Quasiparticle Poisoning of Majorana Qubits

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Qubits based on Majorana zero modes are a promising path towards topological quantum computing. Such qubits, though, are susceptible to quasiparticle poisoning which does not have to be small by topological argument. We study the main sources of the quasiparticle poisoning relevant for realistic devices—nonequilibrium above-gap quasiparticles and equilibrium localized subgap states. Depending on the parameters of the system and the architecture of the qubit either of these sources can dominate the qubit decoherence. However, we find in contrast to naive estimates that in moderately disordered, floating Majorana islands the quasiparticle poisoning can have timescales exceeding seconds.

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Majorana zero modes (MZMs) provide a basis for topologically protected qubits [1–3]. The topological protection means that dephasing of the MZM-based qubit (Majorana qubit) is exponentially small in the separation of the MZMs in space and in the energy gap of the separating region. Both of these quantities can be controlled in experiments which leads to the potential of long dephasing times.

Majorana qubits encode information in the joint parity of MZM pairs. Dephasing of a Majorana qubit can thus only happen via incoherent exchange of parity between the MZMs [4] or via processes of uncontrolled exchange of quasiparticles (QPs) between the Majorana subspace and other fermionic modes [5–7]. The latter is called quasiparticle poisoning (QPP) of the Majorana qubit. QPs may have different origins—they may be inside the Majorana qubit at above-gap energies excited by temperature or external perturbations, they may come from the environment, or they may be located in nontopological subgap states. Exponential suppression of QPP in parameters of the system can be shown for a system decoupled from gapless leads under the assumptions of thermal equilibrium and a spectral gap in the system [8,9]. In the present work we analyze how breaking these assumptions in real systems affects QPP.

Nonequilibrium QPs are present in any realistic physical system and have been explored extensively in the context of superconducting qubits [10–19] and dedicated devices [20–24]. In a conventional superconductor a single QP cannot relax to the condensate consisting of Cooper pairs. Once QPs are created by an external perturbation the only way to get rid of them is to pair them up. This process becomes slow for low densities of the QPs [10,25]. Therefore, even for small rates of QP creation, superconductors typically have a nonequilibrium density of QPs that significantly exceeds the expected thermal occupation.

The effects of nonequilibrium QPs on the Majorana qubits can be described by relaxation events of the QPs into the MZMs. The QPs are described by the density n_{QP}. In order to determine the lifetime (or dephasing time) of the qubit we have to determine the relaxation rate into MZMs given a certain n_{QP} and then find an expression for typical QP densities. The latter can be obtained from steady state solutions of a model with a certain rate of exciting higher-energy QPs, subsequent relaxation, and recombination.

The potential danger of QPP for Majorana qubits is widely acknowledged in the literature [4–7,26] and the timescales for the poisoning influence the design of a Majorana-based quantum computer [27]. However, there exist few quantitative estimates for the corresponding decoherence times. Moreover, the existing estimates [4,6] rely on values for n_{QP} that are typical for conventional superconductors and do not take into account how the presence of the MZM itself will change n_{QP}. Here we show that MZMs act as efficient QP traps which strongly renormalizes n_{QP}.

Finally, the effect of disorder and subgap states is mainly discussed in the literature in terms of the effect on the topological transport gap and transition from the topological to the conventional phase due to disorder [28–31]. The disorder, however, suppresses the spectral gap faster than the transport gap by creating localized states. It thus causes the presence of subgap states well before the topological transition. Such subgap states present a reservoir where parity may leak from the MZMs, thus causing decoherence from equilibrium QPP.

The aim of the present Letter is to provide a self-consistent estimate of QPP taking into account the peculiarities of mesoscopic topological superconductors including the finite volume of the superconductor and equilibrium QPP due to the presence of subgap states.

Poisoning due to nonequilibrium QPs.—We start by considering nonequilibrium QPP resulting from relaxation

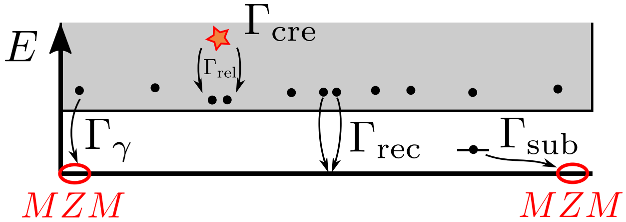

FIG. 1. Schematic representation of the relevant quasiparticle processes in a topological superconductor. Above-gap QPs are created at a rate $\Gamma_{\rm cre}$ which leads to a density of nonequilibrium QPs, which quickly relaxes to an energy window close to the gap edge via the rate $\Gamma_{\rm rel}$. After relaxing to the gap edge, the quasiparticles can relax further either by pairwise recombination into Cooper pairs at a rate $\Gamma_{\rm rec}$ or by relaxing into MZMs at rate Γ_γ. Another source of quasiparticle poisoning can be due the presence of low energy subgap states that may be present in the topological superconductor. Leakage of the parity from the MZMs into such state is described by the rate $\Gamma_{\rm sub}$.

of above-gap nonequilibrium QPs into one of the computational Majoranas with a rate Γ_γ. See Fig. 1 for a schematic representation of the relevant processes. The nonequilibrium QP density in superconductors can result from a steady rate of QP creation $\Gamma_{\rm cre} = 2\gamma_{\rm br} N_{\rm CP}$ via breaking of some of the $N_{\rm CP}$ Cooper pairs. The exact nature of the Cooper pair breaking process may be due to stray radiation, cosmic rays, etc., but is unimportant for our model. High energy QPs relax quickly to the gap edge but only slowly recombine into Cooper pairs by annihilating with another QP with a rate $\Gamma_{\rm rec}$. The latter can be estimated by $\Gamma_{\rm rec} = \tilde\gamma_0 n_{\rm QP}^2 V/n_{\rm CP}$, see, e.g., Ref. [25], where $\tilde\gamma_0^{-1}$ is a characteristic timescale of the electron-phonon coupling, V is the total volume of the island, and $n_{\rm CP}$ is the density of Cooper pairs. The factor $n_{\rm CP}^{-1}$ also describes the average volume of a QP in the energy range of interest, as $n_{\rm CP} = D(E_F)\Delta$ in terms of the density of states $D(E_F)$ and the gap Δ of the superconductor. Crucially, $\Gamma_{\rm rec} \propto n_{\rm QP}^2$ becomes slow for small densities which makes relaxation processes into MZMs that become available in topological superconductors highly relevant.

Following the literature [13,15,16,32], we describe the behavior of above-gap QPs by diffusive dynamics of an energy-independent density $n_{\rm QP}$ of QPs close to the gap edge. Neglecting the energy dependence is based on the observation that the QP relaxation rate $\Gamma_{\rm rel} \gtrsim (10\ {\rm ns})^{-1}$ [32] is typically by far the fastest of the rates introduced in Fig. 1 and can thus be eliminated together with the above-gap energy dependence. Within this model we consider MZMs located at positions $\mathbf{x} = \mathbf{x}_i$ and take into account the remaining creation and relaxation processes defined above. The diffusion equation reads

$$\dot n_{\rm QP} = 2\gamma_{\rm br} n_{\rm CP} + D\nabla^2 n_{\rm QP} - \sum_i \gamma_0 n_{\rm QP} f(\mathbf{x}-\mathbf{x}_i) - \tilde\gamma_0 n_{\rm QP}^2/n_{\rm CP}. \qquad (1)$$

Here, $\gamma_{\rm br}$ is the rate of breaking Cooper pairs, D is the diffusion constant of the above-gap QPs, γ_0^{-1} a characteristic timescale for QP relaxation into a MZM, and $f(\mathbf{x})$ describes the local extent of a MZM located around $\mathbf{x} = 0$. While in general $f(\mathbf{x})$ depends nontrivially on the overlap of MZM and QP wave functions [4], in the limit of a weakly changing QP density [33] relative to the topological coherence length we can write $f(\mathbf{x}) \approx V_{\rm MZM}\delta(\mathbf{x})$, where $V_{\rm MZM} = (\int dV|\psi_{\rm MZM}|)^2$ is a measure of the volume of the MZM with wave function $\psi_{\rm MZM}$. In an effectively one-dimensional system $V_{\rm MZM}^{(1d)} = \xi$.

Note that Eq. (1) neglects the hybrid character of typical realizations of topological superconductors as semiconductor-superconductor heterostructures. Although diffusion in the semiconductor can be faster than in the superconductor, the much larger density of states in the superconductor leads to Eq. (1) being an excellent approximation when using the diffusion constant of QPs in the superconductor as long as the regime of very weak superconductor-semiconductor coupling is avoided. For a more detailed discussion see the Supplemental Material [34].

We first review the argument giving the nonequilibrium QP density in an isolated conventional superconductor. To this end we set $\gamma_0 = 0$. The steady state solution of Eq. (1) then yields a constant density $n_{\rm QP}(x) = n_{\rm SC}$, with

$$n_{\rm SC} = \sqrt{\frac{2\gamma_{\rm br}}{\tilde\gamma_0}} n_{\rm CP} \qquad (2)$$

such that $\Gamma_{\rm cre}(n_{\rm SC}) = \Gamma_{\rm rec}(n_{\rm SC})$.

For the remainder of the Letter we will use parameters for an Al-proximitized nanowire of total volume $V_{\rm SC} = 10\ \mu{\rm m} \times 200\ {\rm nm} \times 10\ {\rm nm} = 2\times 10^{-2}\ \mu{\rm m}^3$ as such a system is the closest to practical applications due to substantial charging energy [3]. We summarize the parameters in Table I. All the estimates are straightforward to perform for other superconductors and host systems [39–41]. In the Al-based system $n_{\rm cp} = D(E_F)\Delta \approx 3\times 10^6/\mu{\rm m}^3$. Using $\gamma_0^{-1}, \tilde\gamma_0^{-1} \sim 50$ ns [4] and $n_{\rm SC} = 0.01, \ldots, 10\ \mu{\rm m}^{-3}$ [10,14,18,19,21,42] we find $\gamma_{\rm br}^{-1} \sim 10^4, \ldots, 10^{10}$ s. Note that this gives quite small total rates of QP creation even when multiplying by the number of Cooper pairs in a typical sample of size $V_{\rm SC}$. Using these numbers we find $\Gamma_{\rm cre}^{-1} \sim 0.1$ s, …, 1 day.

TABLE I. Parameter values used in our estimations.

Parameter	Value	Reference
Volume of superconductor $V_{\rm SC}$	$2\times 10^{-2}\ \mu{\rm m}^3$	[3]
Electron-phonon coupling γ_0^{-1}	50 ns	[4]
QP density in bulk SC $n_{\rm SC}$	$0.01, \ldots, 10\ \mu{\rm m}^{-3}$	[10,19]
Majorana volume $V_{\rm MZM}$	$2\times 10^{-4}\ \mu{\rm m}^3$	[3]
Diffusion coefficient in Al D	$2\ \mu{\rm m}^2\ {\rm ns}^{-1}$	[13]

Note that the above estimate relies on uniformly distributed completely delocalized QPs and thus provides an upper bound for the creation rate. As noted, e.g., in Ref. [25] the annihilation itself can reduce the probability for QPs to be close enough to recombine especially if they are localized due to disorder. This effect makes recombination less efficient and leads to estimates with even smaller rates $\Gamma_{\rm cre}$ that would be consistent with the observed QP densities. Reference [25] also provided an encouraging estimate for the QP creation rate (per volume) due to cosmic radiation $\sim 10^{-4}\ {\rm s}^{-1}\ \mu{\rm m}^{-3}$ which corresponds to $\Gamma_{\rm cre}^{-1} \sim 10$ days.

Let us now consider the rate of relaxation of a density $n_{\rm QP}$ into the localized MZMs given by [4]

$$\Gamma_\gamma = \gamma_0 n_{\rm QP} V_{\rm MZM}. \tag{3}$$

The rate Γ_γ is also the QP poisoning rate of the qubit. A naive estimate of the qubit poisoning rate can be obtained by using typical densities $n_{\rm SC}$ of conventional superconductors as an estimate of $n_{\rm QP}$ in Eq. (3) [4]. As we will show below it is crucial that the QP density is determined self-consistently taking into account the presence of MZMs and estimates based on the density in conventional superconductors vastly overestimate the poisoning rate in mesoscopic supercondcutors.

The importance of the relaxation into MZMs in comparison to the bulk QP recombination can already be revealed by examining for a fixed $n_{\rm QP}$ the ratio of Γ_γ to the rate of pairwise QP annihilation $\Gamma_\gamma/\Gamma_{\rm rec} = n_{\rm cp} V_{\rm MZM}/n_{\rm QP} V$. Using for the estimate of the volume occupied by the Majorana wave function $V_{\rm MZM} \sim 200\ {\rm nm} \times 10\ {\rm nm} \times 100\ {\rm nm} = 2 \times 10^{-4}\ \mu{\rm m}^3$, we obtain that at densities similar to the zero field densities of nonequilibrium QPs $n_{\rm QP} \approx n_{\rm SC}$, $\Gamma_\gamma/\Gamma_{\rm rec} \sim 10^2, \ldots, 10^5\ (1\ \mu{\rm m}^3/V)$. With typical device volumes of order $V_{\rm SC}$ we therefore find that the relaxation into MZMs is by far the dominant relaxation process. The efficient trapping of QPs by MZMs is reminiscent of conventional traps based on superconducting vortices [13,43–47]. Here, however, the trap is directly part of the active area of the qubit.

The above suggests that for topological superconductors of moderate size we can safely neglect the pair recombination in Eq. (1). This allows us to directly integrate Eq. (1). For concreteness we consider in the following a quasi-one-dimensional system of length L with two MZMs located at $x = 0$ and $x = L$ with $f(x) = \xi\delta(x)$, see Fig. 2. Because of the symmetry of the system it is sufficient to focus on the region of $x \in [0, L/2]$. Using boundary conditions of vanishing current $\partial_x n_{\rm QP}(x) = 0$ at $x = 0$ [48] and $x = L/2$ we obtain the steady state solution of the diffusion equation

$$n_{\rm QP}(x) = \left(1 + \frac{\gamma_0 \xi}{D} x\right) n_\gamma - \frac{\gamma_{\rm br} n_{\rm CP}}{D} x^2 \tag{4}$$

with $n_\gamma = n_{\rm QP}(0) = \gamma_{\rm br} n_{\rm CP} L/\gamma_0 \xi$. The expression of the QP density at the MZM reflects the balance of the total relaxation and creation rates.

The solution of Eq. (4) is depicted in Fig. 2 and is characterized by a minimum in the QP density $n_{\rm QP} = n_\gamma$ close to the MZMs and a maximal density $n_{\rm QP} = n_{\rm max}$ in between the MZMs. We now discuss the important length scales of the problem. From Eq. (4) one can extract the length scale $L_\gamma = D/\gamma_0 \xi$ over which the density changes only weakly. In a system of size $L < L_\gamma$ the diffusion time to explore the system is shorter than the typical relaxation time. This leads to a homogeneous density, i.e., $n_{\rm max}/n_\gamma \approx 1$. Using a diffusion constant typical for of Al QPs $D = 2\ \mu{\rm m}^2\ {\rm ns}^{-1}$ [13], $\xi \approx 200$ nm [49,50], and $\gamma_0^{-1} \sim 50$ ns we obtain $L_\gamma \sim 1$ mm. We thus expect that Majorana qubits based on moderately sized islands of topological superconductors [26,51] are in the regime of a small constant density of QPs dominated by relaxation into MZMs. The corresponding QP-limited decoherence times are thus given by

$$\Gamma_{\gamma,\rm meso}^{-1} = \Gamma_{\rm cre}^{-1} \sim 0.1\ {\rm s}, \ldots, 1\ {\rm day}. \tag{5}$$

While the estimate for the creation rate of QPs $\Gamma_{\rm cre}$ involves significant uncertainties, it is reasonable to assume that even in the presence of QPP qubit lifetimes exceeding seconds are possible.

To obtain estimates for the case of Majorana qubits based on bulk superconductors [52] we consider system sizes $L \gg L_\gamma$. Equation (4) suggests that $n_{\rm QP}(x)$ saturates at $n_{\rm max} = \gamma_{\rm br} n_{\rm CP} L^2/4D$ which can be understood as the density of broken Cooper pairs (in the absence of relaxation) created over the time it takes a QP to explore half of the system $L^2/4D$. For large systems with $L > L_{\rm SC}$ this density will be cut off once it reaches $n_{\rm max} = n_{\rm SC}$ and the formerly neglected pair recombination processes become relevant. Increasing the size of the system beyond $L_{\rm SC}$ will not change the density profile around the MZM but only add a larger region of density $n_{\rm SC}$ far away from the MZM. We can therefore estimate the density n_γ in the limit of a bulk superconductor by $n_\gamma(L = L_{\rm SC})$ which

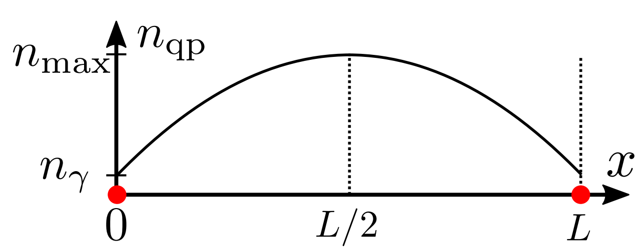

FIG. 2. Density profile of nonequilibrium QPs in a quasi-one-dimensional system of length L with MZMs located at the ends of the system. Close to the MZMs, the density is suppressed to a value n_γ while it reaches $n_{\rm max}$ at the maximal distance from the MZMs.

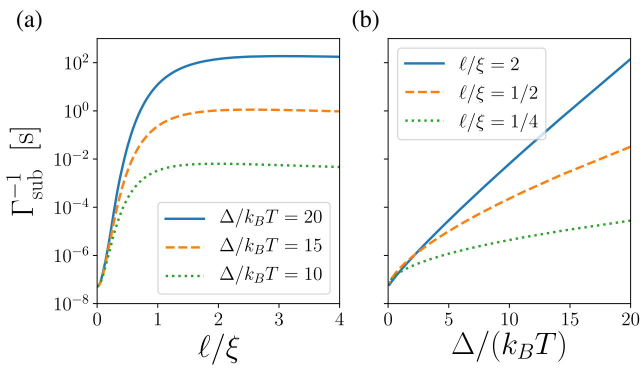

FIG. 3. (a) Equilibrium poisoning time for transport gap $\Delta = 100\ \mu$eV and several values of temperature as a function of ℓ/ξ. (b) Stretched exponential temperature dependence of equilibrium poisoning time for several values of ℓ/ξ.

leads to poisoning rates of $\Gamma_{\gamma,\rm bulk} = 2\gamma_{\rm br} n_{\rm CP} L_{\rm SC}$ with $L_{\rm SC} = \sqrt{n_{\rm SC} D / n_{\rm CP}\gamma_{\rm br}}$. Using the estimates for $\gamma_{\rm br}$ based on Eq. (2) one can rewrite $\Gamma_{\gamma,\rm bulk} = \sqrt{2\gamma_0 D n_{\rm SC}^3 / n_{\rm CP}}$ and with $n_{\rm SC}$ and $\tilde{\gamma}_0$ as quoted above we obtain poisoning times

$$\Gamma_{\gamma,\rm bulk}^{-1} \sim 0.1\ \mu\text{s}, \ldots, 1\ \text{ms}, \tag{6}$$

with corresponding $L_{\rm SC} \sim 1, \ldots, 100$ mm. Note that while the resulting estimated timescales $\Gamma_{\gamma,\rm bulk}^{-1}$ are similar to existing literature estimates using bulk superconductors with constant densities $n_{\rm SC}$ [6], this agreement is accidental as the underlying equations describe different physics [53].

Equations (5) and (6) allow us to compare mesoscopic and bulk superconductors on the same footing. Our results suggests that the poisoning times of Majorana qubits based on bulk superconductors are significantly worse than those of isolated mesoscopic islands. While the above estimates do not make it impossible to build Majorana qubits with bulk superconductors, the resulting coherence times will likely not be able to exceed those of conventional superconducting qubits.

Poisoning due to subgap states.—We now turn to another type of decoherence possible in Majorana devices—leakage of the parity from the MZM subspace into other subgap states. This mechanism does not require nonequilibrium QPs. For simplicity we thus focus on *equilibrium* QPP due to subgap states. The concern here is that realistic Majorana devices may be disordered or inhomogeneous, and while the transport gap can be observed in the widely used transport experiments the (potentially vanishing) spectral gap is less accessible. Thus equilibrium QPP is possible in the topological phase if there are enough subgap states inside the wire that are close enough in position to the MZMs. The equilibrium rate of a jump to a subgap state is

$$\Gamma = \omega_a \exp\left(-\frac{2x}{\xi} - \frac{\delta E}{k_B T}\right). \tag{7}$$

Here x is the distance from the Majorana to the localization center of a subgap state, ξ is the disordered coherence length, δE is the energy difference to the target state, and ω_a is the "attempt frequency" corresponding to the physical process that enables the tunneling event (for example, electron-phonon scattering, charge noise on gates [4,54], etc.).

As a specific scenario, we can take the average number of subgap states in a p-wave wire with finite mean free path ℓ arising from Gaussian disorder, as worked out in Ref. [28]:

$$\langle N(E)\rangle \propto \frac{L}{\xi_0}\left(\frac{E}{\Delta_0}\right)^{\eta}, \quad \eta = 4\ell/\xi_0 - 2 \tag{8}$$

where the proportionality constant is of order one, ξ_0 is the clean coherence length, Δ_0 the gap in the zero-disorder limit, and the parameter η describes the approach to a disorder-driven topological phase transition when $\ell \le \xi_0/2$. The *apparent* coherence length, which governs the spatial overlap of Majorana wave functions and diverges at this transition, is given by $\xi^{-1} = \xi_0^{-1} - (2\ell)^{-1}$. From Eq. (8), we can extract the energy window $[0, E']$, with $E' = \Delta_0[\xi_0/(2x)]^{1/\eta}$, that contains on average 1 state within a distance x from the edge of the system. Thus, to tunnel into a subgap state at distance x an electron typically needs to gain an energy

$$\delta E = \int_0^{E'} E\nu(E)dE = \frac{\Delta_0 \eta}{1+\eta}\left(\frac{\xi_0}{2x}\right)^{1/\eta}. \tag{9}$$

The optimal tunneling distance is then obtained by maximizing the rate Eq. (7) with respect to x,

$$x_{\rm opt} = \frac{\xi}{2}\left(\frac{\eta+2}{\eta}\right)^{\frac{\eta-1}{\eta+1}}\left(\frac{1}{\eta+1}\frac{\Delta}{k_B T}\right)^{\frac{\eta}{\eta+1}}. \tag{10}$$

Note that this expression has been written in terms of the disordered coherence length and transport gap, using $\xi/\xi_0 = (\eta+2)/\eta$. Then, finally, the typical leakage rate is obtained from Eq. (7) at $x_{\rm opt}$:

$$\Gamma_{\rm sub} = \omega_a \exp\left[-g(\eta)\left(\frac{\Delta}{k_B T}\right)^{\frac{\eta}{\eta+1}}\right] \tag{11}$$

with $g(\eta) = (\eta+1)^{1/\eta+1}([\eta+2]/\eta)^{(\eta-1)/(\eta+1)}$. Crucially, this rate is suppressed by a *stretched* exponential in $\Delta/(k_B T)$.

To gain intuition for the timescale in realistic devices we first take a (transport) topological gap of $\Delta \approx 100\ \mu$eV and a temperature $k_B T = 5\ \mu$eV ($\approx$50 mK). We also assume the attempt frequency is the electron-phonon scattering rate

$\omega_a = \gamma_0 \sim (50\text{ ns})^{-1}$. For $\ell/\xi = 1/4$ (or $\ell/\xi_0 = 3/4$) we find a typical leakage rate of $\Gamma_{\text{sub}}^{-1} \approx 30\ \mu$s, and already for $\ell/\xi = 1/2$ ($\ell/\xi_0 = 1$) we find $\Gamma_{\text{sub}}^{-1} \approx 30$ ms. In Fig. 3 we plot the leakage rate as a function of ℓ/ξ and of $\Delta/k_B T$. It is apparent that at high temperatures or at the disorder-driven phase transition the poisoning time is given by the electron-phonon scattering, however the time can exceed seconds (with $\Delta/k_B T \approx 20$) for $\ell/\xi \gtrsim 1$ and this mechanism of QPP becomes essentially inoperative when $\ell/\xi \gtrsim 2$. While these estimates are promising, they rely on a simple model which may be substantially modified in a real topological superconducting heterostructure, for example due to rare but strong impurities like dislocations, or the heterogeneous nature of the system [55].

Conclusions.—We studied the effect of quasiparticle poisoning in Majorana qubits and found that in mesoscopic qubit islands nonequilibrium QPs are less harmful than expected from naive estimates based on the typical bulk quasiparticle concentration in conventional superconductors. We have shown that poisoning due to nonequilibrium QPs is happening at the rate of the QP generation, which we expect to be of the order of seconds (or even days) and thus much slower than the timescales of qubit operations. Another potential source of QPP is the presence of disorder-induced subgap states. Our estimations show that Majorana parity leakage into such states is negligible for weak disorder. Only once the mean free path becomes comparable to the coherence length, QPP may present a problem due to subgap states that proliferate as the system approaches the disorder-induced topological transition.

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