

Numerical Study of a Dual Representation of the Integer Quantum Hall Transition

Kevin S. Huang¹, S. Raghu^{1,2}, and Prashant Kumar^{1,3}

¹Stanford Institute for Theoretical Physics, Stanford University, Stanford, California 94305, USA

²Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

³Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

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We study the critical properties of the noninteracting integer quantum Hall to insulator transition (IQHIT) in a “dual” composite-fermion (CF) representation. A key advantage of the CF representation over electron coordinates is that at criticality CF states are delocalized at all energies. The CF approach thus enables us to study the transition from a new vantage point. Using a lattice representation of CF mean-field theory, we compute the critical and multifractal exponents of the IQHIT. We obtain $\nu = 2.56 \pm 0.02$ and $\eta = 0.51 \pm 0.01$, both of which are consistent with the predictions of the Chalker-Coddington network model formulated in the electron representation.

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Introduction.—The integer quantum Hall (QH) to insulator transition (IQHIT) is one of the most studied topological phase transitions in condensed matter physics [1–6]. Without interactions, the existence of a QH plateau requires quenched disorder, and a magnetic field tunes the system from a QH state to an Anderson insulator. A beautiful representation of the IQHIT known as the Chalker-Coddington model (CCM) involves percolation of droplets of QH and insulating phases [7]. The CCM has been amenable to large scale numerical studies of critical exponents of the noninteracting IQHIT [8].

Nevertheless, all electron representations of the IQHIT suffer with a drawback: it is difficult to include electron-electron interaction effects, which are necessary to account for very basic aspects of the IQHIT. Interactions are necessary to ensure a nonzero finite temperature electrical resistivity [9]. Moreover, interactions determine dynamical scaling laws and superuniversality (the issue of whether or not integer and Abelian fractional QH transitions belong to the same universality class). Thus, there is a need for alternate formulations of the QHIT, which can more easily address such questions.

In this Letter we present a first step in devising alternate formulations of the QHIT, making use of a dual composite-fermion (CF) representation, building on pioneering ideas of flux attachment [10–17] and particle-vortex duality [18,19]. As we show below, in a mean-field approximation, the CF formulation of the IQHIT belongs to the same universality class as the one studied in electron coordinates. However, it offers several distinct advantages: most interestingly, delocalized states occur over all energies [20,21] at the IQHIT in the CF representation enabling a finite dc conductivity as $T \rightarrow 0$. Furthermore, a CF theory can more readily incorporate interaction effects,

and can treat integer and fractional QHITs on equal footing [22].

The phase diagram of the IQHIT is realized in the CF representation as follows. First, the integer QH state of electrons with $\sigma_{xy} = e^2/h$ maps onto an integer QH state of CFs but with opposite Hall conductivity. Second, the electron insulator is a CF insulator. It only remains to show that the critical exponents obtained in the CF representation are identical to those predicted by the CCM.

Using a tight-binding regularization of a CF Hamiltonian, we compute two critical exponents ν and η describing, respectively, the divergence of the localization length and wave function multifractality. We find $\nu = 2.56 \pm 0.02$ and $\eta = 0.51 \pm 0.01$, both of which are in excellent agreement with established results obtained from the CCM [23–34]. Thus, we establish that the IQHIT as viewed in CF coordinates is governed by the same fixed point as the CCM. This observation opens new possibilities in studies of the IQHIT, where interaction effects may be included more readily.

IQHIT in the idealized CF model.—2D electrons in a perpendicular magnetic field B can be transformed, via an exact mapping (“flux attachment”) [12,13] to CFs that couple to the sum of the external and “statistical” flux $B + b(r)$ [14,15]. When two quanta of flux are attached to each electron, there is an exact identity relating the CF density to the statistical flux: $b(r) = -4\pi n(r)$. CF mean-field theory results from “smearing the flux” and the identity is satisfied only on average: $\langle b(r) \rangle = -4\pi \langle n(r) \rangle$. With a quenched random potential $V(r)$ that varies on length scales large compared to the magnetic length, the linear response is a random density $\delta n(r) = \chi V(r)$, where $\chi = m/2\pi$ is the uniform compressibility [35]. Thus, in CF mean-field theory, there is a slaving [20,36] between $V(r)$

and $b(r)$ [37]: $V(r) = -b(r)/2m$. Furthermore, for asymptotic behavior near criticality, we can ignore non-linear response effects and study the following model Hamiltonian density:

$$h(r) = c^\dagger \left[\frac{(-i\partial - \mathbf{a})^2}{2m} + \frac{gb(r)}{2 \cdot 2m} - \mu \right] c, \quad b(r) = \epsilon_{ij} \partial_i a_j. \quad (1)$$

It involves free, spin-polarized fermions with parabolic dispersion coupled to a random vector potential $\mathbf{a}(r)$, along with a “gyromagnetic term” $(g/2)b(r)/2m$. In the above context of CF mean-field theory with long-wavelength disorder, $g = 2$.

Surprisingly, when $g = 2$, the system undergoes an IQHIT as the spatial-average magnetic field $b_0 \equiv \bar{b}(r)$ changes sign. To see why, observe that when $b_0 \neq 0$, all finite energy states are localized in the thermodynamic limit at $T = 0$. However, there are exact zero energy modes [38] for $b_0 < 0$, which behave as a filled Landau level. The zero modes are absent for $b_0 > 0$. As a consequence, the zero temperature phases are IQH (insulating) states for $b_0 < 0$ ($b_0 > 0$). At the critical point ($b_0 = 0$), the Hall conductivity can be computed *analytically* for the Hamiltonian above, and is $\sigma_{xy} = -e^2/2h$ [39]. It follows from the contrapositive of Laughlin’s gauge argument that states at all energies are *delocalized* when $b_0 = 0$. Our present goal is to obtain critical exponents associated with this transition using a lattice realization of the above problem.

Lattice model.—The lattice analog of the above consists of CFs on a square lattice with nearest-neighbor hopping (we set the lattice spacing to unity). Quenched random scalar and vector potentials live on the lattice sites and links, respectively (Fig. 1):

$$\mathcal{H}_{\text{lattice}} = - \sum_{\langle ij \rangle} c_i^\dagger [t e^{ia_{ij}} + \mu \delta_{ij}] c_j - \sum_i V_i c_i^\dagger c_i, \quad (2)$$

where i, j label lattice sites, and $a_{ij} = -a_{ji}$ are associated with the directed nearest-neighbor link connecting sites i

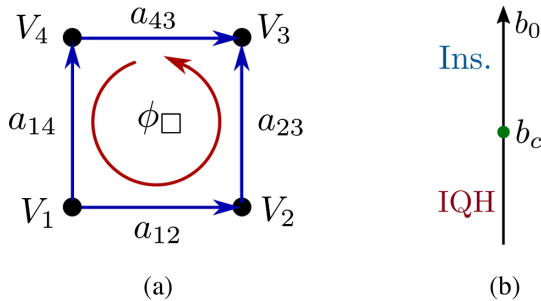


FIG. 1. (a) Tight-binding model on a square plaquette [Eq. (2)]. The flux is proportional to the average potential on the attached vertices [see Eq. (4)]. (b) Phase diagram of the lattice model. In the idealized limit, a topological phase transition between a $\nu_{\text{CF}} = -1$ IQH state ($b_0 < 0$), and insulator ($b_0 > 0$) occurs at $b_0 = b_c = 0$.

and j . Random fluxes are associated with each square plaquette of the lattice.

We slave the random chemical potential to the random flux as follows. Consider a square plaquette, whose vertices are lattice sites labeled 1–4 in a counterclockwise sense (Fig. 1). We equate the flux associated with the plaquette,

$$\phi_{\square} = a_{12} + a_{23} + a_{34} + a_{41}, \quad (3)$$

with the *average* of the 4 random potentials V_i :

$$\phi_{\square} = b_{\square} = -\frac{m}{g} \sum_{i=1}^4 V_i, \quad (4)$$

where, for simplicity, we take the mass to be the effective mass of the clean tight-binding model at the bottom of the band, i.e., $m = 1/2t$. We repeat the procedure for all elementary plaquettes of the lattice. We choose V_i , $i \in 1 \dots 4$, from an independent uniform distribution $V_i \in [-W/2 + V_0, W/2 + V_0]$, where $V_0 = -gb_0/4m$ and W measures the strength of the disorder.

Therefore, for a weak, long-range disorder and Fermi energy close to the bottom of the band, the Hamiltonian in Eq. (2) can be approximated as

$$\mathcal{H}_{\text{lattice}} \approx \frac{(\mathbf{p} - \mathbf{a})^2}{2m} + \frac{gb(r)}{2 \cdot 2m} - 4t. \quad (5)$$

Notice that since the flux through each plaquette is bounded in magnitude by π , we have $|W/2 \pm V_0| \leq g\pi/4m$. In principle one could adopt a more sophisticated procedure whereby the compressibility is determined in a self-consistent manner in equating the potential and flux disorders. We choose not to do so for simplicity: as we show below, the simple procedure employed here is already sufficient to capture the universal properties associated with the critical point, provided the Fermi energy remains sufficiently close to the band bottom to warrant an effective mass approximation.

Figure 2 displays the density of states over the entire bandwidth of the lattice model above, for nonzero b_0 . The lattice model increasingly accurately captures the behavior of the ideal model above in the limit where the Fermi level is close to the band bottom and the disorder is weak. For practical numerical calculations, however, it will be useful to use strong disorder which allows for shorter localization lengths and hence for better finite-size scaling behavior near the critical point. This deviation from weak and long-wavelength disorder, as well as lattice corrections to the effective mass approximation, lead to a shift in the location of the phase transition as a function of b_0 : in general, the IQHIT occurs at a finite value of b_0 , which approaches zero as the idealized limit of the previous section is approached.

Localization length exponent.—Employing the standard transfer matrix techniques [40,41], we study the behavior of

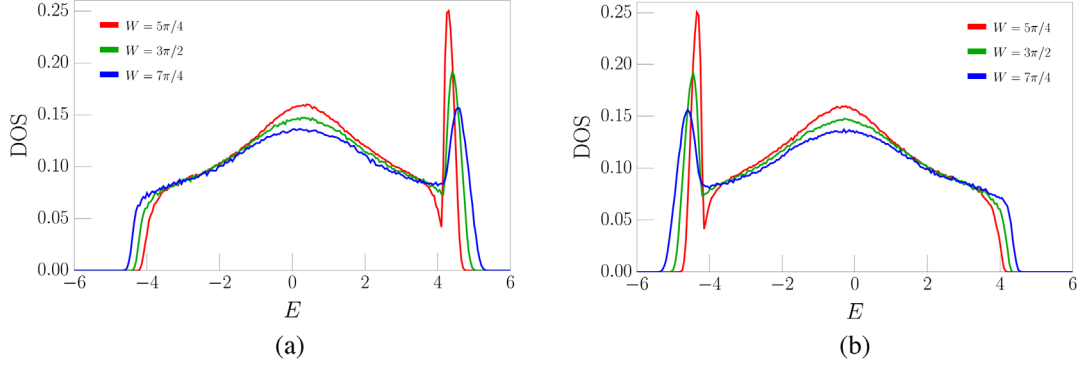


FIG. 2. Density of states of the Hamiltonian (2) for (a) $b_0 = 0.5$ and (b) $b_0 = -0.5$. There are zero modes near the bottom of the band for $b_0 < 0$ that become sharper as disorder strength is reduced. Further, they levitate for $b_0 > 0$.

the localization length in the CF model above. We realize the tight-binding model on a quasi-1D cylinder of dimensions $L \times M$, where L is the length of the cylinder along its axis while M is the circumference. We obtain the localization length $\xi_M(b_0)$ along the axis of the cylinder as a function of b_0 and the system width M with $g = 2$. In the 2D limit, i.e., $M \rightarrow \infty$, it diverges as $\xi_\infty(b_0) \sim |b_0 - b_c|^{-\nu}$ near the critical point with the critical exponent ν . We obtain ν via the finite-size scaling of the dimensionless localization length: $\Lambda_M(b_0) \equiv \xi_M(b_0)/M$ near the critical point [42]. To achieve this, we fit our data to the following polynomial function:

$$\Lambda_M(b_0) = \sum_{n=0}^{N_R} a_n (M^{1/\nu} \Delta)^n + \psi M^{-y} + c_{11} \psi \Delta M^{1/\nu} M^{-y}, \quad (6)$$

where N_R is the degree of the polynomial in the relevant parameter $\Delta \equiv b_0 - b_c$. ψ is the amplitude of the leading irrelevant operator and y is the corresponding correction to scaling exponent. Further, a_n , c_{11} , and b_c are fitting parameters, the last of which gives the location of the transition.

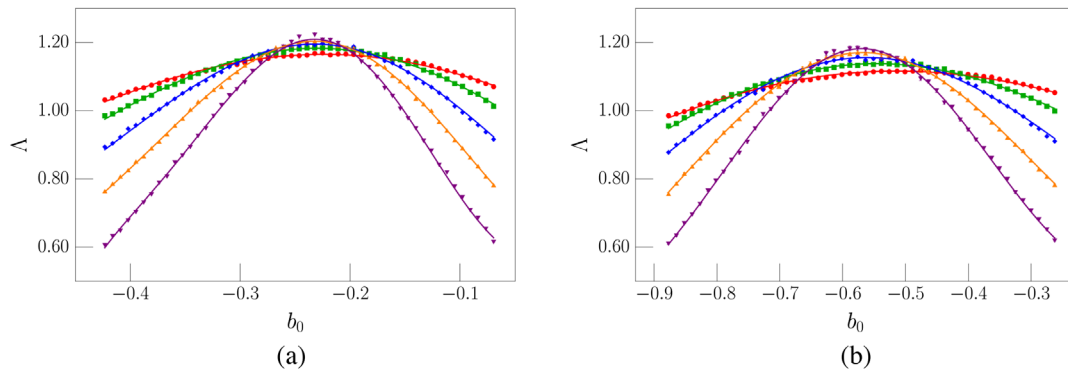


FIG. 3. Scaling of the renormalized localization length as a function of b_0 at $E_F = -4$ and (a) $W = 3\pi/2$, (b) $W = 7\pi/4$. We use $L = 10^7$, and the red to purple data points correspond to $M = 16, 32, 64, 128, 256$ in order. The best fit to Eq. (6) is drawn with solid lines and we obtain the critical exponent: (a) $\nu = 2.56 \pm 0.02$, (b) $\nu = 2.57 \pm 0.02$. Critical points are located at $b_0 = b_c$ where (a) $b_c = -0.229$ and (b) $b_c = -0.558$.

For $W = 3\pi/2$ and Fermi energy $E_F = -4$ at $g = 2$, we plot the calculated $\Lambda_M(b_0)$ in Fig. 3(a). Fitting the data to the above polynomial form using the standard least square error method, we extract $\nu = 2.56 \pm 0.02$. Also, for a stronger disorder, $W = 7\pi/4$, we find $\nu = 2.57 \pm 0.02$ [Fig. 3(b)], suggesting that the exponent is independent of disorder strength. These results are in agreement with the previous studies of the IQHIT using the CCM [7,23–33]. They support the idea that the two descriptions of IQHIT lead to the same universal behavior. In addition, we note that our results are slightly inconsistent with studies based on other models reporting a smaller exponent [43–45].

Multifractal scaling.—In addition to the localization length exponent ν , wave function multifractality represents additional universal characteristics of the IQHIT. They correspond to the finite-size scaling of the inverse participation ratios P_q calculated from the critical wave function ψ :

$$P_q \equiv L^d \langle |\psi|^{2q} \rangle \propto L^{-2(q-1) - \Delta(q)}, \quad (7)$$

where L is the system size and $d = 2$. Employing standard techniques [46,47], we calculate these exponents using the

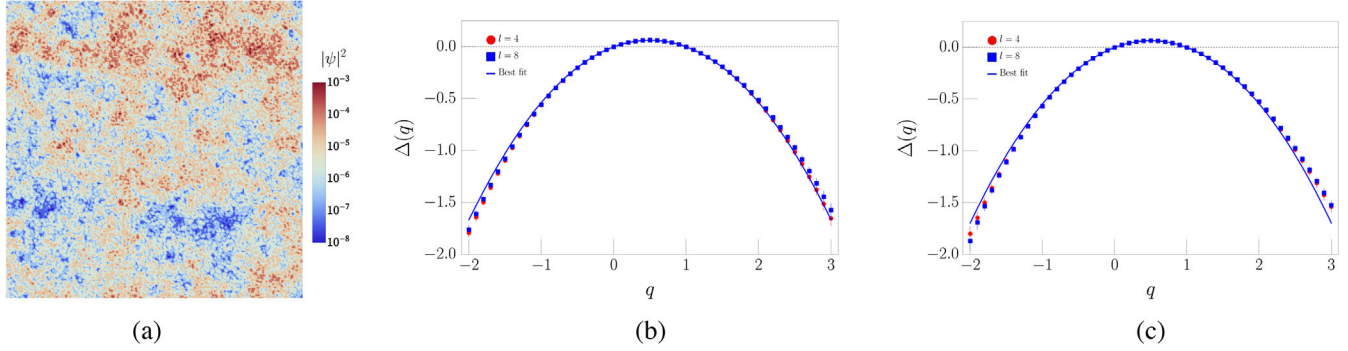


FIG. 4. (a) A critical wave function ψ displaying multifractal behavior. Numerically calculated $\Delta(q)$ at the IQHIT for $E_F = -4$ and (b) $W = 3\pi/2$ and (c) $W = 7\pi/4$. We first average $|\psi|^2$ over a box of dimensions $l \times l$ and then over 1000 wave functions. Using $L = 32, 64, 128$, and 256 , $\Delta(q)$ is obtained by performing finite-size scaling according to Eq. (7). The best fit to Eq. (8) using $l = 8$ data is drawn with the solid blue line and we get (a) $\gamma = 0.129 \pm 0.005$, (b) $\gamma = 0.133 \pm 0.006$.

critical wave functions of a square system of dimensions $L \times L$ with periodic boundary conditions. Since the total flux through the sample is quantized in the units of 2π , we round b_c obtained in the previous section to the nearest integer multiple of $2\pi/L^2$.

For the critical point in Fig. 3(a) at $b_0 = -0.229$, we find $\eta \equiv -\Delta(2) = 0.51 \pm 0.01$. And for the critical point in Fig. 3(b) at $b_0 = -0.558$, we get $\eta = 0.52 \pm 0.01$. These are close to the value $\eta = 0.5425$ obtained in Ref. [34]. Further, they are also consistent with $\eta = 0.5$ predicted in Ref. [48]. We plot the full multifractal spectra in Fig. 4 and fit them to the following form symmetric around $q = 1/2$ [34,49]:

$$\Delta(q) = 2q(1-q)[\gamma_0 + \gamma_1(q-1/2)^2 + \gamma_2(q-1/2)^2]. \quad (8)$$

We find $\gamma_0 = 0.129 \pm 0.005$, $\gamma_1 = 0.003 \pm 0.003$, $\gamma_2 = -0.0002 \pm 0.0004$ and $\gamma_0 = 0.133 \pm 0.006$, $\gamma_1 = 0.002 \pm 0.004$, $\gamma_2 = -0.00005 \pm 0.00050$ for the two critical points. These are in excellent agreement with the corresponding quantities in Ref. [34]. Likewise, we also find evidence for corrections to the proposed parabolic form [50–53] since $\gamma_1 \neq 0$. It should be noted that our data do not show a perfect symmetry around $q = 1/2$. We believe that this is due to finite-size effects. Similar to Ref. [34], as we report in Supplemental Material [54], the asymmetry in $\Delta(q)$ approaches zero in the thermodynamic limit.

TABLE I. Summary of exponents at Fermi energy $E_F = -4$ and $g = 2$.

Parameters	ν	η	γ_0
$W = 3\pi/2$ $b_0 = -0.229$	2.56 ± 0.02	0.51 ± 0.01	0.129 ± 0.005
$W = 7\pi/4$ $b_0 = -0.558$	2.57 ± 0.02	0.52 ± 0.01	0.133 ± 0.006

We summarize the results of all obtained critical exponents in Table I.

While the value $g = 2$ in Eq. (1) is motivated by CF mean-field theory, we can consider the effect of relaxing the value of g on the IQHIT. Such deviations from $g = 2$ can arise from lattice corrections to the effective mass approximation, or from the breaking of particle-hole symmetry in the disorder-averaged theory [58]. As we show in Supplemental Material [58], the localization length exponent decreases monotonically with g . The extent to which such deviations [59] reflect a new universality class for the IQHITs, or are due to substantial finite-size effects, or from large corrections to scaling from irrelevant operators, remain unclear and require further study. We shall return to these questions in future work.

Discussion.—Our results have several important implications for the IQHIT, and suggest several new directions of exploration. The most important implication of our study governs finite temperature dc transport in the quantum critical regime. In electron coordinates, extended states occur at a single energy, and without any interaction effects, $\rho_{xx}(T \rightarrow 0) \neq \rho_{xx}(T = 0)$. By contrast, in the CF representation, this issue does not arise, since extended states occur over a range of energies at criticality. Indeed, a finite CF resistivity implies the same for the electrical resistivity via the exact relation [60]:

$$\rho_{\text{CF}}^{ab} = \rho_{\text{el}}^{ab} + 4\pi\epsilon^{ab}, \quad \epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (9)$$

It is thus the CF representation that guarantees a smooth $T \rightarrow 0$ limit of the resistivity tensor in mean-field theory.

Second, the success of the CF mean-field theory suggests new analytic approaches to describing the noninteracting IQHIT. Recent work has shown that the effective theory governing disorder-averaged quantities in the weak-coupling regime $\sigma_{xx}^{\text{CF}} \gg 1$ is a nonlinear σ model with a topological term, similar to the theory put forward in

electron coordinates [22]. However, such theories run to strong coupling, since the critical point itself occurs at $\sigma_{xx}^{\text{CF}} \sim \mathcal{O}(1)$. Recently, a current algebra description of the IQHIT was proposed in Ref. [48]. Our multifractal scaling results are in excellent agreement with the predictions of Ref. [48]. However, the prediction for the localization length exponent in Ref. [48] requires much larger system sizes than our current simulations. It is likely that the CF representation may give way to new analytic treatments. One possible route is to note that the theory in Eq. (1) is equivalent to a two-component Dirac fermion at finite chemical potential in the presence of a random vector potential. The non-Abelian bosonization of the Dirac fermion may lead to complementary descriptions in terms of Wess-Zumino-Witten models. We shall report progress on such analytic treatments in future studies.

Conclusions.—In summary, we have calculated the critical and multifractal exponents for the IQHIT using a composite-fermion representation, which are in agreement with numerical studies of the CCM. While the electron and CF formulations have distinct origins, they are expected to flow to the same IR fixed point governing the IQHIT: in this sense, the electron and CF formulations are thus “dual” to one another.

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- [1] H. P. Wei, D. C. Tsui, M. A. Paalanen, and A. M. M. Pruisken, Experiments on Delocalization and Universality in the Integral Quantum Hall Effect, *Phys. Rev. Lett.* **61**, 1294 (1988).
- [2] L. W. Engel, D. Shahar, i. m. c. Kurdak, and D. C. Tsui, Microwave Frequency Dependence of Integer Quantum Hall Effect: Evidence for Finite-Frequency Scaling, *Phys. Rev. Lett.* **71**, 2638 (1993).
- [3] D. Shahar, D. C. Tsui, M. Shayegan, R. N. Bhatt, and J. E. Cunningham, Universal Conductivity at the Quantum Hall Liquid to Insulator Transition, *Phys. Rev. Lett.* **74**, 4511 (1995).
- [4] D. Shahar, D. C. Tsui, M. Shayegan, E. Shimshoni, and S. L. Sondhi, Evidence for charge-flux duality near the quantum Hall liquid-to-insulator transition, *Science* **274**, 589 (1996).
- [5] K. Yang, D. Shahar, R. N. Bhatt, D. C. Tsui, and M. Shayegan, Study of universality at integer quantum Hall transitions, *J. Phys. Condens. Matter* **12**, 5343 (2000).
- [6] S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, Continuous quantum phase transitions, *Rev. Mod. Phys.* **69**, 315 (1997).
- [7] J. T. Chalker and P. D. Coddington, Percolation, quantum tunnelling and the integer Hall effect, *J. Phys. C* **21**, 2665 (1988).
- [8] B. Huckestein and M. Backhaus, Integer Quantum Hall Effect of Interacting Electrons: Dynamical Scaling and Critical Conductivity, *Phys. Rev. Lett.* **82**, 5100 (1999).
- [9] Z. Wang, M. P. A. Fisher, S. M. Girvin, and J. T. Chalker, Short-range interactions and scaling near integer quantum Hall transitions, *Phys. Rev. B* **61**, 8326 (2000).
- [10] J. K. Jain, Composite-Fermion Approach for the Fractional Quantum Hall Effect, *Phys. Rev. Lett.* **63**, 199 (1989).
- [11] J. K. Jain, *Composite Fermions* (Cambridge University Press, Cambridge, England, 2007).
- [12] S. C. Zhang, T. H. Hansson, and S. Kivelson, Effective-Field-Theory Model for the Fractional Quantum Hall Effect, *Phys. Rev. Lett.* **62**, 82 (1989).
- [13] A. Lopez and E. Fradkin, Fractional quantum Hall effect and Chern-Simons gauge theories, *Phys. Rev. B* **44**, 5246 (1991).
- [14] V. Kalmeyer and S.-C. Zhang, Metallic phase of the quantum Hall system at even-denominator filling fractions, *Phys. Rev. B* **46**, 9889 (1992).
- [15] B. I. Halperin, P. A. Lee, and N. Read, Theory of the half-filled Landau level, *Phys. Rev. B* **47**, 7312 (1993).
- [16] V. Pasquier and F. D. M. Haldane, A dipole interpretation of the $\nu = 1/2$ state, *Nucl. Phys.* **B516**, 719 (1998).
- [17] N. Read, Lowest-Landau-level theory of the quantum Hall effect: The Fermi-liquid-like state of bosons at filling factor one, *Phys. Rev. B* **58**, 16262 (1998).
- [18] D. T. Son, Is the Composite Fermion a Dirac Particle?, *Phys. Rev. X* **5**, 031027 (2015).
- [19] N. Seiberg, T. Senthil, C. Wang, and E. Witten, A duality web in 2 + 1 dimensions and condensed matter physics, *Ann. Phys. (Amsterdam)* **374**, 395 (2016).
- [20] P. Kumar, M. Mulligan, and S. Raghu, Topological phase transition underpinning particle-hole symmetry in the Halperin-Lee-Read theory, *Phys. Rev. B* **98**, 115105 (2018).
- [21] P. Kumar, Y. B. Kim, and S. Raghu, Self-duality of the integer quantum Hall to insulator transition: Composite fermion description, *Phys. Rev. B* **100**, 235124 (2019).
- [22] P. Kumar, P. A. Nosov, and S. Raghu, Interaction effects on quantum Hall transitions: Dynamical scaling laws and superuniversality, [arXiv:2006.11862](https://arxiv.org/abs/2006.11862).
- [23] D.-H. Lee, Z. Wang, and S. Kivelson, Quantum Percolation and Plateau Transitions in the Quantum Hall Effect, *Phys. Rev. Lett.* **70**, 4130 (1993).
- [24] K. Slevin and T. Ohtsuki, Critical exponent for the quantum Hall transition, *Phys. Rev. B* **80**, 041304(R) (2009).
- [25] H. Obuse, A. R. Subramaniam, A. Furusaki, I. A. Gruzberg, and A. W. W. Ludwig, Conformal invariance, multifractality, and finite-size scaling at Anderson localization transitions in two dimensions, *Phys. Rev. B* **82**, 035309 (2010).
- [26] M. Amado, A. V. Malyshev, A. Sedrakyanyan, and F. Domínguez-Adame, Numerical Study of the Localization Length Critical Index in a Network Model of Plateau-Plateau Transitions in the Quantum Hall Effect, *Phys. Rev. Lett.* **107**, 066402 (2011).

- [27] I. C. Fulga, F. Hassler, A. R. Akhmerov, and C. W. J. Beenakker, Topological quantum number and critical exponent from conductance fluctuations at the quantum Hall plateau transition, *Phys. Rev. B* **84**, 245447 (2011).
- [28] K. Slevin and T. Ohtsuki, Finite size scaling of the Chalker-Coddington model, *Int. J. Mod. Phys. Conf. Ser.* **11**, 60 (2012).
- [29] H. Obuse, I. A. Gruzberg, and F. Evers, Finite-Size Effects and Irrelevant Corrections to Scaling Near the Integer Quantum Hall Transition, *Phys. Rev. Lett.* **109**, 206804 (2012).
- [30] W. Nuding, A. Klümper, and A. Sedrakyan, Localization length index and subleading corrections in a Chalker-Coddington model: A numerical study, *Phys. Rev. B* **91**, 115107 (2015).
- [31] M. Puschmann, P. Cain, M. Schreiber, and T. Vojta, Integer quantum Hall transition on a tight-binding lattice, *Phys. Rev. B* **99**, 121301(R) (2019).
- [32] B. Sbierski, E. J. Dresselhaus, J. E. Moore, and I. A. Gruzberg, Criticality of two-dimensional disordered Dirac fermions in the unitary class and universality of the integer quantum Hall transition, [arXiv:2008.09025](https://arxiv.org/abs/2008.09025).
- [33] N. Charles, I. A. Gruzberg, A. Klümper, W. Nuding, and A. Sedrakyan, Critical behavior at the integer quantum Hall transition in a network model on the kagome lattice, *Phys. Rev. B* **102**, 121304(R) (2020).
- [34] F. Evers, A. Mildenerger, and A. D. Mirlin, Multifractality at the Quantum Hall Transition: Beyond the Parabolic Paradigm, *Phys. Rev. Lett.* **101**, 116803 (2008).
- [35] The mass m that enters the compressibility is the same as the band mass, which is renormalized by Coulomb interactions. However, no further renormalization occurs due to gauge fluctuations, as shown in Ref. [61].
- [36] C. Wang, N. R. Cooper, B. I. Halperin, and A. Stern, Particle-Hole Symmetry in the Fermion-Chern-Simons and Dirac Descriptions of a Half-Filled Landau Level, *Phys. Rev. X* **7**, 031029 (2017).
- [37] From here on, we change our notation so that $b(r)$ and $a(r)$ represent the “effective” magnetic and gauge fields experienced by the composite fermions. The effective field is related by a shift to the “statistical” gauge field, i.e., $a_{\text{eff.}}(r) = a_{\text{stat}}(r) - A(r)$.
- [38] Y. Aharonov and A. Casher, Ground state of a spin- $\frac{1}{2}$ charged particle in a two-dimensional magnetic field, *Phys. Rev. A* **19**, 2461 (1979).
- [39] We assume that all odd moments of $V(r)$ vanish.
- [40] A. MacKinnon and B. Kramer, One-Parameter Scaling of Localization Length and Conductance in Disordered Systems, *Phys. Rev. Lett.* **47**, 1546 (1981).
- [41] A. MacKinnon and B. Kramer, The scaling theory of electrons in disordered solids: Additional numerical results, *Z. Phys. B* **53**, 1 (1983).
- [42] K. Slevin and T. Ohtsuki, Corrections to Scaling at the Anderson Transition, *Phys. Rev. Lett.* **82**, 382 (1999).
- [43] I. A. Gruzberg, A. Klümper, W. Nuding, and A. Sedrakyan, Geometrically disordered network models, quenched quantum gravity, and critical behavior at quantum Hall plateau transitions, *Phys. Rev. B* **95**, 125414 (2017).
- [44] M. Ippoliti, S. D. Geraedts, and R. N. Bhatt, Integer quantum Hall transition in a fraction of a Landau level, *Phys. Rev. B* **97**, 014205 (2018).
- [45] Q. Zhu, P. Wu, R. N. Bhatt, and X. Wan, Localization-length exponent in two models of quantum Hall plateau transitions, *Phys. Rev. B* **99**, 024205 (2019).
- [46] B. Huckestein, Scaling theory of the integer quantum Hall effect, *Rev. Mod. Phys.* **67**, 357 (1995).
- [47] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).
- [48] M. R. Zirnbauer, The integer quantum Hall plateau transition is a current algebra after all, *Nucl. Phys.* **B941**, 458 (2019).
- [49] A. D. Mirlin, Y. V. Fyodorov, A. Mildenerger, and F. Evers, Exact Relations between Multifractal Exponents at the Anderson Transition, *Phys. Rev. Lett.* **97**, 046803 (2006).
- [50] M. R. Zirnbauer, Conformal field theory of the integer quantum Hall plateau transition, [arXiv:hep-th/9905054](https://arxiv.org/abs/hep-th/9905054).
- [51] M. J. Bhaseen, I. I. Kogan, O. A. Soloviev, N. Taniguchi, and A. M. Tsvelik, Towards a field theory of the plateau transitions in the integer quantum Hall effect, *Nucl. Phys.* **B580**, 688 (2000).
- [52] A. M. Tsvelik, Evidence for the PSL(2|2) Wess-Zumino-Novikov-Witten model as a model for the plateau transition in the quantum Hall effect: Evaluation of numerical simulations, *Phys. Rev. B* **75**, 184201 (2007).
- [53] R. Bondesan, D. Wieczorek, and M. Zirnbauer, Gaussian free fields at the integer quantum Hall plateau transition, *Nucl. Phys.* **B918**, 52 (2017).
- [54] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.126.056802> for a finite-size scaling study of the asymmetry in multifractal exponents $\Delta(q)$ and the g dependence of the critical exponent of the lattice model, which includes Refs. [55–57].
- [55] A. Pruisken, On localization in the theory of the quantized Hall effect: A two-dimensional realization of the θ -vacuum, *Nucl. Phys.* **B235**, 277 (1984).
- [56] D. E. Khmel'nitskii, Quantization of Hall conductivity, *Sov. JETP Lett.* **38**, 552 (1983), http://www.jetpletters.ac.ru/ps/1485/article_22668.shtml.
- [57] A. M. M. Pruisken, Dilute instanton gas as the precursor to the integral quantum Hall effect, *Phys. Rev. B* **32**, 2636 (1985).
- [58] For Particle-hole symmetry at $\nu = 1/2$, the Hall conductivity of CFs at $b_0 = 0$ must be $\sigma_{xy}^{\text{CF}} = -e^2/2h$. This was shown to be the case for $g = 2$ in Eq. (1) in Ref. [20]. However, if $g \neq 2$, the particle-hole symmetry can be broken.
- [59] A. Klümper, W. Nuding, and A. Sedrakyan, Random network models with variable disorder of geometry, *Phys. Rev. B* **100**, 140201(R) (2019).
- [60] S. Kivelson, D.-H. Lee, and S.-C. Zhang, Global phase diagram in the quantum Hall effect, *Phys. Rev. B* **46**, 2223 (1992).
- [61] Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Gauge-invariant response functions of fermions coupled to a gauge field, *Phys. Rev. B* **50**, 17917 (1994).