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Ergodic and Nonergodic Dual-Unitary Quantum Circuits with Arbitrary Local Hilbert Space Dimension

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Dual-unitary quantum circuits can be used to construct $1+1$ dimensional lattice models for which dynamical correlations of local observables can be explicitly calculated. We show how to analytically construct classes of dual-unitary circuits with any desired level of (non-)ergodicity for any dimension of the local Hilbert space, and present analytical results for thermalization to an infinite-temperature Gibbs state (ergodic) and a generalized Gibbs ensemble (nonergodic). It is shown how a tunable ergodicity-inducing perturbation can be added to a nonergodic circuit without breaking dual unitarity, leading to the appearance of prethermalization plateaux for local observables.

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Introduction.—The dynamics of isolated systems under general unitary evolution remains one of the fundamental problems in many-body physics. Originating as a model for quantum computation [1], unitary circuits can serve as a minimal model for the study of general unitary dynamics governed by local interactions [2–8]. Such circuits also form the basis of Google's Sycamore processor [9]. The use of minimal models for unitary dynamics is motivated by the fact that analytically tractable models of many-body quantum dynamics remain scarce. While a great deal of understanding has been reached through the study of integrable models, these are nongeneric by definition [10–13]. Although unitary circuit dynamics exhibit many of the features expected of generic many-body dynamics and present a natural realization of a periodically driven (Floquet) system [14,15], exact results generally require the presence of randomness in the circuit.

Recently, *dual-unitary circuits* were identified as a class of unitary circuits for which the dynamics of correlations remains tractable, circumventing the need for integrability or randomness [16,17]. These gates are characterized by the property that the resulting circuit evolution is unitary in both time and space. As a result, correlations vanish everywhere except at the edge of the causal light cone [16], where they can be calculated analytically at all time scales [18]. At long times the resulting correlations can remain constant, oscillate, or decay to either an ergodic or nonergodic value. This makes these models particularly attractive for the study of thermalization: after sufficiently long times, it is expected that all local correlations in a many-body system can be described by a reduced density matrix depending only on the conservation laws present in the system [19–21].

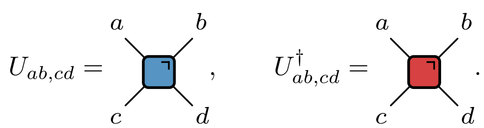
The study of these models started with the realization that the kicked Ising model (KIM) supported an exact calculation of the spectral form factor and entanglement spectrum at particular values of the coupling constants [22,23]. Reference [17] subsequently recast the KIM as a unitary circuit and identified dual unitarity as the underlying reason for the degenerate entanglement spectrum and maximal entanglement growth. Reference [16] introduced the class of dual-unitary gates, and constructed a complete manifold of nontrivial dual-unitary gates for a two-dimensional local Hilbert space. Later works studied more general dynamics, identifying matrix product state initial conditions preserving the solubility of the dynamics [24], correlations of general local operators [25], calculated out-of-time-order correlators [18], later revisited in the general context of scrambling in random unitary circuits [26]. Remarkably, the dynamics generated by perturbed dual-unitary gates can be efficiently described through a path-integral formalism [27].

Almost all such calculations only depend on dual unitarity: these models are solvable for any dimension of the local Hilbert space. Despite this salient feature, systematic realizations of dual-unitary circuits remain relatively restricted. Analytical parametrizations of nontrivial dual-unitary gates are restricted to a two-dimensional local Hilbert space (qubits) [16] and to kicked models built on complex Hadamard matrices for larger Hilbert spaces [25]. Numerically, an iterative protocol has been proposed to generate circuits arbitrarily close to dual unitarity [28], but this does not allow analytic predictions or targeting gates with a desired level of ergodicity.

In this work, we present an analytic parametrization of dual-unitary gates for arbitrary local Hilbert space dimension q, returning circuit models where the dynamics of observables remain analytically tractable. The level of ergodicity is classified through the eigenvalues of quantum channels determining light-cone dynamics, and we show how to systematically realize ensembles of circuits with any desired level of ergodicity. The steady-state values of the correlation functions are shown to be set by either infinite-temperature Gibbs states or generalized Gibbs ensembles (GGEs) in ergodic and nonergodic systems, respectively. In all examples, the total number of free variables scales as q^2. Additionally, we illustrate how an ergodicity-inducing perturbation on top of a nonergodic unitary gate can be introduced without destroying dual unitarity, leading to a class of solvable models illustrating prethermalization to a GGE before eventual thermalization to a featureless infinite-temperature state.

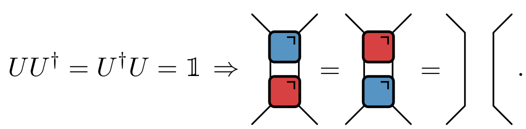
A PYTHON implementation of all presented calculations is available online [29].

Dual-unitary gates.— We consider systems where the time evolution is governed by a unitary circuit consisting of two-site operators, where each gate U and its Hermitian conjugate can be graphically represented as



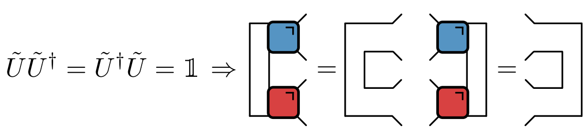
(1)

In this notation each leg carries a local q-dimensional Hilbert space, and the indices of legs connecting two operators are implicitly summed over (see, e.g., Ref. [30]). Unitarity is graphically represented as



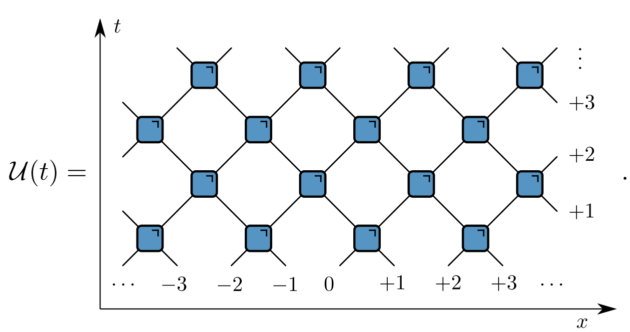
(2)

The dual of U is defined through $\tilde{U}_{ab,cd} = U_{db,ca}$, and dual unitarity is defined as the additional unitarity of $\tilde{U}$ [16,17],



(3)

The full evolution $\mathcal{U}(t)$ at time t consists of the t-times repeated application of staggered two-site gates



Given an infinite lattice with at each site a local Hilbert space $\mathbb{C}^q$, we consider correlation functions

$$c_{\rho\sigma}(x,t) = \mathrm{tr}[\mathcal{U}^\dagger(t)\rho(0)\mathcal{U}(t)\sigma(x)]/\mathrm{tr}(\mathbb{1}), \tag{4}$$

where $\rho, \sigma \in \mathbb{C}^{q\times q}$ are operators acting on a local q-dimensional Hilbert space, and $\rho(x)$, $\sigma(x)$ act as ρ, σ on site x and as the identity everywhere else. We take $\mathrm{tr}(\rho) = 1$ to make the connection with (reduced) density matrices, although this is not a necessary assumption and these can be seen as infinite-temperature correlation functions. Dual unitarity implies that all correlation functions factorize as $c_{\rho\sigma}(x,t) = \mathrm{tr}(\rho)\mathrm{tr}(\sigma)/q$, except on the edges of the light cone $x = \pm t$ [16]. These nontrivial correlation functions can be evaluated as

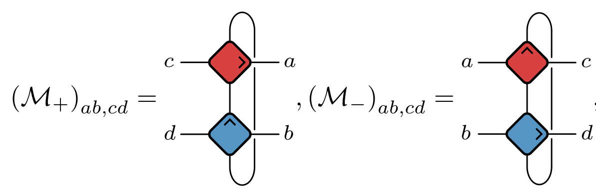
$$c_{\rho\sigma}(\pm t, t) = \mathrm{tr}[\mathcal{M}_\pm^t(\rho)\sigma], \tag{5}$$

where $\mathcal{M}_\pm \in \mathbb{C}^{q^2\times q^2}$ are linear maps defined as

$$\mathcal{M}_+(\rho) = \mathrm{tr}_1[U^\dagger(\rho\otimes\mathbb{1})U]/q, \tag{6}$$

$$\mathcal{M}_-(\rho) = \mathrm{tr}_2[U^\dagger(\mathbb{1}\otimes\rho)U]/q. \tag{7}$$

These are completely positive and trace-preserving maps, acting as a quantum channel. From the unitarity it follows that $\mathcal{M}_\pm(\mathbb{1}) = \mathbb{1}$, such that these channels are unital. Graphically, we can represent



(8)

with matrix elements such that $\mathcal{M}(\rho)_{ab} = \sum_{cd}\mathcal{M}_{ab,cd}\rho_{cd}$. Note that light-cone correlation functions can always be calculated in this way, irrespective of dual unitarity [18].

As shown in Ref. [16], the long-time behavior of all nontrivial correlations and hence the level of ergodicity is fully determined by the number of eigenvalues λ_{ab} of $\mathcal{M}_\pm$ with unit modulus, $|\lambda_{ab}| = 1$, with the corresponding eigenoperators acting as nondecaying modes, excluding the trivial eigenvalue 1 corresponding to the identity operator. At long times, ergodic behavior is evidenced by the convergence of correlations to their thermal value $\lim_{t\to\infty} c_{\rho\sigma}(\pm t, t) = \mathrm{tr}(\sigma)/q$, consistent with thermalization to an infinite-temperature Gibbs state $\rho_{\mathrm{Gibbs}} = \mathbb{1}/q$ such that $\lim_{t\to\infty} c_{\rho\sigma}(\pm t, t) = \mathrm{tr}(\rho_{\mathrm{Gibbs}}\sigma),\ \forall\,\sigma$.

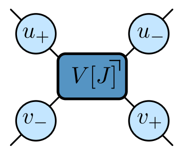
The unitary gates are generally not parity invariant, leading to “chiral” behavior where $\mathcal{M}_\pm$ can have different numbers of nontrivial eigenvalues and corresponding nondecaying modes. In the following, we will consider the behavior along $x = t$, governed by the q^2 eigenvalues of $\mathcal{M}_+$, but this can be immediately extended to $x = -t$.

(1) Noninteracting: All q^2 eigenvalues equal 1, dynamical correlations remain constant. (2) Nonergodic: More than one but less than q^2 eigenvalues are equal to 1,

dynamical correlations decay to a nonthermal constant. (3) Ergodic and nonstationary: All nontrivial eigenvalues are different from 1, but there exists at least one eigenvalue with unit modulus. All correlations oscillate around a time-averaged value corresponding to the thermal value. (4) Ergodic and stationary: All nontrivial eigenvalues lie within the unit disc and all dynamical correlations decay to their thermal value.

Parametrization.—We propose a parametrization of dual-unitary gates $U \in \mathbb{C}^{q^2\times q^2}$ as

$$U = (u_+ \otimes u_-)V[J](v_- \otimes v_+) = \quad (9)$$



with arbitrary one-site unitary gates $u_\pm, v_\pm \in SU(q)$, and where all entanglement is generated by the two-site unitary $V[J]$ defined as

$$V[J]_{ab,cd} = \delta_{ad}\delta_{bc}e^{iJ_{ab}}, \quad (10)$$

with phases set by an arbitrary real matrix $J \in \mathbb{R}^{q\times q}$. Both the unitarity and dual unitarity of $V[J]$ can be readily verified, and the additional one-site unitaries leave both properties intact. We do not expect this parametrization to be exhaustive—the construction from Ref. [28] gives rise to numerically dual-unitary gates that cannot be recast as Eq. (9).

Focusing on $x = t$, we write $\mathcal{M}[U] = \mathcal{M}_+$ and $u, v = u_+, v_+$, but calculations for $x = -t$ are analogous. Plugging the parametrization Eq. (9) in the definition of the quantum channel Eq. (8), we find

$$\mathcal{M}[U] = (v^\dagger \otimes v^T)\mathcal{M}[V](u^\dagger \otimes u^T). \quad (11)$$

The dependence on u_- and v_- drops out and $\mathcal{M}(J) \equiv \mathcal{M}[V]$ can be evaluated as

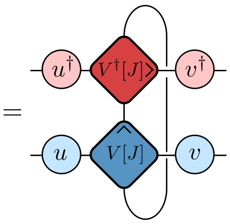
$$\mathcal{M}(J)_{ab,cd} = \frac{1}{q}\sum_{e,f=1}^{q} V[J]^*_{cf,ea}V[J]_{df,eb} = \sigma_{ab}\delta_{ac}\delta_{bd}. \quad (12)$$

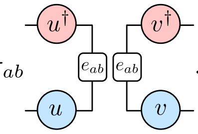
The channel $\mathcal{M}(J)$ is diagonal, with

$$\sigma_{ab} = \frac{1}{q}\sum_{f=1}^{q} e^{-i(J_{af}-J_{bf})}. \quad (13)$$

In this way, Eq. (11) corresponds to a singular value decomposition of $\mathcal{M}[U]$. Graphically, this can be represented as

$$\mathcal{M} = \quad = \sum_{ab}\sigma_{ab} \quad . \quad (14)$$





The singular values $|\sigma_{ab}| \leq 1$ are the absolute values of the diagonal elements of $\mathcal{M}(J)$, and the unitary transformations are fixed by the one-site unitaries. Furthermore, left and right eigenoperators of $\mathcal{M}(J)$ are given by the basis operators $e_{ab} \in \mathbb{R}^{q\times q}$ (defined as the operators with a single nonzero matrix element ab) with corresponding eigenvalue σ_{ab}. There are q guaranteed eigenvalues equal to 1, corresponding to the diagonal matrices e_{aa}, $a = 1, \ldots, q$, and the remaining $q(q-1)$ eigenvalues arise in complex conjugate pairs, $\sigma_{ab} = \sigma^*_{ba}$. J determines the singular values of U, such that all quantum channels are guaranteed to have at least q singular values equal to one [31]. In the following, we show how the parametrization Eq. (9) can be tuned to return classes of dual-unitary models with any given level of ergodicity. The main idea is that $V[J]$ gives rise to a diagonal quantum channel in which we can tune the eigenvalues through J, after which the one-site unitaries can be chosen to leave a subset of these eigenvalues invariant.

Ergodic and mixing gates.—Choosing $J \in \mathbb{R}^{q\times q}$, u, $v \in SU(q)$ arbitrary, the unitaries do not exhibit any additional structure and the resulting channel will generally only have the trivial eigenvalue associated with the identity. Since $\mathcal{M}[U]$ will generally not be Hermitian, its singular values are unrelated to its eigenvalues, and the left and right eigenoperators will differ. All nontrivial eigenvalues have a modulus smaller than one and $\lim_{t\to\infty}\mathcal{M}^t(\rho) = \mathrm{tr}(\rho)\mathbb{1}/q = \mathbb{1}/q$, consistent with thermalization to the infinite-temperature state and thermal correlations $\lim_{t\to\infty}c_{\rho\sigma}(t,t) = \mathrm{tr}(\sigma)/q$. Note that this does not imply that arbitrary multisite operators will thermalize, but nonergodicity in multisite operators would require an additional structure in U that is not generically present.

Nonergodic gates.—Nonergodic gates have n nontrivial unit eigenvalues with $1 \leq n \leq q^2 - 1$. Such nonergodic models can be realized in different ways. First, we consider nonergodic models where $n \leq q - 1$ and the conserved operators are mutually commuting and hence simultaneously diagonalizable. This can be done by imposing a block-diagonal structure on the unitaries u and v, turning n singular values into eigenvalues. Taking

$$u = w\left[\begin{array}{c|c} \mathbb{1}_n & 0 \\ \hline 0 & u_{q-n} \end{array}\right], \qquad v = \left[\begin{array}{c|c} \mathbb{1}_n & 0 \\ \hline 0 & v_{q-n} \end{array}\right]w^\dagger, \quad (15)$$

in which $u_{q-n}, v_{q-n} \in SU(q-n)$ and $w \in SU(q)$, the resulting quantum channel has n additional unit eigenvalues and n mutually commuting eigenoperators $c_a = w e_{aa} w^\dagger, a = 1, \ldots, n$. The block-diagonal matrices preserve the diagonal structure for the eigenvalues σ_{ab} of $\mathcal{M}(J)$ with $a, b \leq n$, whereas w leads to a unitary transformation of the quantum channel and its eigenoperators, leaving the eigenvalues invariant [32].

It follows that $Q_a = \sum_{x \in 2\mathbb{N}} c_a(x)$ are conserved quantities, satisfying $[Q_a, \mathcal{U}(t=2)] = 0$. Even stronger, these behave as solitons that are simply shifted along the light cone during dynamics (see Ref. [33]). The steady-state correlations follow from the overlap of ρ with the conserved charges. As shown in the Supplemental Material, these are described by a GGE if the initial operator represents a density matrix,

$$\lim_{t\to\infty} \mathcal{M}^t(\rho) = \exp\left[\sum_{a=1}^{n} (\mu_a - \mu) c_a + \mu \mathbb{1}\right] = \rho_{\mathrm{GGE}}, \qquad (16)$$

where the μ_a and μ follow from ρ as

$$\mu_a = \ln\left(\mathrm{tr}(\rho c_a)\right), \qquad \mu = \ln\left(\frac{1 - \sum_{b=1}^{n} \mathrm{tr}(\rho c_b)}{q - n}\right). \qquad (17)$$

The GGE state reproduces the initial values of all conserved operators, $\mathrm{tr}(\rho c_a) = \mathrm{tr}(\rho_{\mathrm{GGE}} c_a), a = 1, \ldots, n$, and $\mathrm{tr}(\rho_{\mathrm{GGE}}) = 1$, and all correlations decay to the GGE value $\lim_{t\to\infty} c_{\rho\sigma}(t,t) = \mathrm{tr}(\rho_{\mathrm{GGE}} \sigma),\ \forall\, \sigma$. Since we focus on correlations of one-site operators, ρ_{GGE} is a single-site operator corresponding to the reduced density matrix for a single site. However, this does not guarantee that the reduced density matrix for larger subsystems also corresponds to a GGE.

Additional unit eigenvalues can be obtained by introducing additional unit singular values. However, the additional conserved charges will no longer commute mutually and the steady state can no longer be recast as a GGE. From Eq. (13), a necessary condition for additional unit singular values is for multiple rows of J to be equal. Taking the first $m < n$ rows of J to be equal leads to $m(m-1)$ additional singular values $\lambda_{ab} = 1, a, b \leq m$. For $m \leq n$ the block-diagonal structure of Eq. (15) again leaves these singular values invariant, and the total gate has additional unit-eigenvalue eigenoperators and hence conserved charges $w e_{ab} w^\dagger,\ a,\ b \leq m$. While the final value is no longer described by a GGE, it still converges to a nonthermal value set by the overlap of ρ with the (properly orthonormalized) conserved charges. This can be seen as the limit of the oscillatory behavior: taking rows of J to be equal up to a nonzero constant, e.g., for fixed $a,\ b \leq n$ setting $J_{af} = J_{bf} + \phi, f = 1, \ldots, q$, leads to a pair of complex conjugate eigenvalues $\lambda_{ab} = (\lambda_{ba})^* = e^{i\phi}$. The resulting correlation functions do not decay but exhibit persistent oscillations $\propto e^{i\phi t}$, averaging out to zero for nonzero ϕ, such that the time-average value corresponds to the GGE value Eq. (16) in the absence of equal rows.

Noninteracting models.—Noninteracting models are characterized by all eigenvalues equal to one. This can be done by taking all rows of J to be equal, setting all singular values equal to 1. In this case $V[J]$ corresponds to a swap gate and $\mathcal{M}(J) = \mathbb{1}$, such that $\mathcal{M}[U] = v^\dagger u^\dagger \otimes vu$. All eigenvalues have modulus one, where all eigenvalue are exactly one if $v = u^\dagger$. Dynamical correlations remain constant and $\mathcal{M}[U] = \mathbb{1}$.

Ergodic and nonstationary models.— Ergodic but nonstationary dynamics are characterized by $1 \leq n \leq q^2 - 1$ nontrivial eigenvalues that are all different from one but with unit modulus. This can be done for generic J by setting $u = wP, v = w^\dagger$, with $w \in SU(q)$ and in which P is defined as $P_{a,b} = e^{i\theta_a} \delta_{b,a+1}$, identifying $q + 1 \equiv 1$, and $\theta_a, a = 1, \ldots, q$ are arbitrary phases. Considering the subspace of unit-eigenvalue (diagonal) eigenoperators of $\mathcal{M}[V]$, the effect of P is to set $P^\dagger e_{aa} P = e_{a+1,a+1}$, such that $\mathcal{M}(w^\dagger e_{aa} w) = w^\dagger e_{a+1,a+1} w$. Within this degenerate subspace, $\mathcal{M}$ acts as a shift operator, with known eigenvalues given by $e^{2\pi i f/q}, f = 1, \ldots, q$, where the trivial eigenvalue 1 corresponds to the identity. This leads to $q - 1$ nontrivial eigenvalues given by the remaining roots of unity. At sufficiently long times, all correlation functions remain nonzero and oscillate around the zero ergodic value, satisfying $c_{\rho\sigma}(t+q, t+q) = c_{\rho\sigma}(t,t)$. This effectively realizes a discrete time crystal, where the correlations in a periodically driven system respond with a period that is an integer multiple of the driving period [34–36].

Examples.—In Fig. 1, we present numerical examples for different dynamics. Note that the quantum channel construction does not require all unitaries to be identical [16],

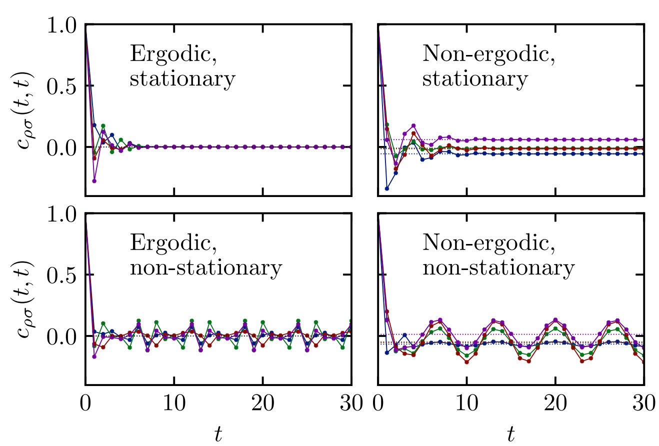


FIG. 1. Correlation functions $c_{\rho\sigma}(t,t)$, where $\rho, \sigma \in \mathbb{C}^{q\times q}$ are randomly generated matrices with $\mathrm{tr}(\sigma) = 0$ leading to a thermal value $c_{\rho\sigma}(t,t) \to 0$. Local Hilbert space dimension $q = 6$ and 4 different operators are considered. After an initial transient regime, in the ergodic models the correlations either exponentially decay to zero (stationary) or oscillate around zero (nonstationary) with period q, whereas in the nonergodic models the correlations decay to a nonzero value (stationary) with possible oscillations around these nonzero values with a tunable period (nonstationary).

such that individual unitary gates can be randomly selected while still keeping the overall level of ergodicity of the full circuit. Level spacing statistics can also be calculated, a common indicator of chaos and ergodicity, returning the expected GUE statistics for ergodic mixing gates and Poisson statistics otherwise, consistent with the proposed classification [37,38].

As an additional example, we consider a dual-unitary model for prethermalization with an arbitrarily large local Hilbert space. For a nonergodic model, the system locally thermalizes to a GGE consistent with the conserved charges. Any perturbation generally destroys all nontrivial conservation laws, inducing thermalization to the infinite-temperature state. However, for small perturbations we expect a separation of time scales: the correlations initially prethermalize to the GGE values of the nonergodic model before eventual thermalization to the infinite-temperature thermal values [39,40].

Given general J, we can introduce an ergodicity-inducing perturbation on top of a nonergodic model starting from Eq. (11), setting

$$u = e^{i\epsilon W_u} w \left[\begin{array}{c|c} \mathbb{1}_n & 0 \\ \hline 0 & u_{q-n} \end{array}\right], \qquad v = \left[\begin{array}{c|c} \mathbb{1}_n & 0 \\ \hline 0 & v_{q-n} \end{array}\right] w^\dagger e^{-i\epsilon W_v},$$

with again $u_{q-n}, v_{q-n} \in SU(q-n), w \in SU(q)$, and where the perturbation is generated by two (nonequal) Hermitian operators $W_{u,v} \in \mathbb{C}^{q\times q}$ and tuned through ϵ. At $\epsilon = 0$, this reduces to a nonergodic model with n conservation laws, whereas any finite ϵ results in an ergodic model. This is illustrated in Fig. 2, where for small ϵ the dynamics of the different circuits are indistinguishable, seemingly converging to the nonthermal steady-state value of the nonergodic model. However, the effect of the perturbation becomes apparent at longer times, where the models with nonzero ϵ eventually thermalize to the infinite-temperature state indicated by vanishing correlations. The time scale needed to reach the eventual thermal state is determined by the subleading eigenvalue of $\mathcal{M}$ and scales as ϵ^{-2}, as it can be verified from degenerate perturbation theory that the first-order correction on the unit eigenvalues vanishes.

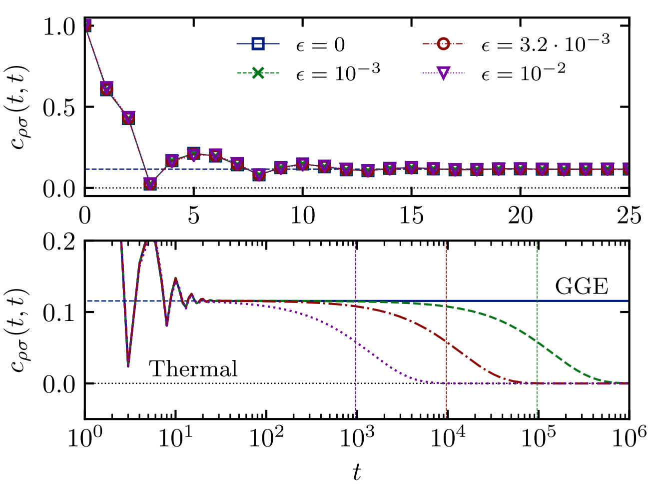


FIG. 2. Correlation function $c_{\rho\sigma}(t,t)$ at short and long time scales for a randomly generated traceless σ and a randomly generated density matrix ρ with $\mathrm{tr}(\rho\sigma) = 1$ for a nonergodic gate U on top of which an ergodicity-inducing perturbation is added with strength ϵ. Local Hilbert space dimension $q = 6$. Vertical dashed lines mark $t = \log(2)/(1-|\lambda|) \propto \epsilon^{-2}$, with λ the dominant nontrivial eigenvalue of $\mathcal{M}$, and horizontal lines denote the thermal and GGE values.

Conclusion.—It was shown how to generate classes of dual-unitary gates with arbitrary local Hilbert space dimension and any desired level of ergodicity. Evolving a local operator under a circuit composed of dual-unitary gates, local correlation dynamics remain analytically tractable, such that these models can be used to study both chaotic and nonergodic dynamics in systems with an arbitrarily large Hilbert space. Focusing on one-site operators, stationary steady-state correlations were analytically shown to be given by the infinite-temperature Gibbs state (ergodic) or a generalized Gibbs ensemble (nonergodic). In both cases, persistent correlations characterizing time crystals can also be included on top of the steady-state value. The proposed construction returns exactly solvable models of thermalization, where we also illustrated prethermalization to the latter before eventual thermalization to the former in a nonergodic model with added ergodicity-inducing perturbation.

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Correction: The inadvertent omission of a marker indicating "Featured in Physics" has been fixed.