltage. Under this assumption, we can immediately compute the conductivity of the whole 2D network out of the "conductance tensor" of a single Y junction.

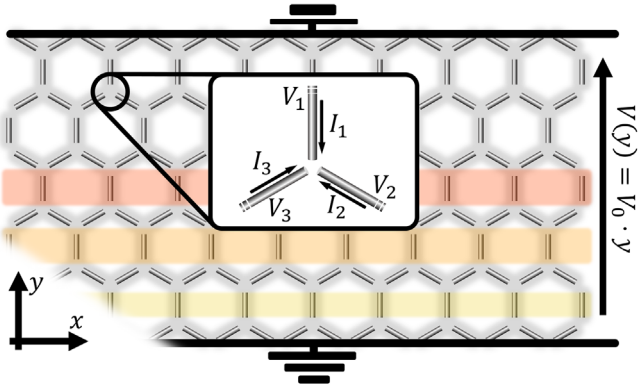

FIG. 2. Conduction through the network. Inset: represents a single Y junction, where voltage and current satisfy $I_a = \sum_b G_{ab} V_b$. The voltage is uniform in x and increases in y, as depicted in the figure. Wires with the regions of the same colors have the same voltage.

Each fixed point of the Y junction can be characterized by its 3×3 conductance tensor G_{ab}, which relates the electric current at the ath wire with the voltage at the bth wire (Fig. 2), i.e., $I_a = \sum_b G_{ab} V_b$. For the spinless fermions, all the known fixed points [24,71] respect the $\mathbb{Z}_3$ permutation symmetry between the three neighboring wires. Hence the conductance tensor can be parametrized only by the two numbers G_S and G_A. Imposing $I_a = \sum_b G_{ab} V_b$ at all the junctions, we can fix the electric current at every wire. From the current, we obtain the conductance of the entire network [18], which is found to be proportional to the width and inversely proportional to the length. (Similar discussions were given in [72,73].) In this way, our network construction leads naturally to the classical Pouillet's law, and the conduction property of the system can be characterized by constant conductivity tensors

$$g_{xx}(T) = \sqrt{3} G_S(T)/4, \qquad g_{xy}(T) = G_A(T)/4. \tag{8}$$

The factors of $1/4$ and $\sqrt{3}/4$ have the geometric origin, e.g., the size of the unit cell. We also checked $g_{xx}(T) = g_{yy}(T)$ and $g_{xy}(T) = -g_{yx}(T)$ [18]. While the classical nature of the conduction is a natural consequence of the assumed local thermalization (and thus decoherence) in each segment, it is remarkable that the macroscopic property (conductivity) is determined by the property of the microscopic junctions, independent of the details of the thermalization. This can be regarded as a generalization of the Landauer-Büttiker approach [74] to extensive macroscopic systems of interacting electrons.

Hence, we next compute G_S and G_A of a Y junction at finite temperatures T. For this, we generalize the results of [23,24,58,71] to the Y junctions. For instance, at the disconnected fixed point, the leading perturbation is the interwire hopping [Fig. 1(a)]

$$H_Y = -t \sum_{j=1,2,3} \psi_j^\dagger(0)\psi_{j-1}(0) + \text{H.c.},$$

where ψ_0 is identified with ψ_3, and $\psi_j(0)$ represents the electron annihilation operator of the jth wire at the Y junction, i.e., $x = 0$. Up to the second order in t, we find[18]

$$G_S(T) \approx 0 + \frac{2e^2t^2}{h}(\pi\tau_c)^{2K}\frac{\Gamma(\frac{1}{2})\Gamma(K)}{\Gamma(\frac{1}{2}-K)}T^{2K-2}, \qquad (9)$$

with τ_c being the inverse of a UV cutoff W of Eq. (2). The first term 0 in G_S is the universal conductance of the disconnected fixed point [24,71]. The second term $\sim T^{2K-2}$ represents the correction from the leading irrelevant perturbation H_Y above. They are evaluated within the standard linear response theory combined with the perturbative expansion in H_Y [18,23,58]. The correction $\sim T^{2K-2}$ is consistent with Eq. (1), because the scaling dimension of H_Y is $\Delta(K) = K$. As we have seen in Eq. (8), this conductance of the single junction directly gives the conductivity of the 2D network and the resistivity exponent is given as in Eq. (7). A fixed point is stable as far as the scaling dimension of the perturbation is larger than 1. Hence the disconnected fixed point is stable for $\Delta(K) = K > 1$. Furthermore, this disconnected fixed point is the only known stable fixed point for repulsive interactions [24,71], and so this strange insulator behavior with the power-law divergence of the resistivity at low temperatures is generic.

For attractive interaction $K < 1$, the interwire tunneling is a relevant perturbation. Equation (9) could describe the conductivity at higher temperatures if the interwire coupling at the junction in the microscopic model is weak. In this case, the same expression (9) now describes the power law with a positive resistivity exponent (7), namely, the typical behavior of a *metallic* NFL. When the attractive interaction is sufficiently strong so that $K < 1/3$, as the temperature is lowered, the junction is governed by the connected fixed point. This fixed point is stable for $K < 1/3$ because the scaling dimension of the leading irrelevant operators at this fixed point is $1/3K$ [18,24]. Taking the operators into account, we can again evaluate the conductance within the standard linear response theory [18,23,58]

$$G_S(T) \approx \frac{2}{3K}\frac{e^2}{h} - C(T^{(2/3K)-2}), \qquad (10)$$

where we suppressed all the unimportant constants into C. As before, the first term is the universal conductance of the connected fixed point [24,71], and the second term $\sim T^{2/(3K)-2}$ is the correction from the perturbative expansion of the leading irrelevant operators (see Supplemental Material [18]). If the interwire coupling at the junction is strong, Eq. (10) would describe the entire temperature range where the 1D TLL description is valid. This also gives the metallic NFL behavior with the decreasing resistivity at lower temperatures, but with the resistivity exponent $\alpha = 2/(3K) - 2 > 0$.

For both the disconnected and connected fixed points, G_A and thus the Hall conductivity g_{xy} vanish, as expected from the time-reversal invariance of the underlying model. On the other hand, for the network of $1/3 < K < 1$ under a uniform magnetic field, the chiral fixed point of $G_A \neq 0$ is stabilized,

$$G_S(T) \approx \frac{4K}{1+3K^2}\frac{e^2}{h}, \qquad G_A(T) \approx \frac{4}{1+3K^2}\frac{e^2}{h}, \qquad (11)$$

up to the perturbative correction $\sim \mathcal{O}(T^{8K/(1+3K^2)-2})$. Such network is metallic and has the "universal" Hall conductivity, which is determined by the Luttinger parameter [18,22]. This result is consistent with [22], which considered the electric conduction across a network consisting of the chiral fixed point. We note that, however, [22] missed much of the NFL physics that we explored here (see the comparison in the Supplemental Material [18]). While the realization of the chiral fixed point requires an explicit breaking of the time-reversal invariance, in principle, the required breaking can be infinitesimal, e.g., by a very weak magnetic field through the junctions [24,71]. This behavior is again quite different from a normal metal, in which the Hall conductivity is proportional to the applied magnetic field. We also note that, for $K > 1$ or $K < 1/3$, the chiral fixed point is unstable. Instead, the connected and disconnected fixed points are stable as discussed above.

Let us give a few remarks on Eqs. (9)–(11), which represent the conductance in the vicinity of three different RG fixed points for the junction. First of all, in all cases, the conductivity of the network exhibits a power law in temperature, whose exponent continuously evolves as the Luttinger parameter K varies. This is the manifestation of the exact marginality of the Luttinger parameter. Essentially, this marginality allows the scaling dimension of electrons vary smoothly, which translates as the continuously changing α in Eq. (7). In experiment, this means that, as the external parameters, e.g., pressure and gating, are tuned, the exponent of the temperature dependence of the resistivity will continuously change. This is markedly different from the behavior of a regular Fermi liquid, where the exactly marginal deformation, i.e., the change of the Fermi velocity, does not alter the temperature dependence of the transport coefficients.

Finally, we comment on possible effects of disorders on transport. There are distinct types of the disorders at different length scales. For instance, microscopic impurities in TLLs will induce a power-law correction $\sim T^{2/K-2}$ to the conductivity [23,58,75], which will add up to those from the junctions. Similarly, randomly missing (completely disconnected) Y junctions will introduce an additional $\sim T^{2K-2}$ correction. Details are given in the Supplemental Material [18].

Conclusions.—We have demonstrated the emergence of a novel class of NFLs in the conducting networks, whose universal properties are controlled by the RG fixed points of the junction of TLLs. This makes our network system a unique theoretical platform, where the isotropic NFL behaviors in $d \geq 2$ dimensions can be deduced from the well-established theory on strongly correlated electrons in 1D. The NFLs we have proposed are potentially already out there [10,17–19,21] in experiments and/or can be easily realized and verified in currently available setups. For instance, one can artificially pattern the network superstructure in experiments [76]. In twisted bilayer graphenes, the crossover temperature T_X, which is related with the length l of the underlying wires, can be controlled by tuning the twisting angle. Hence, in these systems one can look at the dependence of the conductivities on temperature and twisting angles to observe the emergence of the putative NFLs, which will be quite spectacular.

Once the power-law behavior of the resistivity is observed, the Luttinger parameter K for the TLL describing each segment is inferred from the resistivity exponent α. Our scenario can then be verified by a consistency check with an independent determination of the Luttinger parameter of the 1D segment, for example, by angle-resolved photoemission spectroscopy measurement of the local density of states $\sim|\omega|^{(K+1/K)/2-1}$ [77]. The Luttinger parameter and scaling dimensions $\Delta(K)$ of the leading irrelevant operators at the junctions can be also extracted from the specific heat $C_v(T)$ and susceptibility $\chi(T)$. For example, the specific heat has two contributions: one from the 1D TLLs and the other from the junctions [78–80]. The 1D part scales as $\sim T$, but the junction contribution has $\sim T^{2\Delta(K)-2}$ [78–80]; hence $C_v \sim c_1 T + c_2 T^{2\Delta(K)-2}$ in total and similarly $\chi \sim b_1 + b_2 T^{2\Delta(K)-3}$. In the future, it will be interesting to investigate explicitly the effect of the long-range Coulomb interaction on the network NFLs, following the related studies on a single TLL and coupled wires [81–87].

We thank Chenhua Geng, Jung Hoon Han, Jun-Sung Kim, Gil-Ho Lee, Sung-Sik Lee, Jeffrey Teo, and Han Woong Yeom for helpful discussions. J. L. and G. Y. C. are supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1C1C1006048 and No. 2020R1A4A3079707) and also by IBS-R014-D1. This work is supported by the Air Force Office of Scientific Research under Grant No. FA2386-20-1-4029. M. O. is supported in part by MEXT/JSPS KAKENHI Grants No. JP18H03686 and No. JP17H06462, and JST CREST Grant No. JPMJCR19T2. We also thank Claudio Chamon and Dmitry Green for bringing our attention to Ref. [22] and helpful discussions. J. L. was also supported by POSCO Science Fellowship of the POSCO TJ Park Foundation.

*michaelj.lee@postech.ac.kr
†oshikawa@issp.u-tokyo.ac.jp
‡gilyoungcho@postech.ac.kr

[1] P. Nozieres and D. Pines, *Theory of Quantum Liquids*, Advanced Books Classics (Avalon Publishing, Redwood City, 1999), ISBN 9780813346533.
[2] S.-S. Lee, Annu. Rev. Condens. Matter Phys. **9**, 227 (2018).
[3] A. J. Schofield, Contemp. Phys. **40**, 95 (1999).
[4] C. Varma, Z. Nussinov, and W. van Saarloos, Phys. Rep. **361**, 267 (2002).
[5] A. Donos and S. A. Hartnoll, Nat. Phys. **9**, 649 (2013).
[6] A. Donos and J. P. Gauntlett, J. High Energy Phys. 06 (2014) 7.
[7] T. Andrade and A. Krikun, J. High Energy Phys. 05 (2019) 119.
[8] P. Rickhaus, J. Wallbank, S. Slizovskiy, R. Pisoni, H. Overweg, Y. Lee, M. Eich, M.-H. Liu, K. Watanabe, T. Taniguchi *et al.*, Nano Lett. **18**, 6725 (2018).
[9] H. Yoo, R. Engelke, S. Carr, S. Fang, K. Zhang, P. Cazeaux, S. H. Sung, R. Hovden, A. W. Tsen, T. Taniguchi *et al.*, Nat. Mater. **18**, 448 (2019).
[10] S. Xu, A. Berdyugin, P. Kumaravadivel, F. Guinea, R. K. Kumar, D. Bandurin, S. Morozov, W. Kuang, B. Tsim, S. Liu *et al.*, Nat. Commun. **10**, 1 (2019).
[11] Y. Ma, S. Kolekar, H. Coy Diaz, J. Aprojanz, I. Miccoli, C. Tegenkamp, and M. Batzill, ACS Nano **11**, 5130 (2017).
[12] S. Carr, D. Massatt, S. B. Torrisi, P. Cazeaux, M. Luskin, and E. Kaxiras, Phys. Rev. B **98**, 224102 (2018).
[13] A. Weston, Y. Zou, V. Enaldiev, A. Summerfield, N. Clark, V. Zólyomi, A. Graham, C. Yelgel, S. Magorrian, M. Zhou *et al.*, Nat. Nanotechnol. **15**, 592 (2020).
[14] M. Morishita, arXiv:1908.01991.
[15] J. W. Park, G. Y. Cho, J. Lee, and H. W. Yeom, Nat. Commun. **10**, 1 (2019).
[16] A. Spijkerman, J. L. de Boer, A. Meetsma, G. A. Wiegers, and S. van Smaalen, Phys. Rev. B **56**, 13757 (1997).
[17] L. Li, E. Ofarrell, K. Loh, G. Eda, B. Özyilmaz, and A. C. Neto, Nature (London) **529**, 185 (2016).
[18] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.126.186601 for the experimental parameters and detailed review of the junction fixed points, which includes Refs. [10,15,17,19–35].
[19] B. Sipos, A. F. Kusmartseva, A. Akrap, H. Berger, L. Forró, and E. Tutiš, Nat. Mater. **7**, 960 (2008).
[20] C. Chen, L. Su, A. H. Castro Neto, and V. M. Pereira, Phys. Rev. B **99**, 121108(R) (2019).
[21] A. F. Kusmartseva, B. Sipos, H. Berger, L. Forro, and E. Tutiš, Phys. Rev. Lett. **103**, 236401 (2009).
[22] J. Medina, D. Green, and C. Chamon, Phys. Rev. B **87**, 045128 (2013).
[23] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).
[24] M. Oshikawa, C. Chamon, and I. Affleck, J. Stat. Mech. (2006) P02008.
[25] D. Qian, D. Hsieh, L. Wray, E. Morosan, N. L. Wang, Y. Xia, R. J. Cava, and M. Z. Hasan, Phys. Rev. Lett. **98**, 117007 (2007).
[26] S. Yan, D. Iaia, E. Morosan, E. Fradkin, P. Abbamonte, and V. Madhavan, Phys. Rev. Lett. **118**, 106405 (2017).

[27] B. Paredes, A. Widera, V. Murg, O. Mandel, S. Fölling, I. Cirac, G. V. Shlyapnikov, T. W. Hänsch, and I. Bloch, Nature (London) **429**, 277 (2004).

[28] E. Haller, R. Hart, M. J. Mark, J. G. Danzl, L. Reichsöllner, M. Gustavsson, M. Dalmonte, G. Pupillo, and H. C. Nägerl, Nature (London) **466**, 597 (2010).

[29] D. G. Angelakis, M. Huo, E. Kyoseva, and L. C. Kwek, Phys. Rev. Lett. **106**, 153601 (2011).

[30] W. J. Kwon, G. Del Pace, R. Panza, M. Inguscio, W. Zwerger, M. Zaccanti, F. Scazza, and G. Roati, Science **369**, 84 (2020).

[31] T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauß, S. Hild, D. Bellem, U. Schollwöck, T. Giamarchi, C. Gross *et al.*, Nat. Phys. **9**, 235 (2013).

[32] S. Häusler, S. Nakajima, M. Lebrat, D. Husmann, S. Krinner, T. Esslinger, and J.-P. Brantut, Phys. Rev. Lett. **119**, 030403 (2017).

[33] A. Mazurenko, C. S. Chiu, G. Ji, M. F. Parsons, M. Kanász-Nagy, R. Schmidt, F. Grusdt, E. Demler, D. Greif, and M. Greiner, Nature (London) **545**, 462 (2017).

[34] F. Schäfer, T. Fukuhara, S. Sugawa, Y. Takasu, and Y. Takahashi, Nat. Rev. Phys. **2**, 411 (2020).

[35] C. Nayak, M. P. A. Fisher, A. W. W. Ludwig, and H. H. Lin, Phys. Rev. B **59**, 15694 (1999).

[36] Y. Yu, F. Yang, X. F. Lu, Y. J. Yan, Y.-H. Cho, L. Ma, X. Niu, S. Kim, Y.-W. Son, D. Feng *et al.*, Nat. Nanotechnol. **10**, 270 (2015).

[37] Y. Liu, D. F. Shao, L. J. Li, W. J. Lu, X. D. Zhu, P. Tong, R. C. Xiao, L. S. Ling, C. Y. Xi, L. Pi *et al.*, Phys. Rev. B **94**, 045131 (2016).

[38] L. Wang, E.-M. Shih, A. Ghiotto, L. Xian, D. A. Rhodes, C. Tan, M. Claassen, D. M. Kennes, Y. Bai, B. Kim *et al.*, Nat. Mater. **19**, 861 (2020).

[39] P. Fazekas and E. Tosatti, Physica (Amsterdam) **99B+C**, 183 (1980).

[40] J. M. Lee, C. Geng, J. W. Park, M. Oshikawa, S.-S. Lee, H. W. Yeom, and G. Y. Cho, Phys. Rev. Lett. **124**, 137002 (2020).

[41] T. Mizoguchi, M. Maruyama, S. Okada, and Y. Hatsugai, Phys. Rev. Mater. **3**, 114201 (2019).

[42] X.-C. Wu, C.-M. Jian, and C. Xu, Phys. Rev. B **99**, 161405 (R) (2019).

[43] Y.-Z. Chou, Y.-P. Lin, S. Das Sarma, and R. M. Nandkishore, Phys. Rev. B **100**, 115128 (2019).

[44] Y. Cao, V. Fatemi, A. Demir, S. Fang, S. L. Tomarken, J. Y. Luo, J. D. Sanchez-Yamagishi, K. Watanabe, T. Taniguchi, E. Kaxiras *et al.*, Nature (London) **556**, 80 (2018).

[45] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Nature (London) **556**, 43 (2018).

[46] D. K. Efimkin and A. H. MacDonald, Phys. Rev. B **98**, 035404 (2018).

[47] J. Chalker and P. Coddington, J. Phys. C **21**, 2665 (1988).

[48] Y.-Z. Chou, F. Wu, and S. D. Sarma, Phys. Rev. Research **2**, 033271 (2020).

[49] C. De Beule, F. Dominguez, and P. Recher, Phys. Rev. Lett. **125**, 096402 (2020).

[50] G. Ferraz, F. B. Ramos, R. Egger, and R. G. Pereira, Phys. Rev. Lett. **123**, 137202 (2019).

[51] C. Chen, A. H. Castro Neto, and V. M. Pereira, Phys. Rev. B **101**, 165431 (2020).

[52] M. Bockrath, D. H. Cobden, J. Lu, A. G. Rinzler, R. E. Smalley, L. Balents, and P. L. McEuen, Nature (London) **397**, 598 (1999).

[53] Y. Ma, H. C. Diaz, J. Avila, C. Chen, V. Kalappattil, R. Das, M.-H. Phan, T. Čadež, J. M. P. Carmelo, M. C. Asensio *et al.*, Nat. Commun. **8**, 14231 (2017).

[54] N. Y. Kim, P. Recher, W. D. Oliver, Y. Yamamoto, J. Kong, and H. Dai, Phys. Rev. Lett. **99**, 036802 (2007).

[55] Y. Zhen, H. W. Postma, L. Balents, and C. Dekker, Nature (London) **402**, 273 (1999).

[56] S. Zhao, S. Wang, F. Wu, W. Shi, I. B. Utama, T. Lyu, L. Jiang, Y. Su, S. Wang, K. Watanabe *et al.*, Phys. Rev. Lett. **121**, 047702 (2018).

[57] M. Kim, S. G. Xu, A. I. Berdyugin, A. Principi, S. Slizovskiy, N. Xin, P. Kumaravadivel, W. Kuang, M. Hamer, R. Krishna Kumar *et al.*, Nat. Commun. **11**, 2339 (2020).

[58] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).

[59] X. G. Wen, Phys. Rev. B **42**, 6623 (1990).

[60] H. Schulz, Int. J. Mod. Phys. B **05**, 57 (1991).

[61] C. Castellani, C. Di Castro, and W. Metzner, Phys. Rev. Lett. **69**, 1703 (1992).

[62] S. Brazovskii and V. Yakovenko, Zh. Eksp. Teor. Fiz. **89**, 2318 (1985).

[63] C. Bourbonnais and L. Caron, Physica (Amsterdam) **143B+C**, 450 (1986).

[64] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press, Oxford, 2003), Vol. 121.

[65] F. Guinea and G. Zimanyi, Phys. Rev. B **47**, 501 (1993).

[66] S. L. Sondhi and K. Yang, Phys. Rev. B **63**, 054430 (2001).

[67] R. Mukhopadhyay, C. L. Kane, and T. C. Lubensky, Phys. Rev. B **64**, 045120 (2001).

[68] V. J. Emery, E. Fradkin, S. A. Kivelson, and T. C. Lubensky, Phys. Rev. Lett. **85**, 2160 (2000).

[69] A. Vishwanath and D. Carpentier, Phys. Rev. Lett. **86**, 676 (2001).

[70] A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 4631 (1993).

[71] C. Chamon, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. **91**, 206403 (2003).

[72] J. L. D'Amato and H. M. Pastawski, Phys. Rev. B **41**, 7411 (1990).

[73] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).

[74] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, England, 1997).

[75] T. Giamarchi and H. Maurey, arXiv:cond-mat/9608006.

[76] C. Forsythe, X. Zhou, K. Watanabe, T. Taniguchi, A. Pasupathy, P. Moon, M. Koshino, P. Kim, and C. R. Dean, Nat. Nanotechnol. **13**, 566 (2018).

[77] H. Ishii, H. Kataura, H. Shiozawa, H. Yoshioka, H. Otsubo, Y. Takayama, T. Miyahara, S. Suzuki, Y. Achiba, M. Nakatake *et al.*, Nature (London) **426**, 540 (2003).

[78] S. Fujimoto, arXiv:cond-mat/0308046.

[79] S. Fujimoto and S. Eggert, Phys. Rev. Lett. **92**, 037206 (2004).

[80] A. Furusaki and T. Hikihara, Phys. Rev. B **69**, 094429 (2004).

[81] C. Kane, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. **79**, 5086 (1997).
[82] D. W. Wang, A. J. Millis, and S. Das Sarma, Phys. Rev. B: Condens. Matter Mater. Phys. **64**, 193307 (2001).
[83] D. D. Vu, A. Iucci, and S. Das Sarma, Phys. Rev. Research **2**, 023246 (2020).
[84] H. J. Schulz, Phys. Rev. Lett. **71**, 1864 (1993).
[85] S. Sur and K. Yang, Phys. Rev. B **96**, 075131 (2017).
[86] P. Kopietz, V. Meden, and K. Schönhammer, Phys. Rev. Lett. **74**, 2997 (1995).
[87] P. Kopietz, V. Meden, and K. Schönhammer, Phys. Rev. B **56**, 7232 (1997).