Dissipative Engineering of Gaussian Entangled States in Harmonic Lattices with a Single-Site Squeezed Reservoir

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We study the dissipative preparation of many-body entangled Gaussian states in bosonic lattice models which could be relevant for quantum technology applications. We assume minimal resources, represented by systems described by particle-conserving quadratic Hamiltonians, with a single localized squeezed reservoir. We show that in this way it is possible to prepare, in the steady state, the wide class of pure states which can be generated by applying a generic passive Gaussian transformation on a set of equally squeezed modes. This includes nontrivial multipartite entangled states such as cluster states suitable for measurement-based quantum computation.

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The harnessing of quantum many-body dynamics by engineered dissipation is interesting for applications in quantum technology [1–3]. In these approaches, the environment of many interacting quantum systems is designed in such a way that the interplay between controlled dissipation and interactions results in specific controlled system dynamics [3–6], in the simulation of complex quantum system [7–9], and in the robust preparation of nontrivial quantum global stationary states [1,2,10–13], including Gaussian states [14]. In general, the practical realization of these dynamics is hampered by the need to engineer the environment of all the many elements which constitute the system. However, it has been also shown that under certain conditions it is possible to engineer a single localized reservoir to have control over the global properties of the system [15–17].

In this work we are interested in strategies which make use of minimal resources, namely, only one squeezed reservoir and a bosonic lattice with a passive (particle-conserving) quadratic Hamiltonian [17–25]. It has been shown that these systems can be steered into peculiar entangled steady states, when the squeezed reservoir is coupled to single site of the lattice and the Hamiltonian is endowed with specific symmetries [18,21]. Here we characterize the class of Gaussian pure states that can be achieved with this approach, and we show that it is composed of all the states that can be generated by applying any combination of particle-conserving quadratic operations (beam splitters and phase shifts) on a set of equally squeezed modes. We also identify the general properties of the Hamiltonians which enable the generation of these pure stationary states (showing, in particular, that they necessarily satisfy the chiral symmetry identified in Ref. [21]), and, for each state, we discuss how to construct the specific Hamiltonian which sustains such state in the stationary regime. Interestingly, the class of states that can be obtained in this way includes Gaussian cluster states usable for universal measurement-based quantum computation with continuous variables [26,27], and, as a prominent example, we study the performance of the present approach for the preparation of a cluster state in a square lattice. In measurement-based quantum computation a big part of the complexity of the computation is placed into the preparation of the cluster state. In particular, optical setups are very promising and scalable platforms for this task [28–42]. Our proposal suggests that similar results could be achieved also with localized quantum modes in, for example, circuit QED systems [43–45].

In detail, we study the dissipative preparation of a zero-average pure Gaussian state of N bosonic modes $|\Psi\rangle$, considering $N+1$ bosonic modes (including an additional auxiliary mode). They are described by the annihilation operators b_j for $j \in \{0, 1 \cdots N\}$, and we assume that only the auxiliary mode, that is the one with index $j=0$, is coupled to a squeezed reservoir. In the ideal situation the auxiliary mode is the only open mode which is subject to dissipation in the squeezed reservoir. Additional dissipation acting on the other modes reduces the purity of the final state and will be addressed later on. We assume quadratic Hamiltonians H for the $N+1$ modes, with only passive interaction terms, $H = \hbar \sum_{j,k=0}^{N} \mathcal{J}_{j,k} b_j^\dagger b_k$ (with $\mathcal{J}_{j,k} = \mathcal{J}_{k,j}^*$), which conserves the number of excitations, so that the existing quantum correlations in the steady state are a consequence of the correlations in the reservoirs only. The system is described by the master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho, \tag{1}$$

where the effect of the squeezed bath is given by the Lindblad term $\mathcal{L}\rho = \kappa\{(\bar{n}+1)\mathcal{D}_{b_0,b_0^\dagger} + \bar{n}\mathcal{D}_{b_0^\dagger,b_0} - \bar{m}^*\mathcal{D}_{b_0,b_0} - \bar{m}\mathcal{D}_{b_0^\dagger,b_0^\dagger}\}\rho$ with $\mathcal{D}_{x,y}\rho = 2x\rho y - yx\rho - \rho yx$, and $|\bar{m}| = \sqrt{\bar{n}(\bar{n}+1)}$ (this condition corresponds to a reservoir in a pure squeezed state; if $|\bar{m}| < \sqrt{\bar{n}(\bar{n}+1)}$ the reservoir is not pure, and the states that we discuss here are modified, in a straightforward way, by a thermal component [18]). The central result of this work is the following theorem.

Theorem.— A zero-average pure Gaussian state that is factorized between the auxiliary mode ($|\psi_0\rangle$) and the remaining N modes ($|\Psi\rangle$),

$$|\Psi_{\rm tot}\rangle = |\psi_0\rangle|\Psi\rangle, \tag{2}$$

and is generated from the vacuum $|0\rangle$ by the unitary transformations U_0 and U, such that $|\psi_0\rangle = U_0|0\rangle$ and $|\Psi\rangle = U|0\rangle$, is the unique steady state of Eq. (1) if and only if the following three propositions are true: (I) U_0 is the squeezing transformation $U_0 = e^{(z_0/2)(e^{i\varphi_0}b_0^{\dagger 2} - e^{-i\varphi_0}b_0{}^2)}$, where the squeezing strength z_0 and the squeezing phase φ_0 are determined by the squeezing of the reservoir according to the relations $\tanh(z_0) = \sqrt{\bar{n}/(\bar{n}+1)}$, and $e^{i\varphi_0} = \bar{m}/|\bar{m}|$; (II) U can be decomposed as $U = U^{(p)}U^{(S)}$, where $U^{(S)}$ is the product of N single-mode squeezing transformations with squeezing strength equal to that of the transformation U_0, i.e., $U^{(S)} = U_1 \cdots U_N$, with $U_j = e^{(z_0/2)(e^{i\varphi_j}b_j^{\dagger 2} - e^{-i\varphi_j}b_j{}^2)}$, and $U^{(p)}$ is a passive quadratic transformation (note that both $U^{(S)}$ and $U^{(p)}$ do not operate on the auxiliary mode); (III) the passive quadratic Hamiltonian H for the $N+1$ modes of Eq. (1) is given by $H = U^{(p)}H^{(S)}U^{(p)\dagger}$, where $H^{(S)}$ is any passive quadratic Hamiltonian for which the following propositions are true: (a) $H^{(S)}$ remains passive under the effect of the set of single-mode squeezing transformations for the $N+1$ modes $U_0U^{(S)}$, i.e., $U^{(S)\dagger}U_0^\dagger H^{(S)}U_0U^{(S)}$ is passive; (b) all the normal modes of $H^{(S)}$ have a finite overlap with the auxiliary mode (see Ref. [46]).

Proof.—Part 1: If the propositions I–III are true then Eq. (2) is the only steady state.—In the representation defined by the transformation U_0U, the transformed density matrix $\tilde{\rho} = U^\dagger U_0^\dagger \rho U_0 U$, fulfills the master equation $\dot{\tilde{\rho}} = -(i/\hbar)[\tilde{H},\tilde{\rho}] + \tilde{\mathcal{L}}\tilde{\rho}$ where the dissipative term, $\tilde{\mathcal{L}}\tilde{\rho} = \kappa\mathcal{D}_{b_0,b_0^\dagger}\tilde{\rho}$, describes pure dissipation in a vacuum reservoir, and the transformed Hamiltonian $\tilde{H} = U^\dagger U_0^\dagger H U_0 U$, can be written as $\tilde{H} = U^{(S)\dagger}U_0^\dagger H^{(S)}U_0U^{(S)}$. This shows that $\tilde{H}$ is passive because of proposition III(a). The proposition III (b), instead, entails that $H^{(S)}$, and therefore also H and $\tilde{H}$, have no dark modes [46], i.e., all the normal modes are coupled to the reservoir. Thus, the only steady state in the new representation is the vacuum, which is equal to Eq. (2) in the original representation.

Part 2: If Eq. (2) is the only steady state, then the propositions I–III are true.— In the representation defined by the density matrix $\tilde{\rho}$, the transformed steady state, $|\tilde{\Psi}_{\rm tot}\rangle = U^\dagger U_0^\dagger|\Psi_{\rm tot}\rangle = |0\rangle$, is the vacuum. This can be true only if the transformed Hamiltonian $\tilde{H}$ is passive with no dark modes, and the dissipative term $\tilde{\mathcal{L}}\tilde{\rho} = U_0^\dagger[\mathcal{L}(U_0\tilde{\rho}U_0^\dagger)]U_0$ describes pure dissipation in a vacuum reservoir. For this to be true U_0 has to fulfill the proposition I.

Now, in order to demonstrate the validity of the other propositions, we note that it is always possible to decompose a unitary transformation U, which generates a zero-average pure Gaussian state, in a form similar to the one defined in the proposition II, where $U^{(S)}$ is a set of single-mode squeezing transformations which can be, in general, of different strength, and $U^{(p)}$ is a multimode passive transformation. This can be seen by using the Bloch-Messiah decomposition [46]. Thus, Eq. (2) can be always written in the form $|\Psi_{\rm tot}\rangle = U_0U^{(p)}U^{(S)}|0\rangle$. In the representation defined by the transformed density matrix $\rho^{(S)} = U^{(p)\dagger}\rho U^{(p)}$, which fulfill the equation $\dot{\rho}^{(S)} = -(i/\hbar)[H^{(S)},\rho^{(S)}] + \mathcal{L}\rho^{(S)}$, the Hamiltonian $H^{(S)} = U^{(p)\dagger}HU^{(p)}$ is passive (because $U^{(p)}$ and H are passive), and remains passive under the effect of $U_0U^{(S)}$ (in fact $U^{(S)\dagger}U_0^\dagger H^{(S)}U_0U^{(S)} = \tilde{H}$ which, as we have seen, has to be passive), and therefore the proposition III(a) is true. Moreover, $\tilde{H}$ has no dark modes (because we are assuming that the system has a single steady state), and thus the proposition III(b) is true as well [46]. Finally, this also means that all the modes are connected (even if not directly) by the interactions terms of $H^{(S)}$, and this together with the following lemma guarantees that the strength of all the squeezing transformations which constitute $U^{(S)}$ are equal. In particular they have to be equal to the squeezing strength of the auxiliary mode z_0, which is fixed by the squeezing strengths of the reservoir, so also the proposition II is true.∎

Let us now introduce the following lemma that describes the precise structure of the Hamiltonian $H^{(S)}$.

Lemma.—Given a passive quadratic Hamiltonian, $H^{(S)} = \hbar\sum_{j,k=0}^N \mathcal{J}_{j,k}^{(S)} b_j^\dagger b_k$, with $\mathcal{J}_{j,k}^{(S)} = |\mathcal{J}_{j,k}^{(S)}|e^{i\Theta_{j,k}}$ and $\Theta_{j,k} = -\Theta_{k,j}$, the transformed Hamiltonian $\tilde{H} = U_N^\dagger \cdots U_0{}^\dagger H^{(S)}U_0\cdots U_N$, with $U_j = e^{(z_j/2)(e^{i\varphi_j}b_j^{\dagger 2} - e^{-i\varphi_j}b_j{}^2)}$, is passive, if and only if (i) $\mathcal{J}_{j,j}^{(S)} = 0$ for all j with $z_j \neq 0$, (ii) $\Theta_{j,k} = n\pi + (\varphi_j - \varphi_k + \pi)/2$ for $j < k$ (with $n \in \mathbb{Z}$), and $z_j = z_k$ for all $j \neq k$ with $\mathcal{J}_{j,k}^{(S)} \neq 0$. Moreover, if $\tilde{H}$ is passive then $\tilde{H} = H^{(S)}$. (The proof of this lemma is straightforward and is reported in the Supplemental Material [46]).

It is now important to point out that, for any given state $|\Psi\rangle$ which fulfills proposition II, each quadratic Hamiltonian $H^{(S)}$ that fulfills the propositions III(a)–III(b) (and the lemma) can be used to construct a (different) Hamiltonian H (see the proposition *III*) of model (1) which sustain the given state in the stationary regime. Thus the same steady state can be obtained with many different Hamiltonians. The specific form of H can determine how fast (and therefore how efficiently, when additional noise sources affect the system dynamics) the system approaches the steady state. We also note that both $H^{(S)}$ and H satisfy the chiral symmetry identified in Ref. [21] (see Ref. [46]). This implies that the chiral symmetry of H is also a necessary condition (not only a sufficient one, as suggested in Ref. [21]) for the existence of the pure steady state (2) of Eq. (1).

A particularly simple Hamiltonian $H^{(S)}$ that fulfills the propositions III(a)–III(b) (and the lemma) is the Hamiltonian for a linear chain with open boundary conditions (for which the normal modes have always a finite overlap with the end modes)

$$H^{(S)} = i\hbar \sum_{j=1}^{N} J_j^{(S)} (e^{i\theta_j} b_{j-1} b_j^\dagger - e^{-i\theta_j} b_{j-1}^\dagger b_j), \qquad (3)$$

where $\theta_j = (\varphi_j - \varphi_{j-1})/2$, with φ_j the squeezing phases introduced in the proposition II. This means that Eq. (3) can be used to construct the Hamiltonian H corresponding to any state that fulfills the proposition II. Specific examples of multimode entangled states that can be prepared with this strategy have been discussed in Refs. [18–23].

It is interesting to note that the class of states that can be prepared with our approach is wide and it includes also cluster states which are the main resource of measurement-based quantum computation [26,27]. In particular all the cluster states that have been proposed and prepared by manipulating one or two squeezed light beams with a complex interferometer [28–42] can be also generated following our approach. The difference between these results and the present approach is that, while in these works the state is prepared in traveling wave beams of light, our results shows how to generate similar states, in a robust way, as stationary states of a dissipative dynamics. This approach is, hence, attractive in situations in which the quantum modes are localized, as, for example, in a solid-state or atomic device [54,55].

Dissipative generation of a cluster state.—Let us now investigate the potentiality of our result to design a model which sustains in the stationary regime a cluster state in a square lattice [46] which constitutes a universal resource for measurement-based quantum computation [27,38]. To be specific, we consider a cluster state of $N = 25$ modes with a $N \times N$ real symmetric adjacency matrix $\mathcal{A}$ (with nonzero entries equal to one) which represents the square lattice [46]. This state can be generated by the multimode squeezing transformation [56] $U_z = e^{-i(z/2)\sum_{j,k=1}^{N}(\mathcal{Z}_{j,k} b_j^\dagger b_k^\dagger + \mathcal{Z}_{j,k}^* b_j b_k)}$, where the $N \times N$ matrix of interaction coefficients is given by $\mathcal{Z} = -i(\mathcal{A} - i\mathbb{1})(\mathcal{A} + i\mathbb{1})^{-1}$. What characterizes this as cluster state is the fact that the covariance matrix of the N operators $x_j = p_j - \sum_{k=1}^{N} \mathcal{A}_{j,k} q_k$ [with $q_j = b_j + b_j^\dagger$ and $p_j = -i(b_j - b_j^\dagger)$], called nullifiers, approaches the null matrix in the limit of infinite squeezing, $z \to \infty$ [56]. The transformation U_z can be decomposed, similarly to the definition in the proposition II of the theorem, as $U_z = U_z^{(p)} U_z^{(S)}$, with $U_z^{(S)}$ given by the product of N equal single-mode squeezing transformations (where $\varphi_j = 0$ for all j), and with $U_z^{(p)}$ which fulfills the relation $U_z^{(p)\dagger} b_j U_z^{(p)} = \sum_{k=1}^{N} \{(-i\mathcal{Z})^{1/2}\}_{j,k} b_k$ [46]. The fact that $U_z^{(S)}$ describes the equal squeezing of all the modes implies, according to our theorem, that $U_z|0\rangle$ is the steady state of Eq. (1) when

$$H = U_z^{(p)} H^{(S)} U_z^{(p)\dagger}, \qquad (4)$$

where $H^{(S)}$ is the Hamiltonian for the linear chain (3). Note that the same cluster state, given by a specific adjacency matrix, can be generated by many different transformations U_z, which correspond to different $U_z^{(p)}$ [46,56,57], and thus to different H. The specific form of H can be relevant and should be taken into account when considering an experimental implementation of these results.

In Figs. 1 and 2 we show the results for the preparation of this cluster state. We have studied how the present approach performs in nonideal situations that include additional noise sources, with dissipation rate γ, and random deviations from the optimal system Hamiltonian defined in Eq. (4). In particular, in Figs. 1 and 2, we characterize the steady state $\rho'_{\rm st}$ of

$$\dot{\rho}' = -\frac{i}{\hbar}[H, \rho'] + \mathcal{L}\rho' + \gamma \sum_{j=0}^{N} \mathcal{D}_{b_j, b_j^\dagger} \rho', \qquad (5)$$

in terms of its fidelity with respect to the steady state $\rho_{\rm st}$ achievable with $\gamma = 0$ [black solid line, panel (a)], and in terms of the variance of the nullifiers over $\rho'_{\rm st}$, relative to the variance over the vacuum [dark gray lines, panels (b)]. We observe that significant reduction of the variance (squeezing) of the nullifiers (which indicates that the state is close to the cluster state) is observed when $\gamma(N+1) \ll \kappa$, namely, when the total added dissipation is much weaker than the dissipation in the squeezed reservoir. The thin lines in panel (a) describe how the model is sensitive to deviation form the ideal Hamiltonian (4). We have considered both deviation in the amplitude (thin solid gray lines) and in the phase (thin dashed red lines) of the interaction coefficients, and we observe that the system is significantly more stable with respect to the latter. In any case, even when the fidelity is very low, the nullifiers always exhibit significant squeezing [panel (c)].

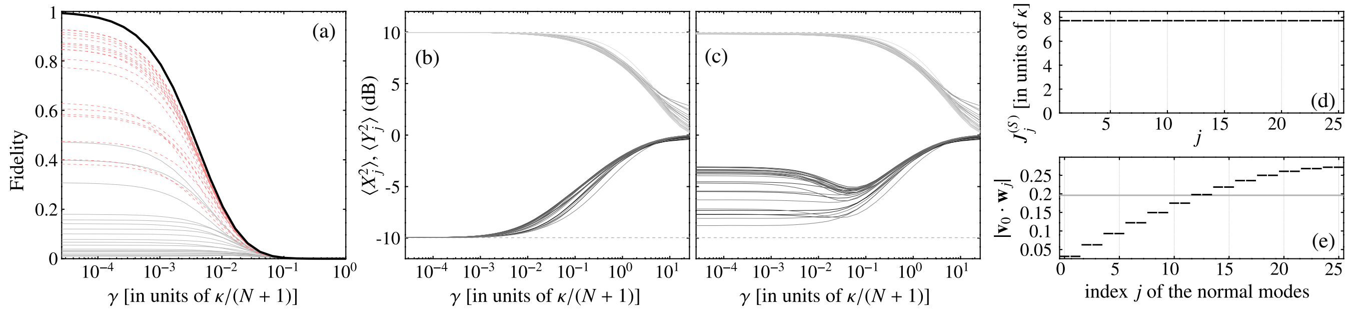

FIG. 1. Dissipative preparation of a cluster state of $N = 25$ modes in a 5×5 square lattice [46]. (a) Fidelity $\mathrm{Tr}\{\rho'_{\mathrm{st}}\rho_{\mathrm{st}}\}$ [53] between the steady state ρ'_{st} of the model (5) and the corresponding steady state ρ_{st} of Eq. (1). The thick black line is evaluated using the Hamiltonian (4) (with $J_j^{(S)} = 7.7\kappa$ [see panel (d)] and $\theta_j = 0,\ \forall\, j$); the thin solid gray lines are evaluated for 20 random realizations of the system Hamiltonian with interaction coefficients $\mathcal{J}_{j,k}^{(\zeta)} = \mathcal{J}_{j,k}(1+\zeta_{j,k})$ where $\mathcal{J}_{jk}$ are the coefficients of H [46], and $\zeta_{j,k}$ are random variables uniformly distributed in the range $[-0.001, 0.001]$; The thin dashed red lines are evaluated for 20 random realizations of the system Hamiltonian with $\mathcal{J}_{j,k}^{(\beta)} = \mathcal{J}_{j,k}e^{i\beta_{j,k}}$ where $\beta_{j,k} = -\beta_{k,j}$ are random variables uniformly distributed in the range $[-0.015, 0.015]$. (b),(c) Corresponding steady state variance of the normalized nullifiers $X_j = r_j x_j$ (lower dark gray lines) and of the orthogonal collective quadratures $Y_j = r_j y_j$ with all the modes rotated by $\pi/2$, such that $y_j = -q_j - \sum_{k=1}^{N}\mathcal{A}_{j,k}p_k$ (upper light gray lines), and where the normalization coefficients r_j are chosen such that X_j and Y_j fulfill the standard commutation relation $[X_j, Y_j] = 2i$. Panel (b) corresponds to the thick black line of (a). Panel (c) corresponds to the realization (thin gray line) with the lowest fidelity of panel (a). The horizontal dashed lines in (b) and (c) indicate the variance of the squeezed and antisqueezed quadratures of the squeezed reservoir, which corresponds to $\bar{n} = 2$. (d) Interaction coefficients $J_j^{(S)}$ of Eq. (3) used to compute the Hamiltonian (4). (e) Corresponding overlap of the normal modes of $H^{(S)}$ and the auxiliary mode, i.e., scalar product $|\mathbf{v}_0 \cdot \mathbf{w}_j|$ between the normalized eigenvectors $\mathbf{w}_j$ of the coefficient matrix $\mathcal{J}^{(S)}$ of the Hamiltonian $H^{(S)}$, and the vector, corresponding to the auxiliary mode, $\mathbf{v}_0 = (1, 0\cdots, 0)$. The horizontal gray line in (e) indicates the value $1/\sqrt{N+1}$.

We note that the overlaps between normal modes and auxiliary mode [see panel (e)] determine the rates at which each normal mode is coupled to the squeezed reservoir. In the ideal case, these overlaps determine how fast each normal mode approaches the steady state. The optimal situation is the one in which all the overlaps are equal and are as large as possible so that all the normal modes are optimally coupled to the reservoir. This is described by Fig. 2, which shows that in this case the system is significantly more resistant to deviations from the ideal configuration. We also note that the overlaps are the same for both $H^{(S)}$ and H (because $U^{(p)}$ does not operate on the auxiliary mode [46]). And this means that, for any state, the time to reach the steady state is entirely determined by the dynamics of the linear chain [Eq. (3)].

In conclusion, we have shown that, by squeezing the local environment of a single site of an harmonic lattice, it is possible to steer the whole system toward any pure Gaussian state that can be generated by a passive multimode transformation which operates on a batch of many equally squeezed modes. In particular, given one of these states, we have shown how to determine a passive quadratic Hamiltonian which sustain it in the stationary regime (and which necessarily fulfills the chiral symmetry identified in Ref. [21]). This Hamiltonian is not unique [46], and we have shown, by studying the generation of a cluster state in a square lattice, that the efficiency for the preparation of the chosen state, in nonideal situations, depends critically on the specific ideal Hamiltonian that one considers. Understanding which Hamiltonian is more suitable to its

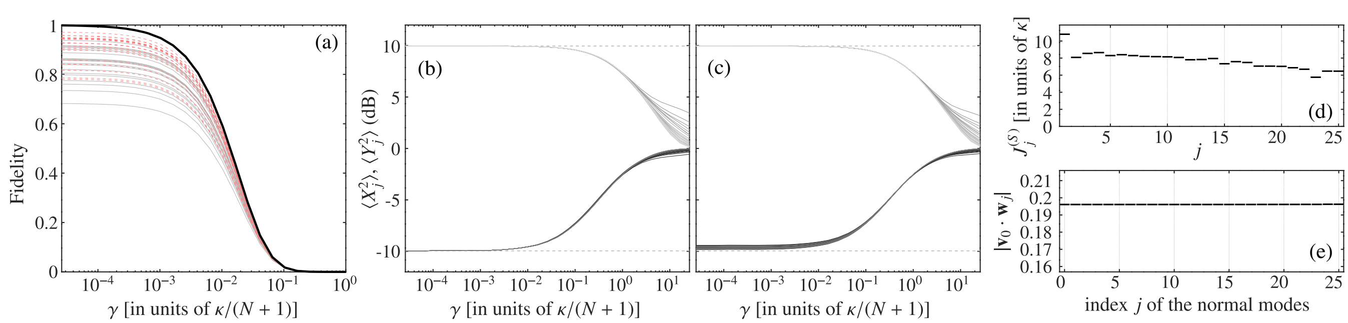

FIG. 2. As in Fig. 1 with the values of the interaction coefficients $J_j^{(S)}$ of $H^{(S)}$ (3) reported in panel (d) (note that the average value of these coefficients is equal to the value of $J_j^{(S)}$ used in Fig. 1). These coefficients have been found by the numerical maximization of the smallest overlap between the normal modes and the auxiliary mode, such that the resulting overlaps are all equal to $1/\sqrt{N+1}$ [see panel (e)].

practical realization, and which Hamiltonian corresponds to a model that is more resistant to imperfections, are questions that deserve further investigation. Another interesting related question regards the possibility to extend this approach to spin systems [17]. Moreover, these findings also suggest how to extend the protocol discussed in Refs. [17,24] to entangle generic distant arrays using a two-mode squeezed field.

We finally note that this approach can be particularly valuable for implementations of quantum information devices with circuit QED systems, which have been recently used to realize various lattice models [43–45]. An experimental implementation of our results would require the ability to design the lattice Hamiltonian with one of these systems, and to combine it with a squeezed field of sufficiently large bandwidth [19,24], produced, for example, with Josephson parametric amplifiers [58,59]. Alternatively, the squeezed reservoir could be also engineered with bichromatic drives [18].

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Correction: Equation (3) contained minor errors and has been fixed.