Dissipative Floquet Majorana Modes in Proximity-Induced Topological Superconductors

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We study a realistic Floquet topological superconductor, a periodically driven nanowire proximitized to an equilibrium s-wave superconductor. Because of the strong energy and density fluctuations caused by the superconducting proximity effect, the Floquet Majorana wire becomes dissipative. We show that the Floquet band structure is still preserved in this dissipative system. In particular, we find that the Floquet Majorana zero and π modes can no longer be simply described by the Floquet topological band theory. We also propose an effective model to simplify the calculation of the lifetime of these Floquet Majoranas and find that the lifetime can be engineered by the external driving field.

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Introduction.—Floquet engineering, which controls quantum systems using periodic driving [1–3], is believed to provide a potentially accessible method to realize topological nontrivial band structures and other exotic quantum states [4–27]. In spite of its success in noninteracting (or isolated) systems, the standard Floquet theorem cannot correctly capture many realistic quantum systems: there is a crossover from a prethermal regime [28–30] to featureless infinite temperature states [31] in nonintegrable interacting Floquet systems. An open Floquet system usually shows complicated statistical behaviors depending on the details of system-bath couplings [32–35]. Certain self-consistent treatments along with realistic conditions are crucial for understanding such elusive nonequilibrium systems.

An example is the Floquet Majorana modes in the periodically driven topological superconductors (SCs) [8,10,11,20,23]. In most experimentally realizable proposals, the topological SC is induced by the proximity effect [36–40]. In the equilibrium case, the low energy physics related to the proximity effect can usually be well approximated by an *ad hoc* pairing term, e.g., the self-energy correction at zero frequency in the nanowire $\Sigma_{sc}(\omega) \simeq \Sigma_{sc}(\omega = 0) = \Delta_{ind}\sigma_y\tau_y$ [41]. Therefore, the resulting minimal model is equivalent to the intrinsic SC, and has been extensively applied in the literature [38–40]. However, when the system is under external driving, a realistic approach is to regard the SC as an external bath, which not only provides the tunneling of Cooper pairs but also acts as a dissipative source that renders the periodically driven nanowire into a nonequilibrium steady state [42]. More importantly, the zero frequency approximation might no longer be suitable due to the existence of Floquet Majorana π modes (FMPMs) at $E = \pm\Omega/2$ [8,11,43,44]; and therefore, the physics around $\omega = \pm\Omega/2$ are also important where Ω is the driving frequency. This calls for a more realistic minimal model to study the corresponding dissipative Floquet Majoranas and topological phase transitions; and it is interesting to ask whether the Floquet picture is still valid if interplay between the nonequilibrium conditions and strong dissipations exists.

In this Letter, we consider a realistic model based on a periodically driven nanowire proximitized to an

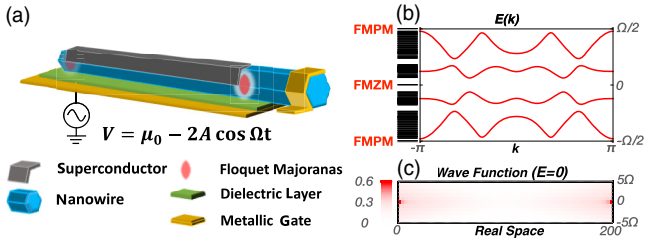

FIG. 1. The closed (or intrinsic) Floquet SC limit. (a) shows the setup. (b),(c) show the open boundary spectra or Floquet band of Eq. (3) under the approximation shown in Eq. (6) and the wave function of the Floquet Majorana zero mode with $E = 0$. Note that the left black spectrum in (b) is for an open finite wire with lattice size $N = 200$, while the right k-dependent spectrum is for a wire with a spatially periodic boundary condition. The parameters are $t_0 = 1$, $\lambda = 1.5$, $\mu_0 = -2$, $V_z = 1.2$, $A = 3/2$, $\Omega = 6$, $V = 0.8$, and $N_F = 5$.

equilibrium s-wave superconductor, as shown in Fig. 1(a). Using the Keldysh Green function's formalism [45–65], we study the physics induced by the self-energy correction beyond the zero frequency approximation and find that under certain conditions, Floquet Majorana zero modes (FMZMs) and FMPMs become dissipative and can no longer be predicted by the Floquet topological band theory in which the pairing term is constant and ω independent. We also propose a Floquet Majorana poisoning model to simulate and evaluate the lifetime of these dissipative Floquet Majoranas.

Realistic Floquet proximity SC.—Consider a periodically driven semiconductor nanowire coupled to a conventional s-wave SC, as shown in Fig. 1(a) (refer to [66–74] for recent experimental progresses of the un-driven scenario). The Hamiltonian has three parts and can be written as follows [8,10,11]:

$$\hat{H}(t) = \hat{H}_{\mathrm{nw}}(t) + \hat{H}_{\mathrm{sc}} + \hat{H}_c. \tag{1}$$

Here $\hat{H}_{\mathrm{nw}}(t) = \sum_k \hat{\Psi}_k^\dagger[(-2t_0\cos k - \mu_0 + 2A\cos\Omega t)\tau_z + V_z\sigma_z\tau_z + \lambda\sin k\sigma_y\tau_z]\hat{\Psi}_k$ is the Hamiltonian of the nanowire driven by the external lead and $\hat{\Psi}_k = (\hat{c}_{k,\uparrow}, \hat{c}_{k,\downarrow}, \hat{c}^\dagger_{-k,\uparrow}, \hat{c}^\dagger_{-k,\downarrow})^t$, where $\hat{c}_{k,\uparrow/\downarrow}$ annihilate spin-up or spin-down electrons with momentum k in the nanowire, and σ_μ/τ_μ represent the Pauli matrices in the spin or Nambu spaces. $\hat{H}_{\mathrm{sc}} = \sum_q \hat{\Phi}_q^\dagger(\epsilon_q\tau_z - \Delta\sigma_y\tau_y)\hat{\Phi}_q$ is the Hamiltonian of the SC bath, and $\hat{\Phi}_q = (\hat{a}_{q,\uparrow}, \hat{a}_{q,\downarrow}, \hat{a}^\dagger_{-q,\uparrow}, \hat{a}^\dagger_{-q,\downarrow})^t$, where $\hat{a}_{q,\uparrow/\downarrow}$ annihilate spin-up or spin-down electrons with momentum q in the SC bath. $\hat{H}_c = \sum_{k,q,\sigma}(V\hat{c}^\dagger_{k,\sigma}\hat{a}_{q,\sigma} + V^*\hat{a}^\dagger_{q,\sigma}\hat{c}_{k,\sigma})$ is the Hamiltonian describing the nanowire-bath coupling. Here the parameters μ_0, V_z, λ, Δ, and V represent the static chemical potential, the Zeeman field, the spin-orbit coupling strength, the SC order parameter of the SC bath, and the nanowire-bath coupling strength, respectively. The external driving is controlled by the amplitude A and frequency Ω. Without loss of generality, Δ and V are assumed to be real positive numbers.

It is widely believed that an open Floquet system coupled to an external thermal bath will eventually reach a nonequilibrium steady state in which the energy absorbed from the external driving field is balanced by the energy flowing out to the environment [1,3]. Theoretically, the physical observables in the nonequilibrium steady state can be conveniently dealt with within the framework of Keldysh formalism [42]. To be more precise, spectral properties and distribution functions can be calculated from the retarded and Keldysh components of the Keldysh Green function. We mainly focus on spectral properties and their quasiparticle lifetimes, so only the retarded component is needed. Using the Floquet theorem, we show in the Supplemental Material (SM) [75] that when the external SC bath degrees of freedom are integrated out, the retarded component of the Keldysh Green function has the following form [81]:

$$\underline{G^R_{\mathrm{nw}}(k,\omega)} = [\underline{\omega} - \underline{\mathcal{H}_{\mathrm{eff}}(k,\omega)}]^{-1}, \tag{2}$$

where $\underline{\omega}$ is proportional to the identity matrix and

$$\underline{\mathcal{H}_{\mathrm{eff}}(k,\omega)} = \begin{pmatrix} \cdots & & & \\ & \mathcal{H}_{\mathrm{nw}}(k) - \Omega + \Sigma_{\mathrm{sc}}(\omega+\Omega) & A\sigma_0\tau_z & 0 \\ & A\sigma_0\tau_z & \mathcal{H}_{\mathrm{nw}}(k) + \Sigma_{\mathrm{sc}}(\omega) & A\sigma_0\tau_z \\ & 0 & A\sigma_0\tau_z & \mathcal{H}_{\mathrm{nw}}(k) + \Omega + \Sigma_{\mathrm{sc}}(\omega-\Omega) \\ & & & \cdots \end{pmatrix}. \tag{3}$$

Here

$$\mathcal{H}_{\mathrm{nw}}(k) = (-2t_0\cos k - \mu_0)\tau_z + V_z\sigma_z\tau_z + \lambda\sin k\sigma_y\tau_z, \qquad \Sigma_{\mathrm{sc}}(\omega) = V^2[-(\omega+i\eta) - \Delta\sigma_y\tau_y]/\sqrt{-(\omega+i\eta)^2 + \Delta^2}, \tag{4}$$

are the static Hamiltonian of the nanowire and the static self-energy correction with $\eta = 0^+$ [42]. The object $\underline{X}$ is a matrix with infinite dimension in the Floquet space, whose basis is spanned by the Harmonic functions $e^{-i(\omega+m\Omega)t}$ with $m = 0, \pm1, \ldots$ [1,75]. In practice, a truncation N_F is necessary for numerics, e.g., $m = 0, \pm1, \ldots, \pm N_F$. We emphasize that in the derivation of Eqs. (2)–(4), we have made a physical assumption, i.e., the external SC density of states (DOS) is constant [75]. However, the Green function method we applied is exact without approximation [41,82]. In the static limit ($A = 0$), the low-energy equilibrium physics of the nanowire can be well described by the following zero frequency approximation:

$$\mathcal{H}_{A=0}(k) \simeq \mathcal{H}_{\mathrm{nw}}(k) + \Sigma_{\mathrm{sc}}(\omega=0) = \mathcal{H}_{\mathrm{nw}}(k) + \Delta_{\mathrm{ind}}\sigma_y\tau_y, \tag{5}$$

where $\Delta_{\rm ind} = -V^2$. Physically, the Eq. (5) Hamiltonian is equivalent to the intrinsic SC with order parameter $\Delta_{\rm ind}$ [41]. When $V_z^2 > (\pm 2t_0 + \mu)^2 + \Delta_{\rm ind}^2$, the system can have Majorana zero modes at two boundaries [39,40]. Under external drivings, when we take the intrinsic SC approximation

$$\Sigma_{\rm sc}(\omega + m\Omega) \simeq \Delta_{\rm ind}\sigma_y\tau_y, \qquad m = -N_F, \ldots, N_F, \quad (6)$$

Eq. (3) reduces to the Floquet Hamiltonian studied in Refs. [8,11]. The emergence or absence of Floquet Majoranas can be well described by the Floquet topological band theory [4–8,27]. Figure 1(b) shows an example of the Floquet band structure and open boundary spectra with $N_F = 5$ (other parameters are shown in the caption of Fig. 1). Both FMZMs and FMPMs exist in the open boundary spectrum [83]. The corresponding wave function of the FMZM with $E = 0$ is plotted in Fig. 1(c). One can also notice that the FMZM is localized not only in the real space but also in the Floquet space. Because of the translational symmetry of the Floquet Hamiltonian, the FMZMs with $E = n\Omega$ must be localized at Floquet sites $n\Omega$ [84].

Now we explain why the above intrinsic SC approximation, Eq. (6), can no longer be applied in Floquet proximity topological SCs. We first discuss the mathematical meaning of Eq. (6). From Eq. (4), $\Sigma_{\rm sc}(\omega + m\Omega) = -V^2(\sigma_y\tau_y + \delta)/\sqrt{1-\delta^2}$, where $\delta = (\omega + m\Omega + i\eta)/\Delta$, one can notice that only when $\delta \to 0$ can the approximation Eq. (4) be applied. This requires $2\Delta \gg (2N_F + 1)\Omega$ for a fixed N_F. Obviously, this requirement cannot be achieved in experiments. Therefore, the self-energy can no longer be approximated by a constant and a ω-independent term. Indeed, from Eq. (4), when $|\omega| > \Delta$, $\Sigma_{\rm sc}(\omega) = iV^2[-|\omega| - \Delta\sigma_y\tau_y]/\sqrt{\omega^2 - \Delta^2}$ becomes purely imaginary, and this imaginary part plays the role of effective dissipations in the nanowire. Therefore, in order to investigate the lifetime of the corresponding Floquet Majoranas, a suitable treatment of the self-energy correction is necessary.

Periodic boundary condition.—In order to investigate the role of the ω-dependent self-energy correction $\Sigma_{\rm sc}(\omega)$, we first apply the numerical calculation of the time-averaged momentum resolved DOS $\nu_k(\omega)$ with a spatially periodic boundary condition as shown in Fig. 2(a), where

$$\nu_k(\omega) = -\frac{1}{\pi}\mathrm{Tr}\,\mathrm{Im}[\underline{G^R_{\rm nw}(k,\omega)}]_{00}, \quad (7)$$

and the subscript 00 represents the 00-Floquet entry. For example, in Eq. (3), $[\underline{\mathcal{H}_{\rm eff}(k,\omega)}]_{00} = \mathcal{H}_{\rm nw}(k) + \Sigma_{\rm sc}(\omega)$. As discussed above, when $\Delta \gg (2N_F + 1)\Omega$, $\nu_k(\omega)$ can be approximately described by the Floquet band theory in the intrinsic SC limit with $\Delta_{\rm ind} = -V^2$, whose band structure has been shown in Fig. 1(b). Comparing Fig. 1(b) to Fig. 2(a1), one can find that the approximation Eq. (6) works well when $\Delta = 200$ [85]. With the decreasing of Δ (blue lines), sharp features in the spectrum continuously broaden due to the dissipation effect. More interestingly, when 2Δ is smaller than Ω, as shown in Fig. 2(a4), $\nu_k(\omega)$ even exhibits discontinue behavior at $\omega = \pm\Delta$ due to the singularity of $\Sigma_{\rm sc}(\omega = \Delta)$. This is a strong nonlinear self-energy effect that will kill FMPMs, as shown later.

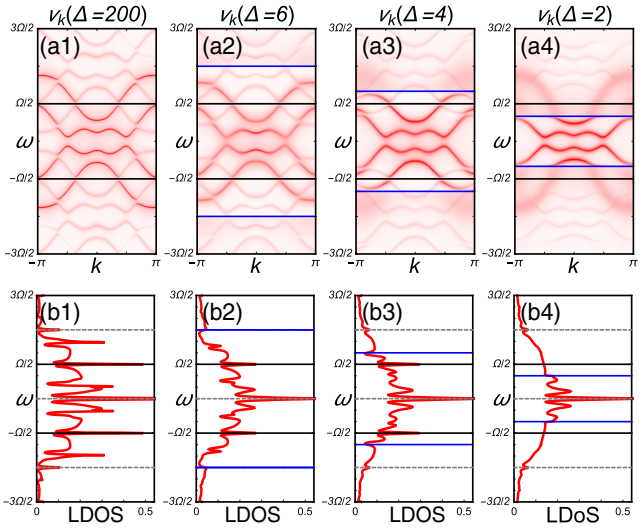

FIG. 2. $\nu_k(\omega)$ (periodic boundary condition) and LDOS (open boundary condition) at the ends of the nanowire. The parameters chosen are the same as Fig. 1 except $\eta = 0.05$ and different values of Δ (blue lines) shown above. (a1) shows the same band structure as Fig. 1(c). When Δ decreases, the bands in (a2)–(a4) change dramatically due to the existence of nonlinear ω terms in the self-energy correction. When $2\Delta < \Omega$, the FMPMs are destroyed.

Open boundary condition.—We now turn to the discussion of Floquet Majoranas. In order to characterize them, we numerically calculate the time-averaged local DOS (LDOS) at the end of the nanowire with open boundary conditions based on the recursive Green function method [86–88] (refer to the SM [75] for Floquet systems). As shown in Figs. 2(b1)–(b4), numerical results verify our observations obtained from the band behavior in (a1)–(a4), namely, the killing of FMPMs at $\omega = (2m+1)\Omega/2$. To be more precise, when Δ decreases to $2\Delta < \Omega$, FMPMs are destroyed, as shown in Figs. 2(b1)–(b4). This can be understood from the gap closing at $\omega = \pm\Omega/2$ shown in Fig. 2(a4) due to the nonlinear self-energy corrections. Therefore, the FMPM can no longer be described by the Floquet topological band theory under the intrinsic SC limit shown in Fig. 1(b) and Eq. (6). Although in our example, FMZMs are not sensitive to the deceasing of Δ, in the SM we show that the topological phase transition of FMZMs is also beyond the description of Floquet topological band theory. We finally note that in the numerical results in Figs. 2(b1)–(b4), for those FMZMs outside the SC gap,

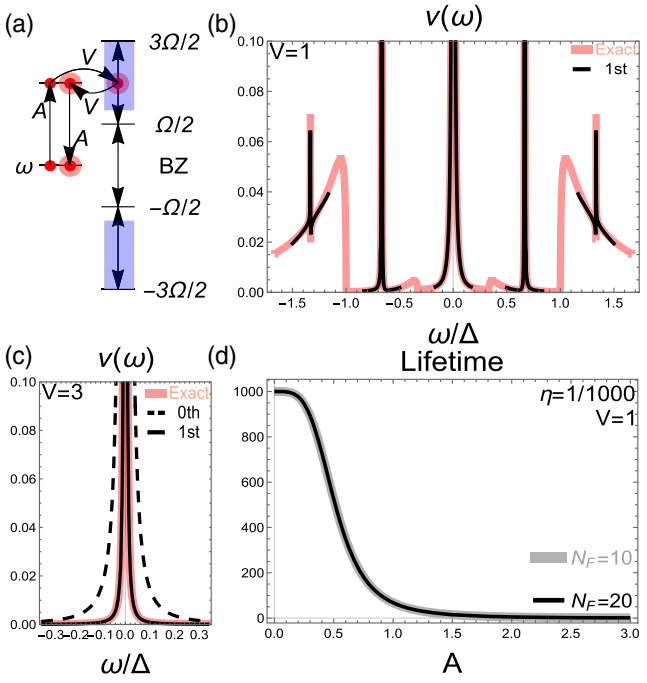

FIG. 3. Floquet Majorana poisoning model. (a) shows the first order procedure. (b) shows the comparison of the exact result and first order approximation of $\nu(\omega)$ in Eq. (9) with $\Omega = 2$, $\Delta = 3$, $A = 1/2$, and $\eta = 1/1000$. (c) shows the exact, 0th, and first order numerical calculations. (d) shows the relation between the lifetime and A with the same parameters shown above.

their peak heights are very tiny comparing with those inside the SC gap. So, it is important to check whether the lifetime of these Floquet Majoranas in different Floquet Brillouin zones (FBZs) [89] are identical or distinct. If their lifetimes are identical, the dissipation-modified Floquet bands are still valid along with a constant finite lifetime acquired from dissipations.

Floquet Majorana poisoning model.—In order to illustrate how the Floquet picture is modified by dissipations provided by the SC, we propose the Floquet Majorana poisoning model shown in Fig. 3(a). We emphasize that the Floquet Majorana poisoning model can also be applied to estimate the lifetime of Floquet Majoranas in the realistic model above [75]. The poisoning model describes a boundary isolated Majorana coupled to a dissipative gapped bath and driven by an external field with frequency Ω and amplitude A. The corresponding ω-dependent Hamiltonian in the Floquet space is

$$\underline{H(\omega)} = \begin{pmatrix} \ddots & & & \\ & \Sigma(\omega+\Omega)-\Omega & A & 0 \\ & A & \Sigma(\omega) & A \\ & 0 & A & \Sigma(\omega-\Omega)+\Omega \\ & & & \ddots \end{pmatrix}, \quad (8)$$

where $\Sigma(\omega) = -V^2(\omega + i\eta)/\sqrt{-(\omega + i\eta)^2 + \Delta^2}$ with $\eta = 0^+$, and V is the Majorana-SC bath coupling strength. The time-averaged DOS $\nu(\omega)$ can be calculated from the 00-Floquet entry of the retarded Green's function $\underline{G^R(\omega)} = [\underline{\omega} - \underline{H(\omega)}]^{-1}$, i.e.,

$$\nu(\omega) = -\frac{1}{\pi}\mathrm{Im}[\underline{G^R(\omega)}]_{00} = -\frac{1}{\pi}\mathrm{Im}\left[\frac{1}{\underline{\omega} - \underline{H(\omega)}}\right]_{00}. \quad (9)$$

As shown in Fig. 3(a), although the zero energy Majorana mode is not directly coupled to the bath due to the SC gap, the mode can be excited (or de-excited) to higher FBZs (which directly couple to the SC bath) and cause finite dissipation or poisoning.

As shown in Figs. 3(b) and 3(c), the exact results of $\nu(\omega)$ are plotted with the red lines. One can find at each FBZ center $\omega = m\Omega$ a Floquet Majorana peak. However, we show in the SM that there only exists a single pole of $G^R(\omega)$, which is at $\omega = i\eta$. This means the lifetimes of the Floquet Majoranas at $\omega = \pm m\Omega$ cannot be defined using the traditional method, i.e., finding the imaginary part of the pole of $G^R(\omega)$. However, we find that these Floquet Majorana peaks at $\omega = m\Omega$ can be well fitted by the first order ω expansion of $\underline{\omega} - \underline{H(\omega)}$ around $\omega = m\Omega$. In Figs. 3(b) and 3(c), the first order results are shown with solid black lines. As a comparison, we also plot the 0th order result, i.e., $1/[\underline{\omega} - \underline{H(m\Omega)}]$, in (c) with dashed black lines. One can notice that the 0th order breaks down, while the first order approximation works well. We also develop a method to compute the quasiparticle lifetime in the open Floquet quantum systems (refer to the SM [75] for more details). Here, we only briefly list the main steps, starting from the expansion of the inverse of the Floquet Green function around ω_0:

$$\det[\underline{G^{R,-1}(\omega)}] \simeq f_0(\omega_0) + f_1(\omega_0)\delta\omega + o(\delta\omega^2), \quad (10)$$

where $\delta\omega = \omega - \omega_0$, and the superscript -1 represents the inverse of the retarded Green function. The lifetime of the quasiparticle around $\omega = \omega_0$ is

$$\tau_{\omega_0} = \Gamma_{\omega_0}^{-1} = -\frac{1}{\mathrm{Im}[f_0/f_1]} \quad (11)$$

[a numerical plot is shown in Fig. 3(d)]. In addition, due to the discrete time translational symmetry, we have $\det[\underline{G^{R,-1}(\omega)}] = \det[\underline{G^{R,-1}(\omega + m\Omega)}]$. This means the expansion of $\det[\underline{G^{R,-1}(\omega)}]$ around $\omega = m\Omega$ must share the same expression coefficients. Therefore, their lifetimes must be the same, i.e., $\tau|_{\omega=0} = \tau|_{\omega=m\Omega}$. This is consistent with the intuition since the total Hamiltonian does not break the discrete time translational symmetry. In addition, the differences between the Floquet Majorana DOSs shown in different FBZs [see Fig. 2(b)] come from the Floquet space

localization nature of the wave function instead of their lifetime difference (refer to the last part of the SM [75] for more details). In the SM, we also provide a comparison of the FMZM peaks of the Floquet Majorana poisoning model and the realistic model.

Discussion and conclusion.—Our work reveals the crucial role of self-energy correction in the Floquet proximity SC. It is widely believed that the self-energy correction at zero frequency can be described by an effective non-Hermitian Hamiltonian [90–95]. The corresponding real and imaginary parts of the eigenvalues can be regarded as the renormalized band structure and quasiparticle lifetime. However, in our model, the linear ω term of the self-energy is important for the definition of lifetime in the Floquet proximity SC. Further numerical results suggest that the Floquet Majorana lifetime can be tuned by the external field, as shown in Fig. 3(d).

In summary, we have shown that, for Floquet proximity SCs, the intrinsic SC approximation [Eq. (6)] is not suitable for understanding the lifetimes of Floquet Majoranas and their topological features. The difficulty comes from the linear and nonlinear effects of the self-energy correction, which can poison and kill Floquet Majoranas, respectively. However, the dissipation structures in different FBZs are identical. Therefore, the Floquet picture is still valid in the presence of dissipations.

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