Comment on "Dispersion Interaction between Two Hydrogen Atoms in a Static Electric Field"

In a motivating paper [1], G. Fiscelli *et al.* show that the interaction between two hydrogen atoms in their fundamental states, submitted to an external electrostatic field $\mathbf{E}_0$, may change into a repulsive one, depending on the distance z_a between the atoms, the value of $E_0 := |\mathbf{E}_0|$, and its orientation θ_0 relative to the atoms. However, we argue that the main conceptual point was not properly emphasized since it is not the dispersion force that changes sign, but the resultant force (sum of the dispersion and the electrostatic forces). It is this electrostatic contribution between the electric dipoles induced in both atoms by $\mathbf{E}_0$ that can become repulsive and overcome the dispersion force. The misunderstanding relies on the fact that the electrostatic and the dispersion interactions are contained in Eq. (6) of their Letter. As we show, the electrostatic contribution is dominant and the dispersion interaction remains attractive because the latter is slightly modified by the field. A similar situation arises if, instead of an external electrostatic field, we consider the interaction between permanent dipoles [2].

To assess the orders of magnitude involved, it suffices to analyze the polarizability $\alpha(\omega) = (2/3\hbar) \times \sum_n \omega_{n0}|\langle n|\hat{\mathbf{d}}|0\rangle|^2/(\omega_{n0}^2 - \omega^2)$. To estimate its variation $\delta\alpha(\omega)$, we investigate the transition from the fundamental state to the first excited state, with $z_a = 1\ \mu$m and $E_0 = 10^5$ V/m. As $z_a \gg c/\omega_0$ (ω_0 is the dominant transition frequency), we can take $\omega = 0$. Denoting by $\alpha_0 := \alpha(\omega = 0)$, $\mathbf{d}_{10} := \langle 1|\hat{\mathbf{d}}|0\rangle$, we have $\delta\alpha_0 = (2/3\hbar)[-(\mathbf{d}_{10}^2\delta\omega_{10}/\omega_{10}^2) + (2\mathbf{d}_{10}\cdot\delta\mathbf{d}_{10}/\omega_{10})]$. The relative contribution of the first term is $(\delta\alpha_0/\alpha_0) = -(\delta\omega_{10}/\omega_{10})$, with $\delta\omega_{10}$ being readily obtained from the Stark effect. Thus, whenever perturbation theory works, the effect of the field on the polarizability can be disregarded. From now on, we consider values of Ref. [1] and it follows that $(\delta\alpha_0/\alpha_0) \sim 10^{-10}$. The second term gives an analogous contribution, ensuring that the correction in the dispersion force is negligible. However, each atom under the influence of $\mathbf{E}_0$ acquires an induced electric dipole moment given by $\mathbf{d}_i = \alpha_0\mathbf{E}_0$, in which we are safely disregarding at a given atom the effect of the electric field created by the induced electric dipole on the other atom, since the ratio between the intensity of the latter field and E_0 is $\sim\alpha_0/(\epsilon_0 z_a^3) \sim (a_0/z_a)^3$ (a_0 is the Bohr radius). We choose the axis so that one atom is at the origin, while the other lies at $(0, 0, z_a)$. Since the induced dipoles point in the direction of $\mathbf{E}_0$, the z component of the electrostatic force acting on the atom at $(0, 0, z_a)$ is $F_z^{\mathrm{el}} = (3\alpha_0^2E_0^2/4\pi\epsilon_0 z_a^4)(1 - 3\cos^2\theta_0)$, which changes sign when $\cos^2\theta_0 = 1/3$. Consider the z component of the resultant force, $F_z^R = F_z^{\mathrm{el}} + F^{\mathrm{CP}}$, where $F^{\mathrm{CP}} = -\{[161\hbar c\alpha_0^2]/[(4\pi\epsilon_0)^2 4\pi z_a^8]\}$ is the Casimir-Polder force. Figure 1 shows the ratio $\Gamma := F_z^R/|F^{\mathrm{CP}}|$ as a function of E_0 and θ_0 for $z_a = 1\ \mu$m. The black line corresponds to $\Gamma = 0$ and it distinguishes regimes of repulsion ($\Gamma > 0$) and attraction ($\Gamma < 0$). Note that repulsion may be achieved for feasible values of E_0 in many configurations.

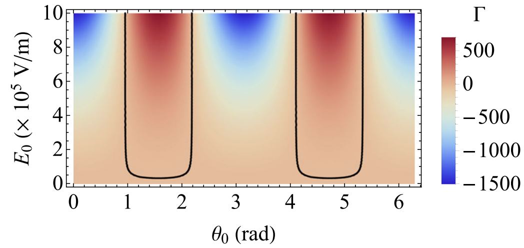

FIG. 1. $\Gamma := F_z^R/|F^{\mathrm{CP}}|$ versus E_0 and θ_0 for $z_a = 1\ \mu$m. The black line separates regions of attractive and repulsive force. $\theta_0 = \pi/2$ furnishes the best repulsion.

Therefore, it is not the dispersion force that changes its attractive character, but the electrostatic force between the induced atomic dipoles that can be altered enough so that the resultant force may change its sign. Nevertheless, the possibility raised by the authors of controlling interatomic interactions through external fields is indeed quite interesting and opens alternative routes for atomic manipulation mechanisms.

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[1] G. Fiscelli, L. Rizzuto, and R. Passante, Dispersion Interaction between Two Hydrogen Atoms in a Static Electric Field, Phys. Rev. Lett. **124**, 013604 (2020).

[2] D. P. Craig and T. Thirunamachandran, *Molecular Quantum Electrodynamics* (Academic Press, New York, 1984); see especially Sec. 7. 8.