Exact Liouvillian Spectrum of a One-Dimensional Dissipative Hubbard Model

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A one-dimensional dissipative Hubbard model with two-body loss is shown to be exactly solvable. We obtain an exact eigenspectrum of a Liouvillian superoperator by employing a non-Hermitian extension of the Bethe-ansatz method. We find steady states, the Liouvillian gap, and an exceptional point that is accompanied by the divergence of the correlation length. A dissipative version of spin-charge separation induced by the quantum Zeno effect is also demonstrated. Our result presents a new class of exactly solvable Liouvillians of open quantum many-body systems, which can be tested with ultracold atoms subject to inelastic collisions.

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In quantum physics, no realistic system can avoid the coupling to an environment. The problem of decoherence and dissipation due to an environment is crucial even for small quantum systems. Furthermore, recent remarkable progress in quantum simulations with a large number of atoms, molecules, and ions has raised a fundamental and practical problem of understanding open quantum many-body systems, where interparticle correlations are essential [1–4]. Within the Markovian approximation, the nonunitary dynamics of an open quantum system is generated by a Liouvillian superoperator acting on the density matrix of the system [5–7]. While interesting solvable examples have been found [8–18], the diagonalization of a Liouvillian of a quantum many-body system is more challenging than that of a Hamiltonian. Extending the class of exactly solvable models to the realm of dissipative systems and discovering prototypical solvable models that can be realized experimentally should promote the deepening of our understanding of strongly correlated open quantum systems.

The Hubbard Hamiltonian provides a quintessential model in quantum many-body physics, where the interplay between quantum-mechanical hopping and interactions plays a key role. In particular, equilibrium properties of the one-dimensional case are well understood with the help of the exact solutions [19–21]. The Hubbard model has been experimentally realized with ultracold fermionic atoms in optical lattices [22], and the high controllability in such systems has recently invigorated the investigation of the effect of dissipation due to particle losses [23]. In this Letter, we show that the one-dimensional Hubbard model subject to two-body particle losses is exactly solvable. On the basis of the exact solution, we obtain an eigenspectrum of the Liouvillian and elucidate how dissipation fundamentally alters the physics of the Hubbard model. Our main findings are threefold. First, we obtain the exact steady states and long-lived eigenmodes that govern the relaxation dynamics after a sufficiently long time. Second, we show that excitations above the Hubbard gap are significantly altered by dissipation and find that the model shows novel critical behavior near an exceptional point [24] that originates from the nondiagonalizability of the Liouvillian. Third, we demonstrate that spin-charge separation, which is a salient feature of one-dimensional systems [25], is extended to dissipative systems by exploiting the fact that the strong correlation is induced by dissipation even in the absence of an interaction. Our result shows that a number of exactly solvable Liouvillians can be constructed from quantum integrable models subject to loss.

Setup.—We consider an open quantum many-body system described by a quantum master equation in the Gorini-Kossakowski-Sudarshan-Lindblad form [5–7]

$$\frac{d\rho}{d\tau} = -i[H,\rho] + \sum_{j=1}^{L}\left(L_j\rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j,\rho\}\right) \equiv \mathcal{L}\rho, \quad (1)$$

where $\rho(\tau)$ is the density matrix of a system at time τ. The system Hamiltonian H is given by the Hubbard model on an L-site chain

$$H = -t\sum_{j=1}^{L}\sum_{\sigma=\uparrow,\downarrow}(c_{j,\sigma}^\dagger c_{j+1,\sigma} + \mathrm{H.c.}) + U\sum_{j=1}^{L} n_{j,\uparrow}n_{j,\downarrow}, \quad (2)$$

where $c_{j,\sigma}$ is the annihilation operator of a spin-σ fermion at site j, and $n_{j,\sigma} \equiv c_{j,\sigma}^\dagger c_{j,\sigma}$. The Lindblad operator $L_j = \sqrt{2\gamma}c_{j,\downarrow}c_{j,\uparrow}$ describes a two-body loss at site j with rate $\gamma > 0$, which is caused by on-site inelastic collisions between fermions as observed in cold-atom experiments [23,26–28].

The formal solution of the quantum master equation can be written down in terms of the eigensystem of the Liouvillian superoperator $\mathcal{L}$ defined in Eq. (1). In this Letter, we aim at diagonalizing the Liouvillian and obtain exact results for the effect of dissipation on correlated many-body systems.

Diagonalization of the Liouvillian.—The one-dimensional Hubbard model, Eq. (2), is known to be solvable with the Bethe ansatz [19–21]. Here, we generalize the solvability of the Hubbard Hamiltonian to that of the Liouvillian on the basis of the existence of a conserved quantity in the Hamiltonian [29]. We first decompose the Liouvillian into two parts as $\mathcal{L} = \mathcal{K} + \mathcal{J}$, where $\mathcal{K}\rho \equiv -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^{\dagger})$ and $\mathcal{J}\rho \equiv \sum_{j=1}^{L} L_j \rho L_j^{\dagger}$. The effective non-Hermitian Hamiltonian H_{eff} is given by $H_{\mathrm{eff}} \equiv H - \frac{i}{2}\sum_{j=1}^{L} L_j^{\dagger} L_j$, and its explicit form is obtained by replacing U in H with $U - i\gamma$, thereby making the interaction strength complex-valued [30–36]. Notably, the one-dimensional Hubbard model with a complex-valued interaction strength is still integrable [12,18,33]. If the interaction strength becomes complex-valued, the SO(4) symmetry of the Hubbard Hamiltonian [37–39] remains intact. In particular, an eigenstate of the non-Hermitian Hubbard model can be labeled by the number of particles. Let $|N, a\rangle_R$ be a right eigenstate of H_{eff} with N particles: $H_{\mathrm{eff}}|N, a\rangle_R = E_{N,a}|N, a\rangle_R$, where a distinguishes the eigenstates having the same particle number. Then, one can diagonalize the superoperator $\mathcal{K}$ as $\mathcal{K}\varrho_{ab}^{(N,n)} = \lambda_{ab}^{(N,n)}\varrho_{ab}^{(N,n)}$, where $\lambda_{ab}^{(N,n)} \equiv -i(E_{N,a} - E_{N+n,b}^{*})$ and $\varrho_{ab}^{(N,n)} \equiv |N, a\rangle_{RR}\langle N + n, b|$. The superoperator $\mathcal{J}$ lowers the particle number but never increases it. Thus, in the representation with the basis $\{\varrho_{ab}^{(N,n)}\}_{N,a,b}$, the Liouvillian $\mathcal{L}$ is a triangular matrix that can easily be diagonalized. This is a general property of Liouvillians of systems with loss [29]. Indeed, because the eigenvalues of a triangular matrix are given by its diagonal elements, the eigenvalues of the Liouvillian are given by $\lambda_{ab}^{(N,n)}$. The corresponding right eigenoperator is given by a linear combination of the basis as $C_{ab}^{(N,n)}\varrho_{ab}^{(N,n)} + \sum_{N'=0}^{N-2}\sum_{a',b'} D_{aba'b'}^{(N,N',n)}\varrho_{a'b'}^{(N',n)}$, where the coefficients $D_{aba'b'}^{(N,N',n)}$ are obtained from the matrix elements ${}_L\langle N' - 2, r|L_j|N', r'\rangle_R$ of the Lindblad operator L_j with $|N', r\rangle_L$ being the left eigenstate dual to $|N', r\rangle_R$ [29,40]. We thus conclude that if the non-Hermitian Hubbard Hamiltonian H_{eff} is integrable, the Liouvillian $\mathcal{L}$ is solvable. Note that this does not mean that the Liouvillian itself has an integrable structure such as the Yang-Baxter relation. Therefore, the mechanism of the solvability here is different from those of previous works on Yang-Baxter integrable Liouvillians [12,16–18].

Steady states.—A steady state of the system is characterized by an eigenoperator of $\mathcal{L}$ with zero eigenvalue. If a state $|\Psi\rangle$ is a right eigenstate of H_{eff} with a real eigenvalue, one can show $L_j|\Psi\rangle = 0$, and hence $|\Psi\rangle\langle\Psi|$ is a steady state [40]. For example, the fermion vacuum $|0\rangle\langle 0|$ is trivially a steady state. Also, in the Hilbert subspace with no spin-down particles, all eigenstates of H_{eff} coincide with those in the noninteracting ($U = \gamma = 0$) case and thus describe steady states. By letting the spin lowering operator act on the spin-polarized eigenstates, one can construct many steady states owing to the spin SU(2) symmetry of H_{eff}, reflecting the fact that magnetization is conserved during the dynamics [42,43]. Clearly, these steady states are ferromagnetic and far from the thermal equilibrium states of the one-dimensional Hubbard model. Physically, the steadiness of the ferromagnetic states can be understood from the Fermi statistics because the spin wave function that is fully symmetric with respect to a particle exchange requires antisymmetry in the real-space wave function and forbids doubly occupied sites that cause a decay, as observed in Refs. [35,44]. In general, a steady state realized after a time evolution becomes a statistical mixture of the above steady states that depends on the initial condition.

Bethe ansatz.—We use the Bethe ansatz to obtain the eigenspectrum of the non-Hermitian Hubbard model H_{eff}. The Bethe equations are [19–21]

$$k_j L = \Phi + 2\pi I_j - \sum_{\beta=1}^{M} \Theta\left(\frac{\sin k_j - \lambda_\beta}{u}\right), \tag{3}$$

$$-\sum_{j=1}^{N} \Theta\left(\frac{\sin k_j - \lambda_\alpha}{u}\right) = 2\pi J_\alpha + \sum_{\beta=1}^{M} \Theta\left(\frac{\lambda_\alpha - \lambda_\beta}{2u}\right), \tag{4}$$

where N is the number of particles, M is the number of down spins, $k_j (j = 1, \ldots, N)$ is a quasimomentum, $\lambda_\alpha (\alpha = 1, \ldots, M)$ is a spin rapidity, $u \equiv (U - i\gamma)/(4t)$ is a dimensionless complex interaction coefficient, and $\Theta(z) \equiv 2 \arctan z$. The quantum number I_j takes an integer (half-integer) value for even (odd) M, and J_α takes an integer (half-integer) value for odd (even) $N - M$. Here we employ a twisted boundary condition $c_{L+1,\sigma} = e^{-i\Phi}c_{1,\sigma}$ for later convenience, but basically set $\Phi = 0$ (i.e., the periodic boundary condition) unless otherwise specified.

Liouvillian gap.—The late-stage dynamics of the system near a steady state is governed by long-lived eigenmodes whose eigenvalues are close to zero [45]. By construction of the steady states, the long-lived eigenmodes correspond to Bethe eigenstates in the $M = 1$ case and their descendants derived from the spin SU(2) symmetry. They consist of ferromagnetic spin-wave-type excitations, and their dispersion relation is obtained by a standard calculation with the Bethe ansatz [40]. Taking consecutive charge quantum numbers $I_j = -(N + 1)/2 + j$, which express the simplest situation of charge excitations from the Fermi surface, we obtain an analytic expression for the dispersion relation of the spin excitations,

$$\Delta E \simeq -\frac{t}{\pi u}\left(Q_0 - \frac{1}{2}\sin 2Q_0\right)\left(1 - \cos\frac{\pi \Delta P}{Q_0}\right), \qquad (5)$$

for the momentum $\Delta P \simeq 0$ where $Q_0 = \pi N/L$ is the Fermi momentum. Since the momentum is discretized in units of $2\pi/L$, the gapless quadratic dispersion around $\Delta P = 0$ leads to the smallest imaginary part of the excitation energy $|\mathrm{Im}[\Delta E]|$ proportional to $1/L^2$. Thus, the Liouvillian gap, which is defined by the largest nonzero real part of eigenvalues of the Liouvillian, vanishes in the thermodynamic limit, implying a power-law time dependence in the decay dynamics [45].

Hubbard gap, correlation length, and exceptional point.—Next, we consider the half-filling case ($L = N = 2M$) and focus on the solution that can be adiabatically connected to the ground state of the Hermitian Hubbard model in the limit of $\gamma \to 0$. Such a solution may not contribute to the late-stage behavior due to a short lifetime, but it can be used to study the early-time decay dynamics of a Mott insulator. We here assume that $U > 0$ and $N(M)$ is even (odd), and set $I_j = -(N+1)/2 + j$ and $J_\alpha = -(M+1)/2 + \alpha$ as in the Hermitian case. In the thermodynamic limit, the Bethe equations, Eqs. (3) and (4), reduce to the integral equations for distribution functions $\rho(k)$ and $\sigma(\lambda)$ as

$$\rho(k) = \frac{1}{2\pi} + \cos k \int_{\mathcal{S}} d\lambda a_1(\sin k - \lambda)\sigma(\lambda), \qquad (6)$$

$$\sigma(\lambda) = \int_{\mathcal{C}} dk a_1(\sin k - \lambda)\rho(k) - \int_{\mathcal{S}} d\lambda' a_2(\lambda - \lambda')\sigma(\lambda'), \qquad (7)$$

where $a_n(z) \equiv (1/\pi)[nu/(z^2 + n^2u^2)]$, and $\mathcal{C}$ and $\mathcal{S}$ denote the trajectories of quasimomenta and spin rapidities, respectively [21]. Figure 1(a),(b) show typical distributions of $\{k_j\}_{j=1,\ldots,N}$ and $\{\lambda_\alpha\}_{\alpha=1,\ldots,M}$ that are obtained from the solution of the Bethe equations, Eqs. (3) and (4). The distributions indicate that if the trajectories $\mathcal{C}$ and $\mathcal{S}$ do not enclose a pole in the integrands of Eqs. (6) and (7), the trajectories can continuously be deformed to those of the $\gamma = 0$ case, i.e., $\mathcal{C} = [-\pi, \pi]$ and $\mathcal{S} = (-\infty, \infty)$. Thus, we obtain the eigenvalue E_0 in the thermodynamic limit from analytic continuation of the solution in the $\gamma = 0$ case [19] as

$$E_0/L = -2t\int_{-\infty}^{\infty} d\omega \frac{J_0(\omega)J_1(\omega)}{\omega(1 + e^{2u|\omega|})}, \qquad (8)$$

where $J_n(x)$ is the nth Bessel function. Similarly, the Hubbard gap Δ_c [19,46] is given as

$$\Delta_c = 4tu - 4t\left[1 - \int_{-\infty}^{\infty} d\omega \frac{J_1(\omega)}{\omega(1 + e^{2u|\omega|})}\right]. \qquad (9)$$

Here E_0 and Δ_c take complex values in general. The lifetime of an eigenmode can be extracted from the imaginary part of the eigenvalue. The absolute value of $\mathrm{Im}[E_0] \le 0$ first increases with increasing γ, takes the maximum at some point, and then decreases [40]. The decreasing behavior at large γ is attributed to the continuous quantum Zeno effect [26,27,47–50], which prevents the creation of doubly occupied sites in eigenstates due to a large cost of the imaginary part of energy. By contrast, the absolute value of $\mathrm{Im}[\Delta_c] \le 0$ monotonically increases with increasing γ [40] since the excitation corresponding to the Hubbard gap creates doubly occupied sites. As the Liouvillian eigenvalues appear as poles of a single-particle Green's function [40,51], the dependence of the Hubbard gap on dissipation can be found from the linear response of the dynamics by, e.g., lattice modulation spectroscopy [40,52,53].

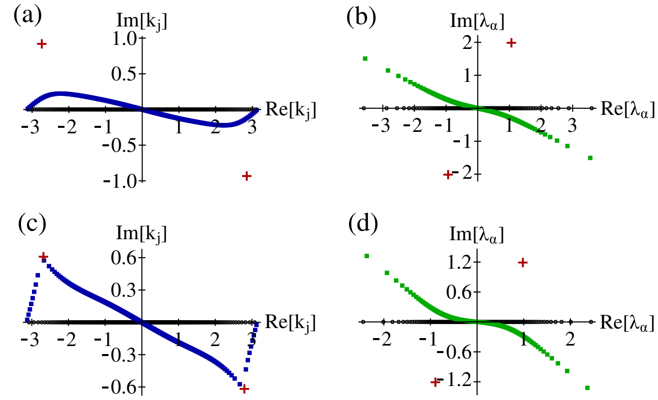

FIG. 1. Numerical solutions of the Bethe equations, Eqs. (3) and (4), for $L = N = 2M = 250$. (a),(c) Blue dots show quasimomenta $\{k_j\}$, and red crosses show the locations of poles at $k = \pm\pi - \arcsin(\pm iu)$. (b),(d) Green dots show spin rapidities $\{\lambda_\alpha\}$, and red crosses show the locations of poles at $\lambda = \pm 2iu$. The interaction strength is set to $u = 1 - 0.5i$ [(a),(b)] and $u = 0.6 - 0.469i$ [(c),(d)]. Points on the real axis show the solutions for the case of $\gamma = 0$ at the same U for comparison.

To further elucidate the physics of the dissipative Mott insulator, we calculate the correlation length ξ of the above eigenstate from the asymptotic behavior of the charge stiffness as $|[d^2E_0(\Phi)]/[d\Phi^2]|_{\Phi=0}| \sim \exp[-L/\xi](L \to \infty)$ [54]. The correlation length quantifies the dependence of the dynamics on the boundary condition and thus measures the spatial correlation in the eigenmode. We find that the correlation length is obtained from the analytic continuation of the result for the $\gamma = 0$ case [54]:

$$\frac{1}{\xi} = \mathrm{Re}\left[\frac{1}{u}\int_1^{\infty} dy \frac{\ln(y + \sqrt{y^2 - 1})}{\cosh(\pi y/2u)}\right]. \qquad (10)$$

Figure 2(a)–(c) show the correlation length for different values of the repulsive interaction. For large γ, the correlation length decreases in all cases, indicating that particles are more localized due to dissipation. This behavior is consistent with the quantum Zeno effect [26,27,47–50]. On the other hand, when U is small, the

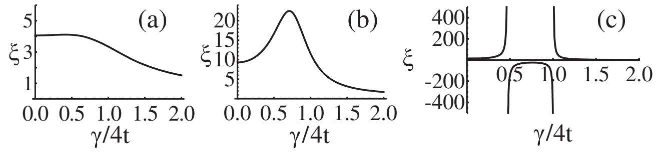

FIG. 2. Correlation length ξ [Eq. (10)] for (a) $U/4t = 1$, (b) $U/4t = 0.7$, and (c) $U/4t = 0.6$.

correlation length grows at an intermediate dissipation strength [see Fig. 2(b)], implying that dissipation facilitates the delocalization of particles. Surprisingly, the correlation length even diverges for small U and takes negative values in between the divergence points [see Fig. 2(c)]. When the correlation length diverges, the trajectory $\mathcal{C}$ crosses poles in the integrand of Eq. (7), thereby preventing the trajectory from deforming to the real axis. This fact can be seen numerically (see red crosses on (off) the trajectory $\mathcal{C}$ ($\mathcal{S}$) in Fig. 1(c) [(d)]) and can also be shown analytically using the Bethe equations [40]. In fact, the solution with $\xi < 0$ is not a solution of the Bethe equations, and the analytic continuation from the Hermitian case breaks down. Similar transitions of Bethe-ansatz solutions have been found in other non-Hermitian integrable models [33,55,56].

The poles in the integrand in the first term on the right-hand side of Eq. (7) are given by $\sin k = \lambda \pm iu$. The same condition appears in the construction of the k-λ string excitations in the Hubbard model [21,57] in which a pair of quasimomenta $k^{(1)}, k^{(2)}$ form a string configuration around a center λ as $\sin k^{(1)} = \lambda + iu$ and $\sin k^{(2)} = \lambda - iu$. Physically, such string excitations describe the creation of a doublon-holon pair [21]. The existence of the poles on trajectory $\mathcal{C}$ indicates that the solution in the thermodynamic limit becomes degenerate with a k-λ string solution. In fact, the excitation energy of a k-λ string is given by [21,46]

$$\varepsilon(k) = 2tu + 2t\cos k + 2t\int_0^\infty d\omega \frac{J_1(\omega)\cos(\omega \sin k)e^{-u\omega}}{\omega \cosh u\omega}, \tag{11}$$

which vanishes at the poles $k = \pm\pi - \arcsin(\pm iu)$. Here not only the eigenvalues but also the eigenstates are the same. This means that the critical point at which the correlation length diverges is an exceptional point in the sense that the non-Hermitian Hamiltonian H_{eff} cannot be diagonalized [24,58]. Importantly, we can show that the nondiagonalizability of H_{eff} leads to the nondiagonalizability of the Liouvillian $\mathcal{L}$ [40]. Thus, the exceptional point is the same for both the non-Hermitian Hamiltonian and the Liouvillian; however, this does not hold true for general Liouvillians [59]. Since a nondiagonalizable Liouvillian leads to a singular time dependence of generalized eigenmodes [59], the exceptional point significantly alters the transient dynamics starting from half filling.

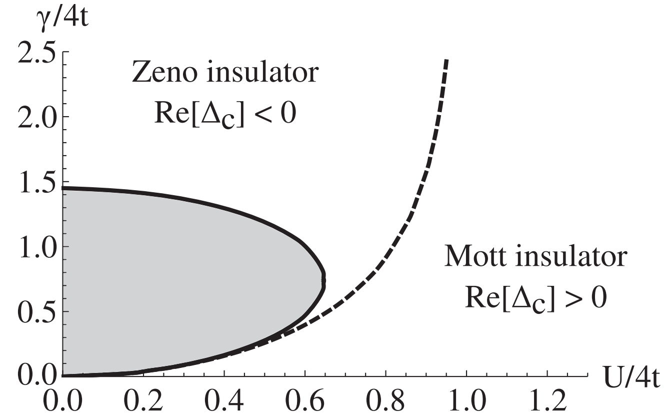

FIG. 3. "Phase diagram" of the Liouvillian eigenmode that governs the transient dynamics at half filling. The solid curve indicates the location of the exceptional point at which the Liouvillian cannot be diagonalized. The shaded region cannot be analytically continued from the case with $\gamma = 0$. The dashed curve shows where the real part of the Hubbard gap $\mathrm{Re}[\Delta_c]$ vanishes.

The solid curve in Fig. 3 shows the position of the exceptional point as a function of U and γ. Outside the shaded region, the analytic continuation of the Bethe-ansatz solution from the $\gamma = 0$ case remains valid. An increase of the correlation length in Fig. 2(b) can be understood as a consequence of the proximity of the system to the exceptional point. For a large repulsive interaction $U > 0$, a Mott insulator is formed as in the Hermitian Hubbard model and it has a finite lifetime due to nonzero γ. On the other hand, for small $U > 0$ and large γ, particles are localized due to dissipation. Because the Hubbard gap becomes negative, $\mathrm{Re}[\Delta_c] < 0$, in this region, the localization should be attributed to the quantum Zeno effect rather than the repulsive interaction, and therefore this localized state may be called a Zeno insulator. Interestingly, the phase diagram looks qualitatively similar to that obtained from a mean-field theory for a three-dimensional non-Hermitian attractive Hubbard model [34] after changing the sign of U via the Shiba transformation [60].

Dissipation-induced spin-charge separation.—Finally, we address an interesting connection between strong correlations and dissipation. The Bethe equations, Eqs. (3) and (4), can be simplified when one takes the large-$|u|$ limit in which one can expand the equations as (here we set $\Phi = 0$)

$$k_j L = 2\pi I_j + \mathcal{O}(1/u), \tag{12}$$

$$N\Theta\left(\frac{\lambda_\alpha}{u}\right) + \mathcal{O}(1/u^2) = 2\pi J_\alpha + \sum_{\beta=1}^{M} \Theta\left(\frac{\lambda_\alpha - \lambda_\beta}{2u}\right). \tag{13}$$

These equations indicate that the quasimomenta and spin rapidities are completely decoupled in the $|u| \to \infty$ limit [46,61]. The quasimomenta in this limit are identical to those of free fermions, and Eq. (13) gives the same Bethe equation as that of the Heisenberg chain after rescaling $\Lambda_\alpha \equiv \lambda_\alpha / u$. This leads to a remarkable fact that the Bethe wave function is factorized into the charge part and the spin part [61]. This argument is parallel to that for the spin-charge separation in the one-dimensional Hermitian Hubbard model. However, the crucial point here is that the spin-charge separation can occur due to large γ even in the absence of the repulsive interaction U. Thus, in a Zeno insulator, the strong dissipation itself induces a strongly correlated state, and holes created by a loss behave as almost free fermions, whereas the spin excitations are described by a non-Hermitian Heisenberg chain with the exchange coupling $4t^2/(U - i\gamma)$ [35]. As spin-charge separation in a Hermitian Hubbard chain has recently been observed in experiments with ultracold atoms [62,63], the dissipation-induced spin-charge separation should be observed with current experimental techniques.

Conclusion.—We have shown that the one-dimensional dissipative Hubbard model is exactly solvable. We have exploited the integrability of a non-Hermitian Hamiltonian to diagonalize a Liouvillian using the generic triangular structure of Liouvillians of systems with loss [29]. We have elucidated how strongly correlated states of the Hubbard model are fundamentally altered by dissipation, yet a number of important issues remain open. For example, the breakdown of the analytic continuation at half filling suggests that a novel state driven by an interplay between strong correlations and dissipation may be realized in the shaded region of Fig. 3. Since the standard solution for the Hermitian Hubbard model cannot be applied to that region, it is worthwhile to investigate the nature of Bethe-ansatz solutions with non-Hermitian interactions, as discussed in Refs. [12,18]. Finally, the solution of Liouvillians based on the non-Hermitian Bethe-ansatz method is not limited to the Hubbard model but applicable to other many-body integrable systems with appropriate Lindblad operators [29]. Examples include one-dimensional Bose [64,65] and Fermi [66,67] gases subject to particle losses [30], quantum impurity models [68,69] with dissipation at an impurity [33], and an XXZ spin chain [70,71] with Lindblad operators that lower the magnetization [72]. We expect that the method proposed in this Letter can be exploited to uncover as yet unexplored exactly solvable models in open quantum many-body systems.

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Note added.—After the submission of this manuscript, a related work [72] appeared in which the Bethe-ansatz approach to triangular Liouvillians is studied for different models.

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