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# Spinning Black Holes Fall in Love

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The open question of whether a black hole can become tidally deformed by an external gravitational field has profound implications for fundamental physics, astrophysics, and gravitational-wave astronomy. Love tensors characterize the tidal deformability of compact objects such as astrophysical (Kerr) black holes under an external static tidal field. We prove that all Love tensors vanish identically for a Kerr black hole in the nonspinning limit or for an axisymmetric tidal perturbation. In contrast to this result, we show that Love tensors are generically *nonzero* for a spinning black hole. Specifically, to linear order in the Kerr black hole spin and the weak perturbing tidal field, we compute in closed form the Love tensors that couple the mass-type and current-type quadrupole moments to the electric-type and magnetic-type quadrupolar tidal fields. For a dimensionless spin $\sim 0.1$, the nonvanishing quadrupolar Love tensors are $\sim 2 \times 10^{-3}$, thus showing that black holes are particularly "rigid" compact objects.

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*Introduction.*—The deformability of a self-gravitating body under the effect of an external tidal field is a question of central interest in gravitational physics. Such tidal deformability can be characterized by a discrete set of tidal Love numbers (TLNs) [1], the gravitational analog of the electric susceptibility in electrodynamics. Importantly, the TLNs of a self-gravitating body encode information about its internal structure, such as its composition or equation of state [2–4]. Those numbers were first introduced by Love [5,6], in the context of Newtonian gravitation, to describe the Earth's ocean tides due to its gravitational interaction with the Moon. They now play an important role in understanding the internal structure of the planets of the Solar System [7,8], and even of exoplanets such as WASP-103b [9] and the TRAPPIST-1 system [10,11].

In the context of relativistic gravitation, current and future gravitational-wave measurements of TLNs in binary inspirals provide a novel way of testing the inspiraling compact objects (neutron stars or black holes) and general relativity in the regime of strong gravitational fields [12]. In events that lead to the coalescence of two neutron stars, such as GW170817 [13] and GW190425 [14], the tidal effects become important for gravitational-wave frequencies of around 600 Hz, by accelerating the coalescence and affecting the gravitational-wave phase. Those two events have been used to set upper bounds on the tidal deformability of neutron stars, thereby constraining their radii and equation of state at supranuclear densities [14–18]. Some universal (i.e., equation-of-state independent) I-Love-Q [19] and I-Love-C [20] relations between the neutron star moment of inertia, quadrupolar TLN, quadrupole moment, and compactness can be used to lift degeneracies among parameters in gravitational-wave signals, enhancing the measurability of the tidal effects. Over the coming decades, the observation by the planned Laser Interferometer Space Antenna (*LISA*) mission [21] of the gravitational-wave signals generated by the inspiral of stellar-mass compact objects into massive black holes might place constraints on the TLNs of the central body that are roughly 8 orders of magnitude more stringent than current ones on neutron stars [22].

It is widely accepted that all astrophysical black holes are rotating and are thus described by the Kerr family [23,24] of solutions of the Einstein field equation. Previous works on the tidal deformability of black holes in general relativity have shown that, differently from the Newtonian case, the tidal field can be decomposed into two sectors, according to their parity, often called electric and magnetic, and, importantly, that the TLNs of nonrotating black holes all vanish under a static tidal field [25–29]. This conclusion was extended to slowly rotating black holes, perturbatively in the spin, for a weak and static quadrupolar tidal field: to quadratic order for an axisymmetric quadrupole of electric-type [30], and to linear order for a generic quadrupole [31]. Given those remarkable results, there appears to be a widespread expectation that the vanishing of black hole static TLNs extends to a generic rotating Kerr black hole in a generic multipolar tidal environment, e.g., in [22,32–38]. In this Letter we will show that, on the contrary, the static TLNs [39] of a Kerr black hole do *not* vanish in general. We show this by fully calculating, for the first time, the induced quadrupole moments on a Kerr black hole due to a static tidal field. The details are given in [41].

Throughout this Letter we use units such that $G = c = 1$, an overbar denotes the complex conjugation, we use the shorthand $\sum_{\ell m} \equiv \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell}$ where $\ell$ is the multipolar index and $m$ the azimuthal index, as well as the notation $L \equiv i_1, \ldots, i_\ell$ for a multi-index made of $\ell$ spatial indices.

*Newtonian Love.*—Consider first an isolated, nonspinning, spherical, Newtonian body of mass $M$ and equilibrium radius $R$. For a weak and slowly varying external tidal field, the induced mass multipole moments $I_L(t)$ are proportional to the applied tidal moments $\mathcal{E}_L(t)$,

$$I_L = \lambda_\ell \mathcal{E}_L, \tag{1}$$

where $\lambda_\ell$ is a constant. For instance, at quadrupolar order the induced mass quadrupole $I_{ij}$ is proportional to the quadrupolar tidal field $\mathcal{E}_{ij}$. This adiabatic approximation holds as long as the typical timescales of the physical processes responsible for adjusting the matter distribution are much shorter than the typical timescale of variation of the tidal environment itself. The tidal deformability parameter $\lambda_\ell$ in (1) depends exclusively on the internal structure of the body and scales as $R^{2\ell+1}$. Introducing the dimensionless TLN $k_\ell$ associated with $\lambda_\ell$, defined via

$$k_\ell \equiv -\frac{(2\ell-1)!!}{2(\ell-2)!}\frac{\lambda_\ell}{R^{2\ell+1}}, \tag{2}$$

the gravitational potential of the tidally perturbed Newtonian body can be expanded over spherical harmonics $Y_{\ell m}(\theta, \phi)$ according to [1]

$$U = \frac{M}{r} - \sum_{\ell m} \frac{(\ell-2)!}{\ell!} \mathcal{E}_{\ell m} r^\ell \left[1 + 2k_\ell \left(\frac{R}{r}\right)^{2\ell+1}\right] Y_{\ell m}, \tag{3}$$

where $r$ is the Euclidean distance to the center of mass, and the $2\ell+1$ coefficients $\mathcal{E}_{\ell m}$ are the spherical-harmonic modes of the tidal moment $\mathcal{E}_L$. The growing term $O(r^\ell)$ corresponds to the external $2^\ell$-polar tidal perturbation, and the decaying term $O(r^{-\ell-1})$ to the body's response, proportional to the TLN $k_\ell$.

*Einsteinian Love.*—In general relativity, the tidal environment of a body is fully characterized by *two* families of tidal moments: the electric-type and magnetic-type tidal fields $\mathcal{E}_L$ and $\mathcal{B}_L$. The former are the relativistic analogs of the Newtonian tidal moments introduced above, while the latter have no counterpart in Newtonian gravity. Similarly, the multipolar structure of that body is now characterized by *two* families of multipole moments: the mass-type and current-type multipole moments $M_L$ and $S_L$, which are defined in a coordinate-independent manner for any asymptotically flat, stationary solutions of the vacuum Einstein equation [42,43].

From now on we consider a weak, stationary tidal perturbation of a given compact body. The associated perturbed metric is $\mathring{g}_{\alpha\beta} + h_{\alpha\beta}$, where the background $\mathring{g}_{\alpha\beta}$ is an exact solution of the Einstein equation and the linear metric perturbation can be decomposed according to

$$h_{\alpha\beta} = h^{\rm tidal}_{\alpha\beta} + h^{\rm resp}_{\alpha\beta}. \tag{4}$$

Here, $h^{\rm tidal}_{\alpha\beta}$ and $h^{\rm resp}_{\alpha\beta}$ are *uniquely* specified as the linearly independent solutions of the linearized Einstein equation in vacuum (outside the body) that have the appropriate asymptotic behavior at large distances. In particular, the growing solution $h^{\rm tidal}_{\alpha\beta}$ is unambiguously associated with the perturbing tidal field, while the decaying solution $h^{\rm resp}_{\alpha\beta}$ is unambiguously associated with the corresponding linear response of the body [44]. The perturbed metric $\mathring{g}_{\alpha\beta} + h^{\rm resp}_{\alpha\beta}$ is an asymptotically flat, stationary solution of the linearized Einstein equation in vacuum, and the corresponding multipole moments are

$$M_L = \mathring{M}_L + \delta M_L, \tag{5a}$$

$$S_L = \mathring{S}_L + \delta S_L, \tag{5b}$$

where a circle over a quantity indicates it is associated with the background $\mathring{g}_{\alpha\beta}$ and a $\delta$ preceding that it is associated with the linear response $h^{\rm resp}_{\alpha\beta}$.

For a nonspinning compact object, the background metric is spherically symmetric. By conservation of parity, the body's linear response contribution to the mass-type (respectively current-type) multipole moments can only couple to the electric-type (respectively magnetic-type) tidal moments,

$$\delta M_L = \lambda^{\rm el}_\ell \mathcal{E}_L \quad \text{and} \quad \delta S_L = \lambda^{\rm mag}_\ell \mathcal{B}_L, \tag{6}$$

where the tidal deformability parameters $\lambda^{\rm el}_\ell$ and $\lambda^{\rm mag}_\ell$ are constant. If $R$ denotes the areal radius of the central body, then its dimensionless gravitoelectric and gravitomagnetic TLNs $k^{\rm el}_\ell$ and $k^{\rm mag}_\ell$ are defined as per the formula (2) above. If the central object is spinning, however, then the spherical symmetry of the background metric is broken. Consequently, (i) the multipoles $(\delta M_L, \delta S_L)$ and the tidal moments $(\mathcal{E}_L, \mathcal{B}_L)$ cannot obey simple proportionality relationships akin to Eq. (6), (ii) the degeneracy of the azimuthal number $m$ is lifted, (iii) fields with different parity can now mix, and (iv) the spherical-harmonic modes $M_{\ell m}$ and $S_{\ell m}$ of $M_L$ and $S_L$ can couple to modes $\mathcal{E}_{\ell' m}$ and $\mathcal{B}_{\ell' m}$ of $\mathcal{E}_L$ and $\mathcal{B}_L$ with $\ell' \neq \ell$ [41].

*Quadrupole moments.*—We consider a Kerr black hole of mass $M$ and spin angular momentum per unit mass $a$ embedded in a weak and stationary, but otherwise completely generic tidal environment. Hence, we work to linear order in the weak tidal perturbation, so that the TLNs are constants. Denoting by $\mathring{M}_{\ell m} = \mathring{M}_\ell \delta_{m0}$ and $\mathring{S}_{\ell m} = \mathring{S}_\ell \delta_{m0}$ the modes of, respectively, the mass-type and current-type multipole moments of the axisymmetric Kerr background spacetime, the multipole moments of the perturbed Kerr geometry read as

$$M_{\ell m} = \overset{\circ}{M}_{\ell m} + \lambda^{M\mathcal{E}}_{\ell m}\mathcal{E}_{\ell m} + \lambda^{M\mathcal{B}}_{\ell m}\mathcal{B}_{\ell m}, \tag{7a}$$

$$S_{\ell m} = \overset{\circ}{S}_{\ell m} + \lambda^{S\mathcal{E}}_{\ell m}\mathcal{E}_{\ell m} + \lambda^{S\mathcal{B}}_{\ell m}\mathcal{B}_{\ell m}, \tag{7b}$$

where $\lambda^{M\mathcal{E}}_{\ell m}$, $\lambda^{M\mathcal{B}}_{\ell m}$, $\lambda^{S\mathcal{E}}_{\ell m}$, and $\lambda^{S\mathcal{B}}_{\ell m}$ are the four families of Kerr TLNs [30,40], which are complex valued. In here, we do not include couplings between different $\ell$ modes because, as we shall show, such couplings are absent in our explicit results for the quadrupole moments $M_{2m}$ and $S_{2m}$.

We computed [44] the quadrupolar Kerr TLNs explicitly up to *linear* order in the dimensionless spin parameter $\chi \equiv a/M$. For convenience, we introduce the symbol "$\doteq$" for an equality that holds to that order. For any azimuthal number $|m| \leq 2$, the result simply reads

$$\lambda^{M\mathcal{E}}_{2m} \doteq \lambda^{S\mathcal{B}}_{2m} \doteq \frac{im\chi}{180}(2M)^5 \quad \text{and} \quad \lambda^{M\mathcal{B}}_{2m} \doteq \lambda^{S\mathcal{E}}_{2m} \doteq 0. \tag{8}$$

The mass-type (respectively current-type) quadrupole moment couples *only* to the electric-type (respectively magnetic-type) quadrupolar tidal perturbation. The coupling between $(\mathcal{E}_{2m}, \mathcal{B}_{2m})$ and $(M_{2m}, S_{2m})$ which arises from (7) and (8) is akin to a Zeeman-like splitting proportional to the azimuthal number $m$ [40]. Remarkably, the nonvanishing quadrupolar TLNs (8) are purely imaginary. In the related context of *nonstatic* tidal perturbations of *nonspinning* black holes [28,33,66], the imaginary part of the linear response function is well known to give rise to purely dissipative effects, such as tidal heating.

Equations (7)–(8) imply that a spinning black hole becomes tidally deformed under the effect of a weak, nonaxisymmetric, static tidal field. In particular, while in a binary system, a spinning black hole falls in Love with its companion. In the nonspinning limit ($\chi = 0$) or for an axisymmetric tidal field ($m = 0$), however, the TLNs in (8) vanish, in agreement with the results in [25–30]. In fact, we have extended those results to an arbitrarily spinning black hole in a generic multipolar tidal environment, as we show in [44]. Our results agree with the previous ones but there is an apparent disagreement with Ref. [31], who found vanishing black hole TLNs for a generic (nonaxisymmetric) quadrupolar tidal perturbation. This disagreement is a consequence of the use of different splits of the full physical solution into tidal and response contributions. As explained in [44], our tidal-response split relies on the analytic continuation of $\ell \in \mathbb{R}$, which allows us to identify uniquely and unambiguously the two large-radius asymptotic behaviors to be matched onto the known Newtonian solution, whereas the tidal-response split of Ref. [31] relies on imposing smoothness of the (nonphysical) tidal solution on the black hole horizon.

The event horizon "radius" of a slowly spinning Kerr black hole is $2M[1 + O(\chi^2)]$. Therefore, by analogy with the TLNs (2) introduced above for a spherical Newtonian body, we define the dimensionless black hole TLNs according to

$$k^{M\mathcal{E}}_{\ell m} \equiv -\frac{(2\ell-1)!!}{2(\ell-2)!}\frac{\lambda^{M\mathcal{E}}_{\ell m}}{(2M)^{2\ell+1}}, \tag{9}$$

and similarly for the $M\mathcal{B}$, $S\mathcal{E}$, and $S\mathcal{B}$ couplings. These TLNs generalize to slowly spinning black holes those for nonspinning compact objects [25,26]. For a Kerr black hole with spin $\chi \sim 0.1$, Eqs. (8) and (9) imply $|k^{M\mathcal{E}}_{2,\pm 2}| = |k^{S\mathcal{B}}_{2,\pm 2}| \sim 2 \times 10^{-3}$. This small number could be compared, for instance, to the values $k_2^{\rm el} \sim 0.05 - 0.15$ and $|k_2^{\rm mag}| \lesssim 6 \times 10^{-4}$ of the gravitoelectric and gravitomagnetic quadrupolar TLNs of a *nonspinning* neutron star, depending on the equation of state [25,26]. Hence, while spinning black holes do deform like any self-gravitating body, they are particularly "rigid" compact objects.

*Tidal Love tensor.*—Having related the spherical-harmonic modes of the quadrupole moments to those of the quadrupolar tidal moments, we now relate $\delta M_{ij}$ and $\delta S_{ij}$ to $\mathcal{E}_{ij}$ and $\mathcal{B}_{ij}$ themselves. Multiplying Eq. (7) for $\ell = 2$ by $Y_{2m}$, using Eq. (8), and summing over modes, we obtain $\hat{r}^i\hat{r}^j\delta M_{ij} \propto \hat{r}^i\hat{\phi}^j\mathcal{E}_{ij}$ and $\hat{r}^i\hat{r}^j\delta S_{ij} \propto \hat{r}^i\hat{\phi}^j\mathcal{B}_{ij}$, where the unit angular vector $\hat{\phi}$ is orthogonal to the unit radial vector $\hat{\mathbf{r}}$. Therefore, $\delta M_{ij}$ and $\delta S_{ij}$ cannot be simply proportional to $\mathcal{E}_{ij}$ and $\mathcal{B}_{ij}$, respectively. Rather, they must obey more general tensorial relations of the form

$$\delta M_{ij} = \sum_{k,l}\lambda_{ijkl}\mathcal{E}^{kl} \quad \text{and} \quad \delta S_{ij} = \sum_{k,l}\lambda_{ijkl}\mathcal{B}^{kl}, \tag{10}$$

where the constant tensor $\lambda_{ijkl} = O(\chi)$ is the quadrupolar tidal Love tensor (TLT) of the Kerr black hole. Such a complication with respect to the nonspinning case [25,26] stems from the fact that the black hole spin breaks the spherical symmetry of the background spacetime. Using Eq. (8), an explicit calculation shows that the quadrupolar Kerr black hole TLT is given by

$$(\lambda_{ijkl}) \doteq \frac{\chi}{180}(2M)^5 \begin{pmatrix} \mathbf{I}_{11} & \mathbf{I}_{12} & \mathbf{I}_{13} \\ \mathbf{I}_{12} & -\mathbf{I}_{11} & \mathbf{I}_{23} \\ \mathbf{I}_{13} & \mathbf{I}_{23} & \mathbf{0} \end{pmatrix}, \tag{11}$$

where we introduced the four symmetric and trace-free matrices

$$\mathbf{I}_{11} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \mathbf{I}_{12} \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{I}_{13} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}, \qquad \mathbf{I}_{23} \equiv \begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}, \tag{12}$$

and with the understanding that the first pair of indices in $\lambda_{ijkl}$ indicates one of these $3 \times 3$ matrices and the second pair refers to an element within it. After rewriting this quadrupolar TLT in a more geometrical form, the tidally induced quadrupole moments (10) of a Kerr black hole explicitly read

$$\delta M_{ij} \doteq \frac{\chi}{90}(2M)^5 \sum_{k,l} \mathcal{E}^k_{(i}\varepsilon_{j)kl}\hat{s}^l, \tag{13a}$$

$$\delta S_{ij} \doteq \frac{\chi}{90}(2M)^5 \sum_{k,l} \mathcal{B}^k_{(i}\varepsilon_{j)kl}\hat{s}^l, \tag{13b}$$

where parentheses around indices denote symmetrization with respect to those indices, $\hat{s}^l$ is a unit vector parallel to the black hole spin, and $\varepsilon_{ijk}$ is the totally antisymmetric Levi-Civita symbol with $\varepsilon_{123} = +1$. The tidally induced quadrupoles (13) are compatible with the known tidal torquing of a Kerr black hole interacting with a tidal gravitational environment [41,67].

Let us consider here the specific but important case that the quadrupolar tidal field $\mathcal{E}_{ij}$ is sourced by a static particle of mass $\mu \ll M$ a distance $r \gg M$ away from the black hole, in the direction $\hat{\mathbf{r}}$. Then, in the Newtonian limit, the formulas (10)–(13) imply

$$\langle \delta M_{ij} \rangle \doteq \frac{\chi}{60}(2M)^5 \frac{\mu}{r^3}[\hat{\mathbf{r}} \otimes (\hat{\mathbf{s}} \times \hat{\mathbf{r}}) + (\hat{\mathbf{s}} \times \hat{\mathbf{r}}) \otimes \hat{\mathbf{r}}], \tag{14}$$

where $\times$ denotes the cross product and $\otimes$ the tensor product. If the particle lies along the axisymmetry axis of the background Kerr geometry (above one of the poles), then $\hat{\mathbf{s}} \times \hat{\mathbf{r}} = 0$ and $\delta M_{ij}$ vanishes, in agreement with the vanishing of the TLNs (8) for an axisymmetric tidal perturbation. If the particle lies on the equatorial plane, then $\langle \delta M_{ij} \rangle \propto \hat{\mathbf{r}} \otimes \hat{\phi} + \hat{\phi} \otimes \hat{\mathbf{r}}$, which is the gravitational analog of the quadrupole moment tensor obtained by setting four electric charges with alternating signs at the corners of a square centered at the origin and whose four sides are tangent to the directions $\hat{\mathbf{r}}$ and $\hat{\phi}$. Interestingly, the purely imaginary TLNs in (8) and the induced mass quadrupole moment (14) suggest that the black hole tidal bulge is rotated by 45∘ with respect to the quadrupolar tidal perturbation, which may be interpreted as a "tidal lag."

*Speculation.*—As suggested by this tidal lag and as argued in Ref. [66], the purely imaginary TLNs (8) may give rise to dissipative effects *only*, such as the Kerr tidal torquing discussed in Ref. [41]. However, under the assumption that the induced quadrupole moments (13) also give rise to conservative effects, there is the exciting prospect that the planned space-based gravitational-wave observatory *LISA* [21] might be able to detect this specific tidal polarization. One of the main sources for *LISA* is the radiation-reaction driven inspiral of a stellar-mass compact object of mass $\mu$ into a massive black hole of mass $M \gg \mu$. An order-of-magnitude *estimate* of the contribution $\Phi_{\rm tidal}$ of the black hole quadrupolar tidal deformability to the total accumulated gravitational-wave phase in such an inspiral is given by applying the formula (11) in Ref. [22], in which we may tentatively use the typical value $k_1 \sim |k_{22}| \doteq \chi/60$ derived from (8) and (9) for a slowly rotating Kerr black hole [68]. For instance, for a mass ratio $M/\mu = 10^7$ and a Kerr black hole spin $\chi = 0.1$, this yields $|\Phi_{\rm tidal}| \simeq 2 \times 10^3$ rad, much larger than the detectability threshold of $|\Phi_{\rm tidal}| > 1$ rad.

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