Universal AdS Black Holes in Theories with 16 Supercharges and Their Microstates

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We provide a universal microscopic counting for the microstates of the asymptotically AdS black holes and black strings that arise as solutions of the half-maximal gauged supergravity in 4 and 5 dimensions. These solutions can be embedded in all M-theory and type II string backgrounds with an AdS vacuum and 16 supercharges and provide an infinite set of examples dual to $\mathcal{N}=2$ and $\mathcal{N}=4$ conformal field theories in four and three dimensions, respectively. The counting is universal and it is performed by either studying the large N limit of the relevant supersymmetric index of the dual field theory or by using the charged Cardy formula.

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Introduction.—The microscopic counting of black hole microstates is a fundamental question for all theories of quantum gravity. String theory has provided a microscopic explanation for the entropy of a class of asymptotically flat black holes [1]. Despite the AdS/CFT correspondence [2], the analogous question for asymptotically anti–de Sitter (AdS) black holes has remained elusive until recently, except for AdS_3. In the last few years there has been some progress, first for a class of magnetically charged supersymmetric black holes in AdS_4 [3], and later for a class of supersymmetric Kerr-Newman (KN) black holes in AdS_5 [4,5]. The AdS/CFT correspondence provides a nonperturbative definition of quantum gravity in asymptotically AdS space in terms of a dual boundary quantum field theory (QFT) and the black hole microstates appear as particular states in the boundary description. The entropy of a supersymmetric black hole with angular momentum J and a set of conserved electric and magnetic charges is reproduced by counting the states with spin J and the same quantum numbers in the dual QFT. Computations in a strongly coupled supersymmetric QFT are difficult but the numbers of states of interest can be extracted from supersymmetric indices that can be often evaluated using exact nonperturbative techniques. Supersymmetric localization [6], for example, allows one to reduce the indices to matrix models that can be evaluated in a saddle-point approximation when the number of colors N is large, which is the regime where holography applies.

Unfortunately, asymptotically AdS black holes are difficult to find, and not so many 4- and 5-dimensional examples are known besides those that can be embedded in $\mathrm{AdS}_4 \times S^7$ or $\mathrm{AdS}_5 \times S^5$ and some universal examples that arise from embedding minimal gauged supergravity into string compactifications [7–10]. Some progress has been made instead in constructing infinite classes of supersymmetric AdS_2 and AdS_3 solutions that can arise as the near horizon limit of AdS black objects [11,12]. In this Letter, in the spirit of the above-mentioned universal examples, we consider a large class of black holes and black strings that arise as solutions of the half-maximal supergravity in AdS_4 and AdS_5. Such solutions can be embedded in all AdS_4 and AdS_5 type II or M-theory backgrounds with 16 supercharges. Indeed, for any supersymmetric solution of 10- or 11-dimensional supergravity of the warped product form $\mathrm{AdS}_D \times_w M$, there is a consistent truncation to pure gauged supergravity in D dimensions containing that solution and having the same supersymmetry [13–16]. This observation calls for a *universal* large N formula for the number of supersymmetric states in conformal field theories (CFT) with 8 supercharges (16 including conformal supersymmetries). We will indeed perform a universal counting of states in CFTs with 8 supercharges using the superconformal index and the charged Cardy formula. Extrapolating from known results about the large N behavior of the index in various limits, we precisely reproduce the Bekenstein-Hawking entropy of the black objects. We will check the results for the known classes of CFTs with 8 supercharges that admit a holographic dual. We will also provide a conjecture for the R-symmetry charge dependence of the S^3 free energy of 3D CFTs with 8 supercharges. A similar universal computation was done in [7] and in [8–10] for theories with 4 supercharges. For other related universal results see [17–19].

$\mathcal{N}=4$ gauged supergravity in 5 dimensions.—The bosonic part of minimal 5D $\mathcal{N}=4$ gauged supergravity [20,21] consists of the metric $g_{\mu\nu}$, a $\mathrm{U}(1)_R$ gauge field a_μ, an $\mathrm{SU}(2)_R$ Yang-Mills gauge field A^I_μ, $I=1, 2, 3$, two antisymmetric tensor fields $B^\alpha_{\mu\nu}$, $\alpha=4, 5$, and one real scalar ϕ. The fermionic components are four gravitini $\psi_{\mu i}$, $i=1, 2, 3, 4$ and four spin-1/2 fermions χ_i. These fields form the $\mathcal{N}=4$ gauged supergravity multiplet $(g_{\mu\nu}, \psi_{\mu i}, a_\mu, A^I_\mu, B^\alpha_{\mu\nu}, \chi_i, \phi)$. The bosonic Lagrangian is given by

$$\begin{aligned}\frac{1}{\sqrt{-g}}\mathcal{L} = &-\frac{R}{4}+\frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi)+\frac{g_2}{8}(g_2\xi^{-2}+2\sqrt{2}g_1\xi)\\ &-\frac{\xi^{-4}}{4}f^{\mu\nu}f_{\mu\nu}-\frac{\xi^2}{4}(F^{\mu\nu I}F^I_{\mu\nu}+B^{\mu\nu\alpha}B^\alpha_{\mu\nu})\\ &+\frac{1}{4\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma\tau}\left(\frac{1}{g_1}\varepsilon_{\alpha\beta}B^\alpha_{\mu\nu}D_\rho B^\beta_{\sigma\tau}-F^I_{\mu\nu}F^I_{\rho\sigma}a_\tau\right),\end{aligned}$$

where $\xi=\exp(\sqrt{2/3}\phi)$, and the field strengths are $f_\mu=\partial_\nu a_\mu-\partial_\nu a_\mu$, $F^I_{\mu\nu}=\partial_\nu A^I_\mu-\partial_\mu A^I_\nu+g_2\epsilon^{IJK}A^J_\mu A^K_\nu$. The theory with $g_2=\sqrt{2}g_1=2\sqrt{2}$ has an AdS vacuum with radius $\ell_5=1$ that preserves all of the 16 real supercharges.

The universal KN black hole in AdS_5.—There is a universal solution of $\mathcal{N}=4$ gauged supergravity corresponding to a supersymmetric, asymptotically AdS black hole with two electric charges Q_1 and Q_2 under $\mathrm{U}(1)^2\subset \mathrm{U}(1)_R\times\mathrm{SU}(2)_R$ and two angular momenta J_1 and J_2. It can be obtained as a particular case of the KN black holes in $\mathrm{AdS}_5\times S^5$ with angular momenta (J_1, J_2) and electric charges (Q_1, Q_2, Q_2) under the Cartan subgroup of SO(6) [22–26]. The latter are solutions of the $\mathrm{U}(1)^3$ truncation of $\mathrm{AdS}_5\times S^5$ and one can easily check that they become solutions of the minimal $\mathcal{N}=4$ gauged supergravity when two U(1) gauge fields are identified. The entropy can be compactly written as [27]

$$S(J_I, Q_i)=2\pi\sqrt{Q_2^2+2Q_1Q_2-2a(J_1+J_2)}, \quad (1)$$

where $a=\{[\pi\ell_5^3]/[8G_{\mathrm{N}}^{(5)}]\}$, with ℓ_5 being the radius of AdS_5 and $G_{\mathrm{N}}^{(5)}$ the Newton constant is the central charge of the dual $\mathcal{N}=2$ CFT [28] at leading order in N. In order for the black hole to have a smooth horizon, the charges must satisfy the nonlinear constraint

$$\begin{aligned}0=&\,2Q_2(Q_1+Q_2)^2+2a(Q_2^2+2Q_1Q_2)\\ &-2a(J_1+J_2)(Q_1+2Q_2)-2aJ_1J_2-4a^2(J_1+J_2).\end{aligned} \quad (2)$$

The entropy (1) can be written as the constrained Legendre transform of the quantity [29]

$$\log\mathcal{Z}(X_i,\omega_i)=-4\pi\mathrm{i}a\frac{X_1X_2^2}{\omega_1\omega_2}. \quad (3)$$

The entropy is obtained indeed by extremizing

$$S=\log\mathcal{Z}(X_i,\omega_i)-2\pi\mathrm{i}(\omega_1J_1+\omega_2J_2+X_1Q_1+2X_2Q_2),$$

with respect to X_i and ω_i with the constraint $X_1+2X_2-\omega_1-\omega_2=\pm1$. Both signs lead to the same critical value (1) [30], which is real precisely when (2) is satisfied. The solution can be embedded in all AdS_5 type II or M-theory backgrounds preserving 16 supercharges.

Examples of $\mathcal{N}=2$ quiver theories with a holographic dual and central charges of order $\mathcal{O}(N^2)$ are provided by orbifolds of $\mathrm{AdS}_5\times S^5$. Another large class of $\mathcal{N}=2$ theories with a holographic dual can be obtained by compactifying N M5-branes on a Riemann surface $\Sigma_\mathfrak{g}$ of genus $\mathfrak{g}$ with regular punctures [31]. We will refer to these theories as holographic class $\mathcal{S}$ theories. The theories are generically non-Lagrangian and their central charge is of order $\mathcal{O}(N^3)$. On general grounds [14,15], one knows that the effective 5D dual gravitational theory can be consistently truncated to the minimal $\mathcal{N}=4$ gauged supergravity. As an example, one can check explicitly that the universal KN solution can be embedded in the compactification with no punctures, corresponding to the M-theory AdS_5 solution originally found in [32]. To this purpose we can use the 5D consistent truncation of 7D $\mathrm{U}(1)^2$ gauged supergravity on $\Sigma_\mathfrak{g}$ derived in [33–35]. The corresponding 5D theory is written as an $\mathcal{N}=2$ gauged supergravity with two vector multiplets and one hypermultiplet. At the AdS_5 vacuum one vector multiplet becomes massive through a Higgs mechanism. The theory depends on a parameter z that specifies the twist along $\Sigma_\mathfrak{g}$ and the preserved supersymmetry is generically $\mathcal{N}=2$ corresponding to a dual $\mathcal{N}=1$ CFT. For the special values $z=\pm1$ supersymmetry is enhanced to $\mathcal{N}=4$, and, for $\mathfrak{g}>1$, one obtains a dual $\mathcal{N}=2$ CFT. One can check that, for $z=\pm1$ and $\mathfrak{g}>1$, by setting the hyperscalars to their AdS_5 value and setting to zero the fields in the massive vector multiplet, equations of motion and supersymmetry variations of the theory in [33] coincide with those of the $\mathrm{U}(1)^2$ sector of the minimal $\mathcal{N}=4$ gauged supergravity. Explicitly, in the notations of [33] we need to set $e^{-10B/3}=2^{-1/3}m^{10/3}\zeta$, $e^{10\lambda_1/9}=e^{-5\lambda_2/3}=2^{-1/3}\zeta$ and $A^{(0)}=m^{-4/3}2^{-1/6}A$, with $g_2=2^{5/6}m^{5/3}$.

We will now show that the entropy (1) matches the prediction of a microscopic computation based on the superconformal index for a generic $\mathcal{N}=2$ CFT with a holographic dual. The number of BPS states with charges Q_1, Q_2 and spin J_1, J_2 in an $\mathcal{N}=2$ CFT can be computed (for large charges and spins) by taking the Legendre transform of the superconformal index $\mathcal{I}$, which is a function of chemical potentials for the electric charges and the angular momenta. The agreement between the gravitational picture and the field theory computation requires $\log\mathcal{I} = \log\mathcal{Z}$. The general expectation for an $\mathcal{N}=1$ CFT with a holographic dual is [36]

$$\log\mathcal{I}(\Delta,\omega_i) = -\frac{4\pi i}{27}\frac{(\omega_1+\omega_2\pm 1)^3}{\omega_1\omega_2}a(\Delta), \qquad (4)$$

valid at large N. In this formula ω_1 and ω_2 are chemical potentials conjugated to J_1 and J_2 and $(\omega_1+\omega_2\pm 1)\Delta/2$ is a set of chemical potentials for the R and flavor symmetries of the theory. They are normalized such that Δ can be interpreted as an assignment of R charges to the fields of the theory with the only constraint that the superpotential has R-charge two. Moreover, $a(\Delta) = \frac{9}{32}\mathrm{Tr}R(\Delta)^3$, where $R(\Delta)$ is the R-symmetry generator and the trace is taken over the fermionic fields, is the trial a charge at large N [37]. For Lagrangian theories with a holographic dual, formula (4) has been derived in the large N limit in [4,10,38,39]. It is also compatible with the Cardy limit performed in [5,8,9,40]. It has not been yet derived in full generality for non-Lagrangian theories. However, the Cardy limit can be also derived by writing the effective theory of the CFT coupled to background fields on $S^3\times S^1$ in the limit where the circle shrinks [5,9] and this method applies also to non-Lagrangian theories. The two signs in (4) arise in the saddle-point evaluation of the index in different regions in the space of chemical potentials. Consider first an $\mathcal{N}=2$ Lagrangian CFT with n_V vector multiplets and n_H hypermultiplets. In $\mathcal{N}=1$ language, the theory can be described by n_V vector multiplets, n_V chiral multiplets ϕ_I, and n_H pairs of chiral multiplets $(q_a,\tilde{q}_a)$. The trial R symmetry can be written as $R(\Delta) = (\Delta_1 r_1 + \Delta_2 r_2)/2$ with $\Delta_1+\Delta_2 = 2$, where r_1 is the $\mathrm{U}(1)_R$ symmetry assigning charge 2 to ϕ_I and zero to $q_a,\tilde{q}_a$ and r_2 is the Cartan generator of $\mathrm{SU}(2)_R$ assigning charge zero to ϕ_I and charge 1 to $q_a,\tilde{q}_a$. In the gravitational dual, Q_1 is the charge under $r_1/2$ and Q_2 under r_2. Notice that the exact R symmetry corresponds to $\hat{\Delta}_1 = \frac{2}{3}$, $\hat{\Delta}_2 = \frac{4}{3}$ leading to canonical dimensions for the fields. We easily compute

$$\mathrm{Tr}R(\Delta) = (n_\mathrm{V}-n_\mathrm{H})\Delta_1,$$
$$\mathrm{Tr}R(\Delta)^3 = \frac{3n_\mathrm{V}}{4}\Delta_1\Delta_2^2 + \frac{n_\mathrm{V}-n_\mathrm{H}}{4}\Delta_1^3. \qquad (5)$$

Holography requires $a=c$ at large N [28]. Using $16(a-c) = \mathrm{Tr}R$ [41], this implies $n_\mathrm{V}=n_\mathrm{H}$ at leading order in N. We then find

$$\log\mathcal{I}(\Delta,\omega_i) = -\frac{\pi i a}{8}\frac{(\omega_1+\omega_2\pm 1)^3}{\omega_1\omega_2}\Delta_1\Delta_2^2, \qquad (6)$$

where $a\equiv a(\hat{\Delta}) = (n_\mathrm{V}/4)$ denotes the *exact* central charge of the CFT at large N. We see that, with the redefinitions $X_1 = (\omega_1+\omega_2\pm 1)(\Delta_1/2)$ and $X_2 = (\omega_1+\omega_2\pm 1)(\Delta_2/4)$, we recover (3) and the constraint $X_1+2X_2-\omega_1-\omega_2 = \pm 1$. For non-Lagrangian theories, we just replace n_V and n_H with an *effective* number of vector multiplets and hypermultiplets, $n_\mathrm{V} = 4(2a-c)$, $n_\mathrm{H} = 4(5c-4a)$ [31]. The structure of (5) is completely fixed by $\mathcal{N}=2$ superconformal invariance [42] and the previous argument still holds.

The universal black string in AdS_5.—We can similarly find a universal black string solution of $\mathcal{N}=4$ gauged supergravity as a special case of the black strings in $\mathrm{AdS}_5\times S^5$ found in [43] with angular momentum J, electric charges (Q_1,Q_2,Q_3), and magnetic charges (p_1,p_2,p_3) when we set $Q_3=Q_2$ and $p_3=p_2$. The horizon geometry of this solution has the topology of a warped product $\mathrm{BTZ}\times{}_wS^2$ and carries an extra conserved charge corresponding to a momentum Q_0 along the Bañados-Teitelboim-Zanelli (BTZ) circle. Upon compactification on the circle we obtain a 4D dyonic black hole with Lifshitz-like asymptotics. In the special case where all of the electric charges Q_i and J are zero, the 5D solution is a domain wall interpolating between AdS_5 and $\mathrm{AdS}_3\times S^2$. There is a constraint on the magnetic charges $p_1+2p_2=-1$, which corresponds to the fact that the dual field theory is topologically twisted along S^2, and a further constraint involving the electric charges: $(1+p_1)Q_2+(1+2p_1)Q_1 = 0$. The entropy of the 4-dimensional black hole reads [43,44]

$$S(J,Q_I,p_i) = 2\pi\sqrt{\frac{c_\mathrm{CFT}}{6}\left(Q_0-\frac{J^2}{2k}-\frac{F^2}{2k_{FF}}\right)}, \qquad (7)$$

where $F = Q_1-Q_2$ and

$$c_\mathrm{CFT} = -24a\frac{(1+p_1)^2}{2+3p_1},$$
$$k = -2ap_1(1+p_1)^2, \qquad k_{FF} = -2a(2+3p_1). \qquad (8)$$

We expect again that the solution can be embedded in any AdS_5 compactification preserving 16 supercharges. The special case of class $\mathcal{S}$ with no punctures has been constructed in [44], as a part of a more general compactifications of M5-branes on a Riemann surface with 8 supercharges. It is not difficult to check that the solution in [44] becomes that of the universal black string when the theory has 16 supercharges ($\kappa=-1$, $z_1=-1$, $\mathfrak{s}_1 = 2-2\mathfrak{g}$, and $\mathfrak{t}_1 = -2p_1$, in the notation of [44]), and the entropy coincides with (7).

The result is indeed universal from the field theory point of view. The black string is dual to a 2D CFT obtained by compactifying the $\mathcal{N}=2$ 4D CFT on S^2 with a topological twist. The microscopic entropy is just the number of states of the 2D CFT with $L_0 = Q_0$, electric charges Q_1 and Q_2 under $r_1/2$ and r_2 and charge J under the additional infrared SU(2) symmetry associated with rotation along S^2. These states are accounted for by the charged Cardy formula. The latter has precisely the form (7), where c_CFT is the central charge of the 2D CFT, k is the level of the rotational SU(2) symmetry, and k_{FF} the level of the flavor symmetry $r_1/2 - r_2$ [44]. All these quantities can be computed with an (equivariant) integration of the 4D anomaly polynomial [44–46] and are universal because, as already noticed, the form of the 4-dimensional anomalies for an $\mathcal{N}=2$ CFT with a holographic dual is universal and only depends on $a = (n_V/4)$. For details on the anomaly integration leading to (8) see [44].

$\mathcal{N}=4$ gauged supergravity in 4 dimensions.—Four-dimensional $\mathcal{N}=4$ SO(4) gauged supergravity can be obtained as the consistent truncation of 11D supergravity on S^7 [47]. The bosonic field content of this theory is the metric $g_{\mu\nu}$, two SU(2) gauge fields $(A^I_\mu, \tilde{A}^I_\mu)$, $I = 1, 2, 3$, a dilaton ϕ, and an axion χ. The fermionic components are four gravitini $\psi_{\mu i}$, $i = 1, 2, 3, 4$, and four spin-1/2 fermions χ_i. These fields form the $\mathcal{N}=4$ gauged supergravity multiplet $(g_{\mu\nu}, \psi_{\mu i}, A^I_\mu, \tilde{A}^I_\mu, \chi_i, \phi, \chi)$. The bosonic Lagrangian is given by

$$
\begin{aligned}
\frac{1}{\sqrt{-g}}\mathcal{L} = {}& R - \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \frac{1}{2}e^{2\phi}(\partial^\mu\chi)(\partial_\mu\chi) \\
& + 2g^2[4 + 2\cosh(\phi) + \chi^2 e^\phi] \\
& - \frac{1}{2}e^{-\phi}F^I_{\mu\nu}F^{I\mu\nu} - \frac{1}{2}\frac{e^\phi}{1+\chi^2 e^{2\phi}}\tilde{F}^I_{\mu\nu}\tilde{F}^{\mu\nu I} \\
& - \frac{\chi}{2\sqrt{-g}}\varepsilon_{\mu\nu\rho\sigma}\left(F^{\mu\nu I}F^{\rho\sigma I} - \frac{e^{2\phi}}{1+\chi^2 e^{2\phi}}\tilde{F}^{\mu\nu I}\tilde{F}^{\rho\sigma I}\right).
\end{aligned}
$$

The universal KN black hole in AdS_4.—The universal 4D KN solution can be obtained by specializing the general KN black holes in $\mathrm{AdS}_4 \times S^7$ to have angular momentum J, electric charges (Q_1, Q_2, Q_1, Q_2), and magnetic charges (p_1, p_2, p_1, p_2) under the Cartan subgroup of SO(8) [48–50]. The magnetic charges are restricted to satisfy $p_1 = -p_2 \equiv p$, which corresponds to the absence of a topological twist, and there is a further constraint among charges

$$
J = \frac{Q_1 + Q_2}{2}\left(-1 + \sqrt{1 - 16p^2 + \frac{4\pi^2}{F^2_{S^3}}Q_1 Q_2}\right). \quad (9)
$$

The entropy, given by $S(p, Q_1, Q_2, J) = 2F_{S^3}\frac{J}{(Q_1+Q_2)}$, where $F_{S^3} = \{[\pi\ell_4^2]/[2G_N^{(4)}]\}$ is the S^3 free energy of the dual CFT, can be obtained by extremizing the functional [51,52]

$$
S = -F_{S^3}\frac{(\Delta_1\Delta_2 - 4p^2\omega^2)}{\omega} + \pi\mathrm{i}\sum_{i=1}^{2}\Delta_i Q_i + \pi\mathrm{i}\omega J, \quad (10)
$$

with the constraint $\Delta_1 + \Delta_2 - \omega = 2$. This solution can be embedded in any AdS_4 solution of M-theory or type II string theory with 16 supercharges.

The general expectation for a 3D $\mathcal{N}=4$ CFT with a holographic dual is that the entropy is the Legendre transform of [52]

$$
\log\mathcal{I}(\Delta,\omega) = -\frac{F_{S^3}(\Delta_i - 2\omega p_i)}{2\omega} - \frac{F_{S^3}(\Delta_i + 2\omega p_i)}{2\omega}, \quad (11)
$$

where $F_{S^3}(\Delta_i)$ is the S^3 free energy as a function of the trial R symmetry [53] and $\Delta_1 + \Delta_2 = 2$. Δ_1 and Δ_2 are conjugated to the Cartan generators of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ R symmetry. This formula follows from gluing gravitational blocks in gravity [52], which is the counterpart of gluing holomorphic blocks in the dual field theory [54,55]. For $p = 0$ and the Aharony, Bergman, Jafferis, Maldacena (ABJM) theory, (11) has been derived in the Cardy limit in [56,57] by factorizing the superconformal index into vortex partition functions. It is expected to hold for more general theories and for $p \neq 0$. We then find a general *prediction* for the trial free energy of a generic $\mathcal{N}=4$ CFT with a holographic dual in the large N limit

$$
F_{S^3}(\Delta_i) = F_{S^3}\Delta_1\Delta_2. \quad (12)
$$

We can explicitly check this prediction in various examples. Holographic $\mathcal{N}=4$ CFTs arise as wold volume theories of M2-branes probing $\mathbb{C}^2/\Gamma_1 \times \mathbb{C}^2/\Gamma_2$, with Γ_i discrete subgroups of SU(2), where the role of Γ_1 and Γ_2 can be exchanged by mirror symmetry [58]. Consider, for simplicity, the case where $\Gamma_2 = \mathbb{Z}_p$. The world volume theory is based on an $\mathcal{N}=4$ ADE quiver with gauge groups $\mathrm{U}(n_a N)$ corresponding to the nodes of the extended Dynkin diagram of Γ_1 and bifundamental hypermultiplets associated with the links, flavored with the addition of p fundamental hypermultiplets (n_a are the comarks: see [59] for conventions and details). Denote by n_V, n_B, and n_F the total number of vector multiplets, bifundamental and fundamental hypers, respectively. The large N limit of the S^3 partition function can be computed with the methods in [60–62]. In the large N limit the eigenvalue distribution for a group $\mathrm{U}(n_a N)$ is given by n_a copies of the segment $\lambda(t) = N^{1/2}t$ with density $\rho(t)$ $[\int dt\rho(t) = 1]$. In the large N limit, using the rules in [61,62], we obtain

$$
\begin{aligned}
F_{S^3}(\Delta_i) = {}& \frac{n_\mathrm{B}}{N^{1/2}}\frac{\pi^2}{6}\Delta_2(\Delta_2 - 2)(\Delta_2 - 4)\int\rho(t)^2 dt \\
& + \frac{n_\mathrm{V}}{N^{1/2}}\frac{2\pi^2}{3}\Delta_1(\Delta_1 - 1)(\Delta_1 - 2)\int\rho(t)^2 dt \\
& + \frac{n_\mathrm{F}N^{1/2}}{2}(2 - \Delta_2)\int\rho(t)|t|dt, \quad (13)
\end{aligned}
$$

where we assign charge Δ_1 to the adjoint chiral in the vector multiplet and $\Delta_2/2$ to the chiral fields $q_a, \tilde{q}_a$ in the hypermultiplets, in $\mathcal{N}=2$ notations. Since the ADE quivers are balanced (the number of hypers for each group is twice the number of colors), we have $n_{\rm V} = n_{\rm B}$ and we find the saddle-point distribution

$$\rho(t) = \frac{\pi\sqrt{2n_{\rm F}n_{\rm V}N}\Delta_2 - n_{\rm F}N|t|}{2\pi^2 n_{\rm V}\Delta_2^2}, \tag{14}$$

with free energy

$$F_{S^3}(\Delta_i) = \frac{\pi}{3}\sqrt{2n_{\rm F}n_{\rm V}}\Delta_1\Delta_2, \tag{15}$$

which reproduces (12) with $F_{S^3} = (\pi/3)\sqrt{2n_{\rm F}n_{\rm V}}$. The previous computation for $\Gamma_1 = \mathbb{Z}_q$ was already done in disguise in [63]. Notice that $F_{S^3} = \mathcal{O}(N^{3/2})$, as expected for M2-brane theories. Formula (15) can be also derived from the identification of the trial free energy with the volume functional of the transverse Calabi-Yau [61,64]. M2-branes probing Abelian hyper-Kähler orbifolds of $\mathbb{C}^4$ can be also realized in terms of $\mathcal{N} = 4$ circular quivers with nonzero Chern-Simons terms [65]. The simplest example is actually ABJM itself, whose free energy has been computed in [61] and reads $F_{S^3}(\delta_i) = 4F_{S^3}\sqrt{\delta_1\delta_2\delta_3\delta_4}$, where δ_i are conjugated to the Cartan subgroup of SO(8) and satisfy $\sum_{i=1}^4 \delta_i = 2$. This reduces to (12) for $\delta_3 = \delta_1 = \Delta_1/2$ and $\delta_4 = \delta_2 = \Delta_2/2$. Using and extending the results in [63,66], one can also compute the free energy for the more general $\mathcal{N} = 4$ quivers discussed in [65] and check that (12) is valid. Notice that in all these examples the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ R symmetry acts differently from the case with no Chern-Simons and is fully visible once the theory is written in terms of both standard and twisted hypermultiplets [67–69]. Another large class of $\mathcal{N} = 4$ holographic quivers are the $T^\rho_\sigma(G)$ theories [70] whose gravitational dual was found in [71,72]. The refined free energy on S^3 for $T(\mathrm{SU}(N))$ has been recently computed in the large N limit in [73] and it reads $F_{S^3} = \frac{1}{2}\Delta_1\Delta_2 N^2 \log N$ which also agrees with (12). The prediction (12) can be also checked for a larger class of $T^\rho_\sigma(G)$ theories [74].

The universal twisted black hole in AdS_4.—We can obtain dyonic black holes with a twist (magnetic charge for the R symmetry) and horizon $\mathrm{AdS}_2\times\Sigma_{\mathfrak{g}}$, where $\Sigma_{\mathfrak{g}}$ is a Riemann surface of genus $\mathfrak{g}$, by specializing the corresponding solution in $\mathrm{AdS}_4\times S^7$ [75–77]. For $\mathfrak{g} = 0$ we can add an angular momentum J [78]. The case of static solutions of minimal gauged supergravity has been already discussed in [7,18]. Solutions with a generic $\mathcal{N} = 4$ choice of charges are not regular and we will not discuss them further. One can check however that the comparison between the gravity entropy functional and the large N limit of the (refined) topologically twisted index would still agree [since it is also based on (12) [52]], although the extremization leads to a nonphysical value for the entropy. For $J = 0$ one can find an off-shell agreement by considering the Euclidean black saddles discussed in [79].

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