Large Anisotropies of the Stochastic Gravitational Wave Background from Cosmic Domain Walls

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We investigate the stochastic gravitational wave background (SGWB) from cosmic domain walls (DWs) caused by quantum fluctuations of a light scalar field ϕ during inflation. Large-scale perturbations of ϕ lead to large-scale perturbations of DW energy density and anisotropies in the SGWB. We find that the angular power spectrum of this SGWB is scale invariant and at least of the order of 10^{-2}, which is a distinctive feature of observational interest. Since we have not detected primordial gravitational waves yet, anisotropies of the SGWB provide a nontrivial opportunity to verify the rationality of inflation and detect the energy scale of inflation, especially for low-scale inflationary models. Square kilometer array has the opportunity to detect the anisotropies of such SGWBs. The common-spectrum process observed recently by NANOGrav could also be interpreted by the SGWB from cosmic DWs.

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Introduction.—The direct detection of gravitational waves (GWs) produced by black hole binary merger events opens a new era of GW astronomy and GW cosmology [1]. Primordial GWs caused by vacuum fluctuations during inflation also receive extensive investigation and can be detected by the B-mode polarizations of cosmic microwave background (CMB) temperature fluctuations. We can determine the inflationary energy scale by detecting primordial GWs. However, primordial GWs have not been detected yet, and the current constraint on the tensor-to-scalar ratio is $r \lesssim 0.09$ at the 95% level by Planck 2018 [2], and inflationary models with cubic and quartic potentials are excluded by the constraint on r. Since r is proportional to the fourth power of the inflationary energy scale, it is hard to detect primordial GWs in low-scale inflationary models [3–5]. If the trans-Planckian censorship conjecture [6] indeed makes sense, the inflationary energy scale is much lower than the Grant unified theory scale, so that the tensor-to-scalar ratio r would be smaller than 10^{-30} [7], which is almost impossible to detect primordial GWs by CMB experiments. Since the determination of the inflationary energy scale is an essential issue in modern cosmology, it is quite important to explore new methods and observations.

Analogous to CMB, the GWs produced after inflation can also form a stochastic gravitational wave background (SGWB). We find that energy density anisotropies in SGWBs can also contain key information of inflation. Such GW sources in the early Universe can be phase transition [8–10], preheating [11,12], topological defects [13–16], and large amplitude scalar perturbations [17–21], etc. Because of the weakness of gravitational interaction, GWs produced by those sources can be directly observed by the GW detectors and then provide essential clues about the early evolution history of the Universe, the standard model of particle physics, and new physics beyond the standard model. The peak frequencies of those SGWBs could be within the sensitivity bands of various detectors such as aLIGO [22], Laser Interferometer Space Antenna (*LISA*) [23], *Taiji* [24], and square kilometer array (SKA) [25]. Moreover, a SGWB could explain the stochastic common-spectrum process just detected by the NANOGrav Collaboration [26].

The study of anisotropies in SGWBs has received a lot of attention recently, which could be generated by the sources [27–30] and the processes during propagation [31–34]. But it is found that the anisotropies in most cases are a challenge to be observed [35–40]. In this Letter, we explore the SGWB produced from cosmic domain walls (DWs) and propose a novel mechanism to generate large anisotropies in the SGWB, which encodes the information of inflation. DWs are predicted by the models where discrete symmetry is spontaneously broken [41–44], and GWs can be produced by dynamics of nonspherical DWs [15,45–48]. Since stable DWs will finally dominate the Universe, we consider a model where one of vacua is slightly lifted so that DWs will annihilate before becoming domination [44,49]. The model is motivated by and can be realized in, for example, the Higgs field models [47,48], axion models [50–54] and supersymmetric models [55,56], etc.

The formation of DWs is caused by quantum fluctuations of an extra light scalar field ϕ during inflation. The effective mass of ϕ, m_{eff}, is smaller than the Hubble parameter H during inflation so that perturbations of ϕ remain constant on superhorizon scales. The light fields received much attention for their ability to generate both curvature perturbations and entropy perturbations [57–59], and the nature of these light fields might be the Higgs field and the string axions. ϕ could cross the potential barrier because of quantum fluctuations. After inflation, H decreases with time and once H becomes smaller than m_{eff}, in different regions ϕ settles down in different vacua and DWs form. Large-scale perturbations of ϕ remain constant at superhorizon scales, then result in large-scale perturbations of the energy density of DWs and anisotropies in SGWBs. We find strong anisotropies of at least ~ 0.1 variations at the CMB scales that are predicted in our model, which could be used as a novel method to probe inflation. We set $c = 8\pi G = 1$ throughout this Letter.

GWs from cosmic DWs.—Consider a scalar field ϕ with an effective potential $V(\phi)$ in Einstein gravity, where the minima of $V(\phi)$, $\phi = \pm v$, are separated by the potential barrier V_0 as depicted in Fig. 1. Let $\phi(z)$ denote the static planar DW solution perpendicular to the z axis in Minkowski space. The tension of DWs σ is obtained by integrating the energy density $(d\phi/dz)^2/2 + V(\phi)$ along the direction perpendicular to the wall, which is also the surface energy density of DWs.

Since the energy density of DWs ρ_{DW} is proportional to t^{-1}, while the critical energy density of the Universe ρ_c scales as t^{-2} in the radiation- and matter-dominated periods, $\rho_{\mathrm{DW}}/\rho_c$ is increasing as t. To prevent DWs from overclosing the Universe, one can lift one of the degenerate vacuum by ΔV, so that DWs annihilate at time $t_{\mathrm{ann}} \sim \mathcal{A}\sigma/\Delta V$ [15], where $\mathcal{A} \approx 0.8$ is fixed by numerical simulation [46]. (Here the annihilation time is just a typical timescale of the collapse of wall network [45], from which the energy density of domain walls starts to obviously deviate from the scaling law $1/t$. The whole annihilation precess of domain walls can be approximately treated as it happens at around t_{ann} [46].)

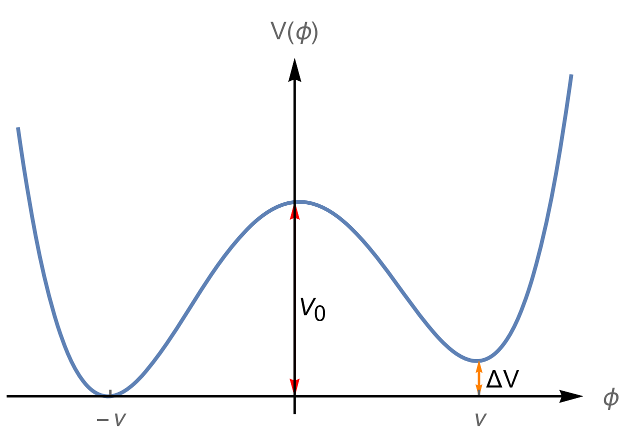

FIG. 1. The discrete symmetry breaking effective potential with a bias term ΔV.

The numerical and analytical results of GWs from DWs in the radiation-dominated era is obtained in Refs. [15,46], where ϕ settles in different vacua almost equiprobably. According to Ref. [15], the energy density of GWs ρ_{GW} is a constant before t_{ann}. Since ρ_c is proportional to t^{-2}, the energy spectrum of GWs $\Omega_{\mathrm{GW}}(k)$ is proportional to t^2, so most energy in GWs is produced around the annihilation time t_{ann}. Then Ω_{GW} at t_{ann} reads

$$\Omega_{\mathrm{GW,peak}}(t_{\mathrm{ann}}) = \frac{\tilde{\epsilon}_{\mathrm{GW}} \mathcal{A}^2 \sigma^2}{24\pi H^2(t_{\mathrm{ann}})}, \tag{1}$$

where $\tilde{\epsilon}_{\mathrm{GW}} \approx 0.7$ is a constant given by numerical results [46]. According to Ref. [46], $\Omega_{\mathrm{GW}}(k)$ scales as k^3 for $k < k_{\mathrm{peak}}$ and k^{-1} for $k > k_{\mathrm{peak}}$.

The wavelength at the peak is the same order of the Hubble horizon size at t_{ann}, and then redshifted by the expansion of the Universe, so the peak frequency f_{peak} and the peak amplitude at the present time t_0 reads

$$f_{\mathrm{peak}} = \left[\frac{H^2(t_{\mathrm{ann}})}{H_0^2 \Omega_{\mathrm{rad}}(t_0)} * \left(\frac{g_{*\mathrm{ann}}}{g_{*0}}\right)^{1/3}\right]^{-1/4} H(t_{\mathrm{ann}}),$$
$$\Omega_{\mathrm{GW,peak}}(t_0) h^2 = \Omega_{\mathrm{rad}}(t_0) h^2 \left(\frac{g_{*0}}{g_{*\mathrm{ann}}}\right)^{1/3} \Omega_{\mathrm{GW,peak}}(t_{\mathrm{ann}}), \tag{2}$$

where g_{*0} and $g_{*\mathrm{ann}}$ are the effective relativistic degrees of freedom at t_0 and t_{ann}, respectively. $\Omega_{\mathrm{rad}} h^2 = 4.2 \times 10^{-5}$ is the current density fraction of radiation, and $H_0 = 68\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant from the results of Planck 2018 [2].

Anisotropies.—Let us then focus on how anisotropies of the SGWB are generated. Quantum fluctuations of ϕ during inflation lead to perturbations $\delta\phi(\mathbf{x})$ at superhorizon scales. The initial background value of ϕ is set to be ϕ_i, and the initial cosmic time t_i is defined by $a(t_0)H_0 = a(t_i)H_{\mathrm{inf}}$, where $a(t_0) = 1$ and H_{inf} is the Hubble parameter during inflation. Let $\mathcal{P}(\tilde{\phi}, t)$ denote the probability of $\tilde{\phi}$ at t, where $\tilde{\phi}$ is the value of spacial averaged ϕ inside one Hubble horizon. For the light field during inflation, $\mathcal{P}(\tilde{\phi}, t)$ reads

$$\mathcal{P}(\tilde{\phi}, t) = \sqrt{\frac{2\pi}{H^3(t - t_i)}} \exp\left(-\frac{2\pi^2}{H^3(t - t_i)} (\tilde{\phi} - \phi_i)^2\right), \tag{3}$$

which is the solution of the Fokker-Planck equation [60]. During inflation, H_{inf} is almost a constant, so the e-folding numbers $N(t) \equiv \ln[a(t)/a(t_i)]$ can be expressed as a function of t as $N(t) = H_{\mathrm{inf}}(t - t_i)$. Without loss of generality, setting $\phi_i > 0$, the probability of $\tilde{\phi}(t) < 0$ at t is

$$P(t) \equiv \int_{-\infty}^{0} d\tilde{\phi}\mathcal{P}(\tilde{\phi}, t) = \frac{1}{2}\mathrm{erfc}\left(\frac{\sqrt{2}\pi\phi_i}{H_{\mathrm{inf}}\sqrt{N(t)}}\right). \quad (4)$$

The regions where $\tilde{\phi} < 0$ and $\tilde{\phi} > 0$ fall in different vacua when the DWs form, so Eq. (4) implies that the possibilities of the two domains are different. Note that Eq. (4) is valid only during inflation. Since ϕ remains constant at superhorizon scales, for any scale that leaves and reenters the horizon at t in the inflation phase and t' in the radiation-dominated era, the probability remains the same. Thus, we can relate the probability $P(t')$ to the probability $P(t)$ with the same mode $a(t)H_{\mathrm{inf}} = a(t')H(t')$. We define $\alpha(t) \equiv [\sqrt{2}\pi\phi_i/H_{\mathrm{inf}}\sqrt{N(t)}]$, and in the observationally interesting case $\alpha \sim 1$. [In principle, α is a free parameter since ϕ_i and H_{inf} are not related. It is not natural to consider the case $\phi_i \ll H_{\mathrm{inf}}$ because of the quantum kicks before t_i. In the case of $\alpha \gg 1$, Ω_{GW} is exponentially suppressed by the error function in Eq. (4), which is hard to be detected.] Large-scale perturbations of ϕ result in large-scale perturbations of $P(t)$,

$$\delta P(t, \mathbf{x}) = \frac{1}{2}\mathrm{erfc}\{\alpha(t)[1 + \delta\phi(\mathbf{x})/\phi_i]\} - \frac{1}{2}\mathrm{erfc}[\alpha(t)]. \quad (5)$$

[Since $\delta\phi(\mathbf{x})$ is caused by quantum fluctuations in the first several e-foldings after t_i, $N(t)$ overestimates the e-folding numbers. This deviation is negligible if we focus on anisotropies at large scales.] Here we emphasize that around $\mathbf{x}$ we apply spacial average at a scale much larger than the wavelength of GWs but much smaller than the CMB horizon scale. Note that Eq. (4) is valid as long as $[(dV/d\phi)/3H_{\mathrm{inf}}]$ is negligible during inflation, even if the thermal correction term may push the scalar field back to the origin after inflation.

Since the averaged radius of curvature of DWs at t' is comparable to the Hubble horizon size at the same time, the area of DWs can be estimated as $4\pi H^{-2}$ for each Hubble volume where the averaged value of ϕ is negative. Therefore, the energy density of DWs ρ_{DW} is proportional to its area in unit volume $\rho_{\mathrm{DW}}(t') \propto P(t')$. Then, the energy spectrum of GWs is proportional to the energy density of the source ρ_{DW},

$$\Omega_{\mathrm{GW},P}(k) = 2P(t_{\mathrm{ann}})\Omega_{\mathrm{GW}}(k), \quad (6)$$

where we use $P(t_{\mathrm{ann}})$ since most of the GW energy is produced near t_{ann}. [$P(t_{\mathrm{ann}})$ is calculated by using Eq. (4), but some cautions should be added here because, as mentioned above, Eq. (4) is only valid during inflation. Here one first identifies t_{ann} with the potential of the scalar field and then the scale that just reenters the horizon at t_{ann}. The next one is able to fix the e-folding number in Eq. (4) when the same scale leaves the Hubble horizon during inflation. This is what $P(t_{\mathrm{ann}})$ means in this Letter.] We have set $\phi_i > 0$ in advance, so $P(t_{\mathrm{ann}})$ should be smaller than 1/2. If the possibilities of $\phi < 0$ and $\phi > 0$ are equal, then $P(t_{\mathrm{ann}}) = 1/2$ and $\Omega_{\mathrm{GW},P}(k)$ reaches the maximum $\Omega_{\mathrm{GW}}(k)$.

The averaged comoving radius of DWs at t_{ann} is close to k_{peak}, which implies $N(t)$ in Eqs. (4) and (5) can be determined by k_{peak}. The peak mode leaves the Hubble horizon when $k_{\mathrm{peak}} = a(t)H_{\mathrm{inf}}$, and the Hubble constant satisfies $H_0 = a(t_i)H_{\mathrm{inf}}$, so the e-folding number for the peak is

$$N_{\mathrm{peak}} = \ln(k_{\mathrm{peak}}/H_0), \quad (7)$$

and $\alpha_{\mathrm{peak}} \equiv (\sqrt{2}\pi\phi_i/H_{\mathrm{inf}}\sqrt{N_{\mathrm{peak}}})$.

Taking into consideration the inability of the GW detectors to probe high angular resolution of the SGWB, we focus on large-scale anisotropies of the SGWB. Since for each k mode anisotropies of $\Omega_{\mathrm{GW}}(k)$ are proportional to perturbations of ρ_{DW} at t_{ann}, anisotropies are independent of k and we omit the variable k in $\Omega_{\mathrm{GW}}(k)$ in the following. Perturbations of $\Omega_{\mathrm{GW},P}$ are defined by $\delta\Omega_{\mathrm{GW},P}(\mathbf{x}) \equiv [\Omega_{\mathrm{GW},P}(\mathbf{x}) - \overline{\Omega_{\mathrm{GW},P}}]/\overline{\Omega_{\mathrm{GW},P}}$, where the overline denotes the spacial average in total space. $\delta\Omega_{\mathrm{GW},P}(\mathbf{x})$ is proportional to large-scale perturbations of ϕ in the first order,

$$\delta\Omega_{\mathrm{GW},P}(\mathbf{x}) = c_1\delta\phi(\mathbf{x}). \quad (8)$$

Here coefficient c_1 is given by Eqs. (5) and (6),

$$c_1 = \frac{2}{\sqrt{\pi}\phi_i}\frac{\exp(-\alpha_{\mathrm{peak}}^2)\alpha_{\mathrm{peak}}}{\mathrm{erfc}(\alpha_{\mathrm{peak}})}. \quad (9)$$

Using the approximation $\mathrm{erfc}(x) \approx (1/x\sqrt{\pi})e^{-x^2}$ for $x \gg 1$, we have $c_1 \approx 2\alpha_{\mathrm{peak}}^2/\phi_i$ for $\alpha_{\mathrm{peak}} \gg 1$.

Analogous to the Sachs-Wolfe plateau for CMB temperature fluctuations for small multiple l [61], the angular power spectrum can be expressed in terms of the power spectrum of $\delta\Omega_{\mathrm{GW}}(\mathbf{x})$ by

$$l(l+1)C_l = \frac{\pi}{2}\mathcal{P}_{\mathrm{GW}}, \quad (10)$$

which is valid for all small l, and $\mathcal{P}_{\mathrm{GW}} \equiv \langle\delta\Omega_{\mathrm{GW}}^2\rangle$. Since the angular power spectrum is frequency independent, combining Eqs. (8)–(10) and the relation $\langle\delta\phi^2\rangle = (H_{\mathrm{inf}}^2/4\pi^2)$, we obtain

$$l(l+1)C_l \approx \begin{cases} \frac{\pi}{N_{\mathrm{peak}}}\alpha_{\mathrm{peak}}^2, & \alpha_{\mathrm{peak}} \gg 1, \\ \frac{1}{N_{\mathrm{peak}}}, & \alpha_{\mathrm{peak}} \ll 1. \end{cases} \quad (11)$$

N_{peak} must be smaller than 60, so $l(l+1)C_l$ is larger than 10^{-2}. Setting ϕ_i near the true vacuum of the potential, ϕ_i and ν are of the same order, so the inflationary scale can also be derived using Eq. (11), if ν is determined by the particle physics models.

Examples.—(1) Consider the formation of DWs in the spontaneous breaking of discrete R symmetries discussed in Ref. [55]. DWs form after the gauge interaction becomes strong at the scale Λ_c; the tension of DWs is $\sigma \sim \Lambda_c^3$. The bias term ΔV is relevant to the mass of gravitinos $\Delta V \sim m_{3/2}\Lambda_c^3$. Choosing the parameters as $\Lambda_c = 5\times 10^{10}$ GeV, $\phi_i = H_{\rm inf}$, and $m_{3/2} = 1$ MeV, the GW energy spectrum peaks at $f = 1$ Hz with the peak value $\Omega_{{\rm GW},P}(t_0)h^2 = 10^{-11}$, and the angular power spectrum $l(l+1)C_l = 0.085$ for small l.

In Fig. 2 we show the result of $\Omega_{{\rm GW},P}(k,t_0)h^2$ and the random realizations of the SGWB. *LISA* [23] and *Taiji* [24] have the ability to detect such SGWB and its anisotropies.

(2) In the presence of monodromy, the discrete symmetry of the axion is explicitly broken by a quadratic term, and the effective potential reads $V(\phi) = \frac{1}{2}M^2\phi^2 + m^2\nu^2[1 - \cos(\phi/\nu)]$ [66–68]. With the parameters $\nu = 10^{-6}$, $m = 8.7\times 10^{-33}$, $M = 3.8\times 10^{-38}$, and $\phi_i = 0.5H_{\rm inf}$, the tension of DWs is $\sigma = 6.9\times 10^{-44}$, $\Omega_{{\rm GW},P}(t_0)h^2$ peaks at 6.3×10^{-11} Hz with the peak value 6.4×10^{-8}, and the angular power spectrum $l(l+1)C_l = 0.12$ for small l, respectively. We find that the common-spectrum process observed by NANOGrav could be interpreted by the SGWB from DWs at the 68% level. The SGWB produced in our model could be distinguished from the SGWBs from other sources [69–76] by characteristic large anisotropies with the improving sensitivity of pulsar-timing arrays such as SKA.

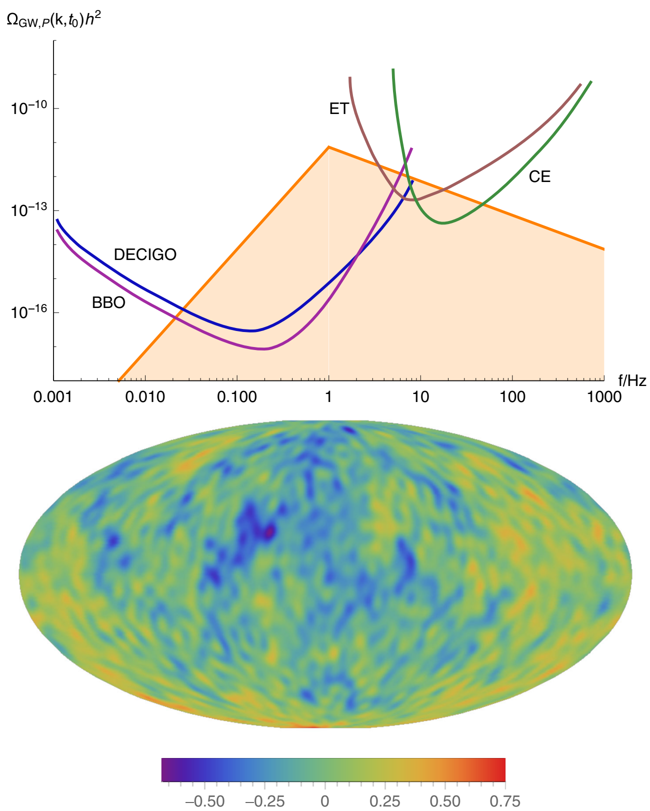

FIG. 2. Upper: the orange line presents $\Omega_{{\rm GW},P}(k,t_0)h^2$ in the model discussed in Ref. [55] with the parameters we choose, using the approximation method in Ref. [15]. This SGWB can be observed by decihertz laser interferometer gravitational wave observatory [62], big bang observer [63], Einstein telescope [64], and cosmic explorer [65]. Lower: the random realizations of the SGWB using the first 50 l modes.

Conclusion and discussion.—In this Letter, we have investigated the anisotropic SGWB from cosmic DWs when a discrete symmetry is spontaneously broken. Quantum fluctuations of the light scalar field remain constant at superhorizon scales, then induce large-scale perturbations of the energy density of cosmic DWs, and finally lead to the anisotropies in the SGWB. The angular power spectrum of the SGWB in this scenario is larger than 10^{-2}, which is a distinctive feature of observational interest. Since primordial GWs are too weak to be detected in low-scale inflationary models, observing the anisotropic SGWB provides a potential method to detect the inflationary energy scale, even though it is several orders of magnitude lower than the grand unified theory scale. Additionally, we find that the SGWB from the cosmic DWs can also explain the common-spectrum process detected by NANOGrav in some parameter space.

Since $\rho_{\rm DW}$ at large scales is highly suppressed by $P(t)$, in principle, ΔV is not required to prevent DWs from becoming dominant. In this case, $\Omega_{{\rm GW},P}$ reaches the maximum when the increase of $\Omega_{\rm GW}$ and the decrease of $P(t)$ in Eq. (6) cancel each other out. However, the peak frequency is lower than 10^{-9} Hz; otherwise the GW signal is too weak to be observed. The anisotropic k^{-1} slope of $\Omega_{{\rm GW},P}$ might be detected by SKA.

In the case of $H_{\rm inf} \gg \nu$, ϕ may cross the potential barrier after it rolls down the potential, which implies our assumption $\rho_{\rm DW}$ and $\Omega_{{\rm GW},P}$ being proportional to $P(t)$ is violated. The linear approximation in Eq. (8) will be replaced by a complicated form and non-Gaussianity arises. Even though the angular power spectrum could be smaller than 10^{-2} in this case, it provides us a good chance to detect $H_{\rm inf}$ by the non-Gaussianity and the relation between $l(l+1)C_l$ and $H_{\rm inf}/\nu$.

It would be quite interesting to generalize our model with $\mathbb{Z}_2$ symmetry discussed in this Letter to the case of $\mathbb{Z}_n$ symmetry. In the latter case, one would expect that various DWs with different tensions and different annihilation times would appear and a network of DWs would form. If one kind of DWs is dominated, the result will be same as that discussed here. In a general case, some enhanced effect may appear due to the nonlinearity of the Einstein equations and the spectrum structure of GWs would be quite rich. A simulation of DWs dynamics and resulted GWs is needed. We leave this topic to a future work.

When $H(t)$ becomes smaller than $m_{\rm eff}$, ϕ begins to oscillate around the minimum of the potential, leading to a resonant amplification of perturbations of ϕ inside the Hubble horizon, which is similar to the preheating scenario. The amplified perturbations of ϕ will produce an extra SGWB, and large-scale perturbations of ϕ induces anisotropies in the SGWB. $\Omega_{\rm GW}(k)$ of such a SGWB contains useful information about $V(\phi)$, which helps us to further distinguish the particle physics model. GWs becomes stronger for a larger ν, so for string axions with $\nu \sim 10^{16}$ GeV, the SGWB is more likely to be detected.

The light field during inflation may also make contributions to entropy perturbations, depending on its decay products. In turn, the constraint on entropy perturbations from the detection of such SGWB could be stronger than that from the CMB experiments.

Finally, let us mention that, in principle, our mechanism is applicable to cosmic strings as well. Anisotropies of the SGWB from cosmic strings encode abundant information of the source in a wide range of scale, and detecting the frequency-dependent angular power spectrum will help us to reconstruct the potential and distinguish the inflationary models. We leave this interesting topic for future investigation.

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