Precision Determination of Pion-Nucleon Coupling Constants Using Effective Field Theory

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The pion-nucleon coupling constants determine the strength of the long-range nuclear forces and play a fundamental part in our understanding of nuclear physics. While the charged- and neutral-pion couplings to protons and neutrons are expected to be very similar, owing to the approximate isospin symmetry of the strong interaction, the different masses of the up and down quarks and electromagnetic effects may result in their slightly different values. Despite previous attempts to extract these coupling constants from different systems, our knowledge of their values is still deficient. In this Letter, we present a precision determination of these fundamental observables with fully controlled uncertainties from neutron-proton and proton-proton scattering data using chiral effective field theory. To achieve this goal, we use a novel methodology based on the Bayesian approach and perform, for the first time, a full-fledged partial-wave analysis of nucleon-nucleon scattering up to the pion production threshold in the framework of chiral effective field theory, including a complete treatment of isospin-breaking effects and our own determination of mutually consistent data. The resulting values of the pion-nucleon coupling constants are accurate at the percent level and show no significant charge dependence. These results mark an important step toward developing a precision theory of nuclear forces and structure.

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The pion-exchange mechanism drives the low-energy interaction between protons and neutrons and is of utmost importance for our understanding of atomic nuclei that make up 99.9% of the visible Universe and for answering big science questions such as the origin of the elements, the limits of nuclear stability, searches for physics beyond the standard model, and physics of neutron stars. The interaction of a charged and neutral pion with protons (p) and neutrons (n) is characterized by three coupling constants $f_{\pi^0 pp}$, $f_{\pi^0 nn}$, and $f_{\pi^\pm pn}$, whose precise definitions will be given below. These three constants determine the strength of the long- and intermediate-range nuclear forces originating from exchange of virtual pions. Their precise knowledge with controlled uncertainties is, therefore, of fundamental importance for a quantitative understanding of nuclear physics.

While possible in principle, the *ab initio* precision determination of the pion-nucleon (πN) coupling constants from lattice quantum-chromodynamics (QCD) and quantum-electrodynamics (QED) calculations is presently out of reach [1]. Attempts have been made to extract $f_{\pi^\pm pn}$ from experimental data on pion-nucleon (πN) scattering [2–4], pionic atoms [5,6] and proton-antiproton scattering [7], but the best way to determine all three constants is by analyzing the abundance of proton-proton (pp) and neutron-proton (np) scattering data. However, previous studies along this line [8–12] relied on phenomenological models and offered no way of a reliable uncertainty quantification apart from estimating statistical errors.

In this Letter, we determine the values of all three πN coupling constants from nucleon-nucleon (NN) scattering data using the model-independent framework of chiral effective field theory (EFT) [13,14]. This method has already been applied to NN scattering, and the EFT expansion of the NN force has been recently pushed to fifth order ($\mathrm{N^4LO}$) [15–17]. The crucial new aspects of the current investigation include the following.

(i) For the first time, a determination of all three πN coupling constants and a partial wave analysis of NN data up to the pion production threshold including the determination of mutually consistent data are carried out in the framework of chiral EFT. We have taken into account *all* charge-independence-breaking (CIB) and charge-symmetry-breaking (CSB) isospin-violating NN interactions up through $\mathrm{N^4LO}$. This allowed us to achieve a statistically perfect description of mutually consistent pp and np scattering data in the framework of chiral EFT that is unprecedented in its precision.

(ii) We have succeeded in overcoming the computational challenge of performing a *Bayesian* determination of the πN coupling constants. Contrary to the computationally much less demanding frequentist methods used in all previous determinations [8–12,18], the Bayesian approach provides a *rigorous* way to calculate the joint conditional probability density of the πN coupling constants given NN data.

(iii) A careful uncertainty analysis, facilitated by the usage of Bayesian methods, is performed to estimate not only statistical errors in the calculated πN coupling constants, but also systematic uncertainties from the truncation of the EFT expansion and the choice of the highest energy of the included NN data—a feature lacking in the earlier determinations of these quantities.

Definitions of the πN coupling constants.—Consider first the interaction of the nucleon with the isovector weak current A_i^μ, which is described in terms of the axial and induced pseudoscalar form factors G_A and G_P, respectively. In the limit of exact isospin symmetry corresponding to the equal masses of the up and down quarks and in the absence of electromagnetic interactions, the matrix element of $A_i^\mu(x=0)$ between nucleon states can be parametrized via

$$\langle N(p')|A_i^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu G_A + \frac{q^\mu}{2m_N}G_P\right]\gamma_5\frac{\tau_i}{2}u(p),$$

with $u(p)$ and $\bar{u}(p')$ the corresponding Dirac spinors, τ_i the isospin Pauli matrices, and m_N the nucleon mass. Furthermore, $q^\mu = (p'-p)^\mu$ refers to the momentum transfer of the nucleon. The form factors $G_A(q^2)$ and $G_P(q^2)$ carry important information about the internal structure of the nucleon. For example, the axial charge of the nucleon $g_A \equiv G_A(0) = 1.2756(13)$ [19] controls the decay rate of a neutron to a proton. Recently, this quantity was calculated from first principles at a percent level using lattice QCD [20]; see also [21] for a review of lattice QCD calculations of g_A. While $G_A(q^2)$ is a smooth function near $q^2=0$, the induced pseudoscalar form factor possesses a pion-pole contribution, $G_P(q^2) = 4m_N g_{\pi NN} F_\pi/(M_\pi^2 - q^2) + \text{nonpole terms}$, whose residue is determined by the (pseudoscalar) πN coupling constant $g_{\pi NN}$. The pion decay constant $F_\pi = (92.1 \pm 0.8)$ MeV [19] determines the rate of weak decays $\pi^\pm \to \mu^\pm \nu_\mu$. The strong-interaction constant $g_{\pi NN}$ is connected to g_A and F_π entering weak processes via the celebrated Goldberger-Treiman relation $F_\pi g_{\pi NN} = g_A m_N(1+\Delta_{\text{GT}})$, where the small Goldberger-Treiman discrepancy Δ_{GT} is driven by the nonvanishing masses of the up and down quarks.

Away from the isospin limit and in the presence of QED, one has to distinguish between protons and neutrons and between the charged and neutral pions by introducing three coupling constants $g_{\pi^0 pp}$, $g_{\pi^0 nn}$, and $g_{\pi^\pm pn}$ or, equivalently, the corresponding pseudovector couplings $f_p \equiv f_{\pi^0 pp} = M_{\pi^\pm} g_{\pi^0 pp}/(2\sqrt{4\pi}m_p)$, $f_n \equiv f_{\pi^0 nn} = M_{\pi^\pm} g_{\pi^0 nn}/(2\sqrt{4\pi}m_n)$, and $f_c \equiv f_{\pi^\pm pn} = M_{\pi^\pm} g_{\pi^\pm pn}/[\sqrt{4\pi}(m_p + m_n)]$. The determination of these fundamental constants from NN scattering data is the main subject of this study.

Chiral EFT for nuclear forces.—We use chiral EFT, an effective field theory of QCD, to describe the low-energy interactions between two nucleons and employ the resulting NN potential to extract the πN coupling constants from a combined Bayesian analysis of np and pp scattering data below the pion production threshold. Chiral EFT utilizes an expansion in powers of momenta and pion masses to describe interactions between pions and nucleons in a systematically improvable way [13,14,22]. The corresponding effective Lagrangian contains all possible terms compatible with the symmetries of QCD. The nonperturbative dynamics of QCD is encoded in the so-called low-energy constants (LECs), which control the strength of the interactions in the effective Lagrangian and can be determined from experiments or lattice QCD calculations. Chiral EFT has also been extended to include virtual photons. Figure 1 shows examples of contributions to the NN force in chiral EFT. The most important terms at leading order (LO) include one-pion exchange [Fig. 1(b)] and contact interactions [Fig. 1(e)]. Two-pion exchange [Fig. 1(d)] and one-photon exchange [Fig. 1(a)] start to contribute at next-to-leading order (NLO), while pion-photon exchange [Fig. 1(c)] appears first at fourth order (N^{3}LO).

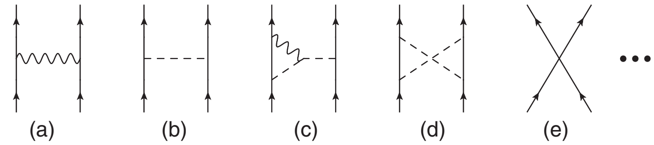

FIG. 1. Diagrammatic illustration of the NN interaction in chiral EFT. Photons, pions, and nucleons are shown by wavy, dashed, and solid lines, respectively. Diagrams (a)–(e) are representative examples of the one-photon exchange, one-pion exchange (OPE), electromagnetic corrections to the OPE, two-pion exchange (TPE), and NN short-range contributions, respectively. The range of interactions decreases from the left to the right.

In recent years, the chiral expansion of the NN force has been pushed to fifth order [15–17]. All relevant isospin-invariant πN LECs have been reliably determined from a dispersion theory analysis of πN scattering in Ref. [23]. Therefore, the long-range part of the NN interaction is parameter-free. To avoid distortions of the long-range forces due to a finite cutoff Λ, we introduced in Ref. [17] an improved local regulator which respects the analytic structure of the interaction. The LECs accompanying short-range operators [Fig. 1(e)] were determined in Ref. [17] from a fit to the 2013 Granada database [24] of mutually compatible np and pp data. Furthermore, we have introduced a N^{4}LO$^+$ NN potential, where the leading F-wave short-range interactions, formally appearing at sixth order, were taken into account in order to achieve a statistically satisfactory description of certain very precisely measured pp data; see also Ref. [16]. This allowed us to achieve a description of NN data on par with or even better than that based on the most precise phenomenological potentials but with a much smaller number of adjustable parameters. However, the treatment of isospin-breaking (IB) effects in Ref. [17] was incomplete and limited to the one of the Nijmegen [25] and Granada 2013 [24] partial wave analyses (PWA).

In this Letter, we include the CIB and CSB IB NN interactions complete up through N^{4}LO. In particular, we employ the most general form of the OPE potential including the leading electromagnetic corrections [26] and take into account the leading and subleading IB two-pion-exchange contributions [27–29]. These long-range interactions are expressed in terms of known LECs, the πN coupling constants f_p^2, f_c^2, and $f_0^2 \equiv f_p f_n$ to be determined, the nucleon mass difference $\delta m = m_n - m_p \simeq 1.29$ MeV, and its QCD contribution $\delta m^{\rm QCD} = 2.05(30)$ MeV [30]; see Ref. [31] for an update and Ref. [32] for a recent *ab initio* calculation using lattice QCD and QED. We also include short-range IB interactions in the $^1S_0$, $^3P_0$, $^3P_1$, and $^3P_2$ partial waves. Details of the employed NN interaction are given in Supplemental Material [33].

Determination of the πN coupling constants.—We end up with 33 parameters that need to be determined from NN data, comprising of three πN LECs $f^2 \equiv \{f_c^2, f_p^2, f_0^2\}$ and $25 + 5$ LECs C_i from isospin-invariant + IB short-range interactions, collectively denoted as $C \equiv \{C_i\}$. For normally distributed errors, the likelihood of data D given f^2, C, and Λ is given by

$$p(D|f^2 C\Lambda) = \frac{1}{N} e^{-\chi^2/2}, \qquad (1)$$

where N is a normalization constant. The data D employed in our analysis include mutually compatible np and pp scattering data according to our own selection as detailed in Supplemental Material [33], where we also provide the definition of the χ^2 measure. Using Bayes' theorem to relate the probability density function (PDF) $p(f^2 C\Lambda|D)$ of the parameters given the data to $p(D|f^2 C\Lambda)$ and integrating over the nuisance parameters C and Λ, we obtain the quantity we are actually interested in, namely, the PDF of f^2 given the data D:

$$p(f^2|D) = \int d\Lambda dC \frac{p(D|f^2 C\Lambda) p(f^2 C\Lambda)}{p(D)}. \qquad (2)$$

For the case at hand, $p(D)$ is a (normalization) constant. Furthermore, we use independent priors for f^2, C, and Λ so that $p(f^2 C\Lambda) = p(f^2)p(C)p(\Lambda)$ and employ a Gaussian prior for C and uniform priors for Λ and f^2 specified in Supplemental Material [33]. To determine f^2, we need to find the maximum of $p(f^2|D)$ in Eq. (2). However, for each set of f^2, this requires integrating over a 31-dimensional space spanned by Λ and C, which is not feasible. Instead, we employ the Laplace approximation by fitting C to D for fixed values of f^2 and Λ and expressing the likelihood $p(D|f^2 C\Lambda)$ as

$$p(D|f^2 C\Lambda) \approx \frac{1}{N} e^{-(1/2)[\chi^2_{\rm min} + (1/2)(C - C_{\rm min})^T H (C - C_{\rm min})]}. \qquad (3)$$

Here, $\chi^2_{\rm min} \equiv \chi^2_{\rm min}(f^2, \Lambda)$ at $C_{\rm min} \equiv C_{\rm min}(f^2, \Lambda)$ and the Hessian $H \equiv H(f^2, \Lambda)$ is given by $H_{ij} = [(\partial^2\chi^2)/(\partial C_i \partial C_j)]|_{C=C_{\rm min}}$. Performing an analytical integration over C then allows us to cast Eq. (2) into a numerically tractable form; see Supplemental Material [33] for details. The remaining integration over $\Lambda \in [400, 550]$ MeV is performed numerically. We emphasize that reducing the amount of information in the employed priors for C, f^2, and Λ has a negligible effect on our results; see Ref. [33] for details.

To account for the uncertainty inherent in the choice of the energy range of our PWA, we performed separate analyses of NN data up to the laboratory energies of $E_{\rm lab}^{\rm max} = 220$, 240, 260, 280, and 300 MeV. Furthermore, to address the systematic error stemming from the truncation of the EFT expansion for IB interactions, we considered two additional models of the NN interaction that include IB pion-photon- and two-pion-exchange contributions beyond N^{4}LO. Our final PDF for the πN coupling constants are obtained by performing averaging over five values for $E_{\rm lab}^{\rm max}$ and three models for IB interactions in order to account for the truncation uncertainty at N^{4}LO as detailed in Supplemental Material [33]. For all considered cases, the self-consistency of our results is verified by comparing the quantiles of the χ^2 residuals with those of the assumed normal distribution [33]. Although no further assumptions regarding the shape of the distributions $p(f^2|D)$ have been made, the calculated PDFs $p(f^2|D)$ are found to follow a multivariate Gaussian distribution to a very high accuracy. This is exemplified in Fig. 2 for the case of the central model and the energy range of $E_{\rm lab} = 0$–280 MeV. The distributions $p(f^2|D)$ can, therefore, be accurately characterized by the central values and errors of the f_i^2's along with the corresponding correlation coefficients (see Ref. [33]), which greatly facilitates their averaging as explained in Supplemental Material, Sec. 6 [33].

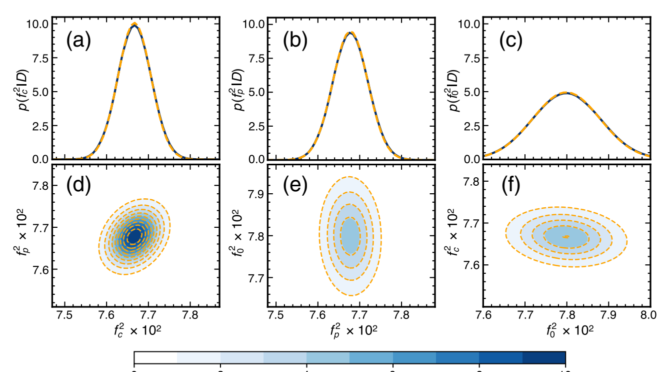

FIG. 2. Marginal posteriors for the central model and the energy range of $E_{\rm lab} = 0$–280 MeV. (a)–(c) show the probability distributions $p(f_i^2|D)$ in units of 10^2. (d)–(f) show the joint distributions $p(f_i^2, f_j^2|D)$ in units of 10^5. Blue solid lines and filled contours are based on the exact numerical evaluation, while orange dashed lines and contours represent its approximation by a multivariate Gaussian distribution as described in the text.

We can verify the statistical validity of our results by studying the likelihood at the optimal values of the parameters. For example, fitting the LECs C for $\Lambda = 463.5$ MeV (see Fig. S1 in Supplemented Material [33]) and the central values of the f^2's from Eq. (4) in the range of $E_{\mathrm{lab}} = 0$–280 MeV yields $\chi^2 = 4950.72$ for $N_{\mathrm{dat}} = 4926$, leading to $\chi^2/N_{\mathrm{dat}} = 1.005$. The quantity $\chi^2/(N_{\mathrm{dat}} - N_{\mathrm{par}}) - 1 = 0.012$, where $N_{\mathrm{par}} = 34$ (including Λ), is comparable to half of the standard deviation (s.d.), $\sqrt{2/(N_{\mathrm{dat}} - N_{\mathrm{par}})} = 0.020$, expected for a perfect model.

We have also investigated the robustness of our results with respect to the variation of input parameters. Specifically, the uncertainty from higher-order πN LECs entering the two-pion-exchange potential is quantified by repeating our analysis for 50 sets of these LECs generated from the central values and covariance matrix of Ref. [23]. Furthermore, the uncertainty of the QCD contribution δm^{QCD}, even taking its conservative estimate of ± 0.30 MeV [30], is found to induce errors in f_i^2 that are negligibly small compared to the ones given below. Our final result for the πN coupling constants after the averaging reads

$$\begin{aligned} f_p^2 &= 0.0770(5)^a(0.8)^b, \\ f_0^2 &= 0.0779(9)^a(1.3)^b, \\ f_c^2 &= 0.0769(5)^a(0.9)^b, \end{aligned} \tag{4}$$

where the first error (a) is obtained from the marginal posteriors $p(f^2|D)$ and includes the statistical and systematic errors due to the truncation of the EFT expansion, the choice of the energy range, and the associated data selection. The second error (b) reflects the uncertainty in the higher-order πN LECs. Notice that the present study can, in principle, be extended to obtain a joint posterior probability distribution for f_i's and the higher-order πN LECs that can be useful for uncertainty quantification in chiral EFT via a combined analysis of the NN data and the experimental or empirical information on the πN scattering amplitude; see Ref. [71] for a related work.

Discussion of the results.—Our results for f_i^2 are compared in Fig. 3 with selected earlier determinations. Similarly to the Granada PWA, we find considerably larger values for the coupling constants as compared to the ones recommended by the Nijmegen group [9]. As already found in Ref. [12], this difference cannot be explained by the new experimental data since 1993 (see Supplemental Material [33] for more details), thus pointing toward a possible sizable systematic uncertainty from the interaction modeling in the Nijmegen PWA. Our value for f_c^2 is consistent with the determinations from the πN system in Refs. [3–6] (at the 1.3σ level). The results for f_0^2 and f_c^2 agree within errors with the recent determination by the Granada group [12], while for f_p^2 we obtain a slightly larger value. However, contrary to the Granada group that found

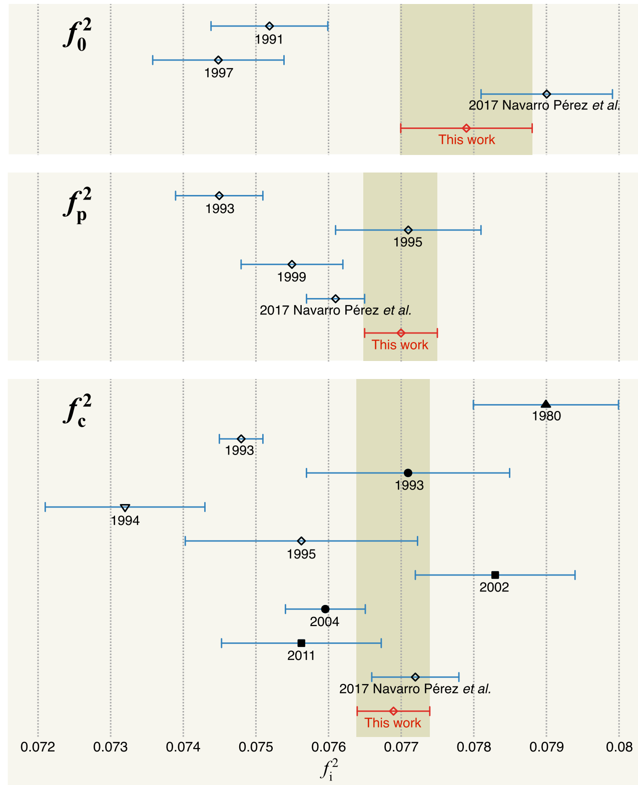

FIG. 3. Values for the πN coupling constants. The data points show selected determinations of the πN coupling constants f_0^2, f_p^2, and f_c^2. The results were obtained using πN PWA [2] (filled triangle), fixed-t dispersion relations of πN scattering [3,4] (filled dots), πN scattering lengths in combination with the GMO sum rule [5,6] (filled squares), proton-antiproton PWA [7] (open triangle), and NN PWA [8–11,18] including the 2017 Granada PWA from Navarro Pérez, Amaro, and Ruiz Arriola [12] (open diamonds). When provided separately, the statistical and systematic uncertainties are added in quadrature. The vertical bands show our full uncertainty. Uncertainties are one s.d.

evidence that the coupling of neutral pions to neutrons is larger than to protons, $f_0^2 - f_p^2 = 0.0029(10)$ [12], our result $f_0^2 - f_p^2 = 0.0010(10)^a(2)^b$ is consistent with no charge dependence. This difference may point to significant systematic uncertainties in the analysis by the Granada group [12] which are not quantified in that paper, in particular, due to the cutoff radius r_c and phenomenological modeling of the interaction. In contrast, our analysis relies on the systematically improvable EFT framework and takes into account model-independent long- and intermediate-range nuclear interactions due to exchange of virtual pions and photons. This allows us to substantially reduce the number of adjustable parameters (33 in our analysis versus 55 in Ref. [12]) while still achieving at least a comparable description of NN data below the pion production threshold. Compared to the Granada analysis, we also do not find a large anticorrelation between f_0^2 and f_c^2; see Table V in Ref. [33].

In summary, our Bayesian determination of the πN coupling constants from np and pp scattering data in the framework of chiral EFT yields new reference values for these fundamental observables, accurate at the percent level. It provides new insights into the isospin symmetry of the strong interaction at the hadronic level by quantifying the charge dependence of these quantities. Our work also establishes important benchmarks for future first principles calculations using lattice QCD and QED (see [32] for a first step along this line) and opens the door for precision studies of nuclear structure and reactions by fixing the strengths of the long-range nuclear forces. As a very recent example, we mention the high-accuracy calculation of the deuteron charge and quadrupole form factors [72], where the isospin-breaking corrections considered in this work have been taken into account and were found to play an important role for the determination of the quadrupole moment, for which the value of $Q_d = 0.2854^{+0.0038}_{-0.0017}$ fm^2 was obtained. Redoing the same analysis with the IB corrections from this paper being switched off, i.e., using the NN interactions from Ref. [17], the central value of the quadrupole moment would change significantly to $Q_d = 0.2803$ fm^2. These calculations are currently being extended to other light nuclei, and we expect the considered IB corrections to be relevant at the desired accuracy level. It would also be interesting to explore the implications of our study for the understanding of charge symmetry breaking in binding energy differences of mirror nuclei [73] and for certain low-energy three-nucleon scattering observables such as the doublet scattering length [74] and vector analyzing power A_y [75]. Furthermore, since our analysis also results in an accurate determination of the IB short-range operators in the $^1S_0$ partial wave, it may shed new light on the ongoing studies of neutrinoless double-beta decay in chiral EFT; see Ref. [76] for a related discussion. This will be addressed in a separate publication. Last but not least, we emphasize that the value for f_c can be related to the pionic hydrogen width Γ_{1s}, whose measurement at PSI is currently being analyzed; see [77] for a preliminary result.

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