Vaporization Dynamics of a Dissipative Quantum Liquid

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(Received 3 July 2020; accepted 7 December 2020; published 30 December 2020)

We investigate the stability of a Luttinger liquid, upon suddenly coupling it to a dissipative environment. Within the Lindblad equation, the environment couples to local currents and heats the quantum liquid up to infinite temperatures. The single particle density matrix reveals the fractionalization of fermionic excitations in the spatial correlations by retaining the initial noninteger power law exponents, accompanied by an exponential decay in time with an interaction dependent rate. The spectrum of the time evolved density matrix is gapped, which collapses gradually as $-\ln(t)$. The von Neumann entropy crosses over from the early time $-t \ln(t)$ behavior to $\ln(t)$ growth for late times. The early time dynamics is captured numerically by performing simulations on spinless interacting fermions, using several numerically exact methods. Our results could be tested experimentally in bosonic Luttinger liquids.

DOI: [10.1103/PhysRevLett.125.266803](https://doi.org/10.1103/PhysRevLett.125.266803)

Introduction.—While dissipation is traditionally viewed as detrimental due to causing decay and randomization of phase, recent years have witnessed tremendous progress both in experiment and theory, as a result of which dissipation can now be considered as a useful tool or probe. Coupling to environment, combined with the ability to create and manipulate quantum systems $[1-3]$ $[1-3]$ in a controlled manner has provided us with unique states of matter [4–[13\]](#page-4-2), where dissipation plays a major role. Such states also hold the promise of being relevant for quantum technologies [\[14\]](#page-4-3).

Besides the properties of the steady state, the route toward reaching it can also reveal a plethora of peculiar phenomena. The most prominent example includes quantum effects near the event horizon of a black hole which give rise to the celebrated Hawking radiation [\[15,16\]](#page-4-4) and, eventually, to black hole evaporation. In condensed matter and cold atom contexts, it is rather natural to consider the dynamics of open quantum systems, as these are never perfectly isolated from the environment. Consequently, several dissipative manybody systems were investigated [\[17](#page-4-5)–23], focusing on the propagation and spreading of correlations, quantum information loss, exponential vs power law temporal relaxation toward the steady state, as well as the stability of various phases when coupled to a bath [\[24,25\].](#page-5-0)

Quantum many-body effects are particularly amplified in one spatial dimension [\[26,27\].](#page-5-1) In the resulting Luttinger liquid (LL) phase, the original fermionic excitations fractionalize [\[28\]](#page-5-2) into bosonic collective modes due to interactions. This phase of matter is realized in a variety of fermionic, bosonic, anyonic, etc. systems, including condensed matter [\[26\]](#page-5-1) and cold atomic systems [\[29\]](#page-5-3), quantum optics [\[30\]](#page-5-4), and even in black holes [\[31\]](#page-5-5) and promises to be a building block in possible application in topological quantum computation, spintronics, and quantum information theory. This motivated us to combine dissipation with strong correlations and focus on the stability and evaporation dynamics of LLs by coupling it to a dissipative environment, modeled by the Lindblad equation. We find that the fermionic single particle density matrix retains its initial LL correlations in space in terms of noninteger power law exponents, but the amplitude is reduced in time due to dephasing. This indicates that fractionalization persists in spatial correlations.

The von Neumann entropy crosses over from $-t \ln(t)$ for early times to $ln(t)$ growth for late times. The early time dynamics is benchmarked numerically with dissipative interacting fermions. Our results are also relevant for bosonic Luttinger liquids [\[29\]](#page-5-3).

Dissipation in the interacting Luttinger model.—The low-energy behavior of one-dimensional systems is described by the Luttinger model whose Hamiltonian reads

$$
H = \sum_{q>0} \omega_q (b_q^+ b_q + b_{-q}^+ b_{-q}) + g_q (b_q^+ b_{-q}^+ + b_q b_{-q}), \quad (1)
$$

where $\omega_q = v|q|$, $g_q = g_2|q|$, and b_q annihilates a bosonic excitation. Here, $v = v_0 + g_4$ is the sound velocity, where v_0 is the bare sound velocity and g_2 and g_4 describe forward scattering between fermions with different and same chiralities, respectively [\[26\].](#page-5-1) Since the Hamiltonian is quadratic in the bosonic operators, it can be diagonalized by the Bogoliubov transformation, yielding

$$
H = E_{\rm g.s.} + \sum_{q>0} \tilde{\omega}_q (d_q^+ d_q + d_{-q}^+ d_{-q}), \tag{2}
$$

where $E_{\text{g.s.}} = \sum_{q>0} (\tilde{\omega}_q - \omega_q)$ is the ground state (g.s.) energy and $\tilde{\omega}_q = \tilde{v}|q|$ is the spectrum of elementary excitations with the renormalized sound velocity $\tilde{v} = \sqrt{v^2 - g_2^2}$.
We consider a LL prepared in the ground state of the

We consider a LL, prepared in the ground state of the interacting Hamiltonian, thus, no excitations are present. At $t = 0$, the coupling between the LL and its environment is switched on, and for $t > 0$, the time evolution is governed by the Lindblad equation [\[32](#page-5-6)–34]. The coupling to environment is modeled by local current operators, as in Refs. [\[20,35](#page-4-6)–38]. Such dissipators arise naturally when considering fluctuating vector potential or gauge field as the environment. The Lindblad equation reads as

$$
\partial_t \rho = -i[H, \rho] + \gamma \int dx ([j(x), \rho j(x)] + \text{H.c.}), \quad (3)
$$

where $\rho(t)$ is the density matrix of the system and $j(x)$ is the current operator playing the role of the jump operator. Using bosonization [\[26\],](#page-5-1) the current operator is [\[39\]](#page-5-7)

$$
j(x) = \sum_{q \neq 0} \sqrt{\frac{|q|}{2L\pi}} \text{sgn}(q) e^{-iqx} (b_{-q} - b_q^+), \qquad (4)
$$

with L the system size, and the spatial integral in Eq. (3) results in

$$
\partial_t \rho = -i[H, \rho] + \frac{\gamma}{2\pi} \sum_{q \neq 0} ([L_q, \rho L_q^+] + \text{H.c.}), \quad (5)
$$

with $L_q = \sqrt{|q|} (b_q - b_{-q}^+)$. The spectrum of Eq. [\(5\)](#page-1-1) is
expected to be gapless since the energy scale in both the expected to be gapless since the energy scale in both the Hamiltonian and the dissipator $\sim |q|$. After Bogoliubov transformation, the jump operator is rewritten as $L_q =$ $\sqrt{\frac{(|q|}{K}}\left(d_q - d_q^+d_q\right)$, where $K = \sqrt{\frac{v - g_2}{v + g_2}}\left(\frac{v + g_2}{v + g_2}\right)$ is the Luttinger parameter [\[26\]](#page-5-1) and $K < 1$ ($K > 1$) for repulsive (attractive) interaction. The presence of the interaction induces a renormalization of the dissipative coupling $\gamma \rightarrow \gamma/K$. This indicates that dissipation becomes effectively stronger or weaker for repulsive or attractive interaction for the density matrix, respectively.

Based on the Lindblad equation, the expectation values of the occupation number and the anomalous operator are obtained as

$$
n_q(t) = \text{Tr}[\rho(t)d_q^+ d_q] = \gamma|q|t/(\pi K),\tag{6a}
$$

$$
m_q(t) = \text{Tr}[\rho(t)d_q^+ d_{-q}^+] = \frac{\gamma}{2\pi i K \tilde{v}} (e^{i2\tilde{v}|q|t} - 1), \quad (6b)
$$

in accordance with Ref. [\[42\]](#page-5-8). The linear increase of the occupation number implies that the system heats up to infinite temperatures [\[43\]](#page-5-9) and the LL eventually evaporates during the Lindblad dynamics, unlike the related problem with localized loss [\[44,45\].](#page-5-10) This also follows from the observation that the jump operator is Hermitian.

Green's function.—To have a deeper understanding of correlations, we study the time evolution of the single particle density matrix or equal time Green's function defined as

$$
G(x,t) = \operatorname{Tr}[\rho(t)\Psi_R^+(x)\Psi_R(0)],\tag{7}
$$

where $\Psi_R(x) = (1/\sqrt{2\pi\alpha}) \exp[i\sum_{q>0}\sqrt{(2\pi/qL)}(e^{iqx}b_q + e^{-iqx}b_q]$ $e^{-iqx}b_q^+$] is the fermionic field operator of right-moving $e^{i\phi}$ ϕ_{q}) is the fermionic field operator of right-moving
electrons. By evaluating the trace in Eq. [\(7\)](#page-1-2), the single particle density matrix is obtained as [\[39\]](#page-5-7)

$$
\ln \frac{G(x,t)}{G_0(x)} = \sum_{q>0} \frac{8\pi}{L|q|} \left(\frac{g_2}{\tilde{v}} \operatorname{Re} m_q(t) - \frac{v}{\tilde{v}} n_q(t)\right) \sin^2\left(\frac{qx}{2}\right),\tag{8}
$$

where $G_0(x) = [i/2\pi(x + i\alpha)](\alpha/\sqrt{x^2 + \alpha^2})^{[(K+K^{-1})/(2)]-1}$
is the initial Green's function obeying the well-known is the initial Green's function obeying the well-known [\[26\]](#page-5-1) power-law decay for $x \gg \alpha$ with the exponent of $(K + K^{-1})/2$. The momentum summation is regularized with the exponential cutoff $exp(-\alpha|q|)$ with α the short distance cutoff.

It is important to note that the time dependence of the single particle density matrix occurs only through the quantities $n_q(t)$ and $m_q(t)$ which have been calculated in Eqs. [\(6\).](#page-1-3) Substituting these into Eq. [\(8\)](#page-1-4), the summation over q is carried out analytically as

$$
\ln \frac{G(x,t)}{G_0(x)} = -\frac{\gamma t}{\pi \alpha} \frac{K^{-2} + 1}{(\frac{\alpha}{x})^2 + 1} + \frac{\gamma}{2\pi \tilde{v}} \left(\frac{1}{K^2} - 1\right) I\left(\frac{\tilde{v}t}{\alpha}, \frac{x}{\alpha}\right),\tag{9}
$$

where $I(y, z) = \arctan(2y) - \sum_{s=\pm} [\arctan(2y - sz)/2]$. In
the scaling limit, when $(x, \tilde{y}t) \gg \alpha$, the time evolution of the scaling limit, when $(x, \tilde{v}t) \gg \alpha$, the time evolution of the single particle density matrix is summarized as

$$
G(x,t) = \frac{i}{2\pi\alpha} \left(\frac{\alpha}{x}\right)^{[(K+K^{-1})/(2)]} \exp\left(-\frac{(K^{-1}+K)\gamma t}{\pi\alpha K}\right)
$$

$$
\times \begin{cases} \exp\left[\frac{\gamma}{4\bar{v}}(K^{-2}-1)\right] & \text{for } 2\tilde{v}t \ll x \\ 1 & \text{for } 2\tilde{v}t \gg x \end{cases} (10)
$$

It exhibits two peculiar phenomena: the power law spatial decay of the single particle density matrix is preserved throughout the time evolution with the initial LL exponent of $(K + K^{-1})/2$. This noninteger exponent indicates that part of the original fermionic excitations remain fractionalized during the nonunitary time evolution. In addition, the spatial correlations are uniformly suppressed, exponentially in time, in accord with Ref. [\[35\]](#page-5-11). The characteristic time scale of the dephasing is set by the dissipative coupling and the interaction strength as $K\pi\alpha/[{\gamma(K+K^{-1})}]$, as found
numerically in Fig. 1. The decay rate decreases from attractive numerically in Fig. [1](#page-2-0). The decay rate decreases from attractive $(K > 1)$ to repulsive $(K < 1)$ interaction: even though γ itself is renormalized to γ/K in the Lindblad equation, the original bare fermion, $\Psi_R(x)$ is also dressed by the interaction, thus, reverting the trend for the Green's function. It is rather remarkable that, in spite of the gapless spectrum of the Lindbladian [\[46\]](#page-5-12), the fermionic Green's function still decays exponentially in time. On top of this, one may observe a kink in the single particle density matrix which travels with the velocity $2\tilde{v}$, which is the only light-cone effect, though this is rather minor and is expected to be hardly observable. The behavior in Eq. [\(10\)](#page-1-5) is rather generic and occurs for other correlation functions as well [\[39\].](#page-5-7)

Time evolved density matrix and entropy.—Another interesting quantity which characterizes the time evolution governed by the Lindblad equation is the von Neumann or thermodynamic entropy defined as $S(t) = -\text{Tr}[\rho(t)\ln\rho(t)]$.
With the bosonized version of $\rho(t)$ [39] the trace is With the bosonized version of $\rho(t)$ [\[39\],](#page-5-7) the trace is evaluated as

FIG. 1. The early time scaling of the Green's function for various x values, obtained using three distinct numerical methods. The Green's function decays with the same interaction dependent exponent at each spatial separation, x. Top panel: $J_z/J = 0.3$, $\Gamma/J = 0.04$, and $N = 22$ (thick solid line) using the quantum jump method with ED and PBC and 6000 averages over quantum trajectories and for $N = 14$ (thin dashed line) using ED with PBC for the Lindblad equation. Bottom panel: $J_z/J = -0.5$, $\Gamma/J=0.4$ and $N=41$ using TDVP (thick solid line) with OBC and for $N = 14$ (thin dashed line) using ED with PBC for the Lindblad equation. The agreement between various methods indicates that the data is relatively free from finite size effects. Here, $x = 1, 3, 5, 7, 9, 11, 13$ (blue, red, black, green, magenta, gold, and light blue, respectively), but not all x 's are shown.

$$
S(t) = 2\sum_{q>0} \{ [N_q(t) + 1] \ln[N_q(t) + 1] - N_q(t) \ln N_q(t) \},\tag{11}
$$

where $N_q(t) = \sqrt{[n_q(t) + \frac{1}{2}]^2 - |m_q(t)|^2}$ $\sqrt{[n_q(t) + \frac{1}{2}]^2 - [m_q(t)]^2} - \frac{1}{2}$. Interestingly, the time dependence occurs again only through the functions given in Eq. [\(6\)](#page-1-3). Its early and long time limits are calculated as

$$
S(t) \sim \frac{L}{\pi \alpha} \begin{cases} -\frac{\gamma t}{K \pi \alpha} \ln(\frac{\gamma t}{K \pi \alpha}) & \text{for } \gamma t \ll K \pi \alpha \\ \ln(\frac{\gamma t}{K \pi \alpha}) & \text{for } \gamma t \gg K \pi \alpha \end{cases} (12)
$$

The early time growth agrees with numerics on dissipative interacting fermions in Fig. [2](#page-2-1), while the latter [\[47\]](#page-5-13) is reminiscent of the behavior of the entanglement entropy in many-body localized systems [\[48,49\].](#page-5-14)

In order to understand more closely the origin of this behavior, we can also evaluate the eigenvalues of the time evolved density matrix at each time instant, denoted by $\lambda_0 \geq \lambda_1 \geq \lambda_2$ Formally, we can also assign an instantaneous Hamiltonian to the time evolved density matrix, $\rho(t) = \exp[-H_{\rho}(t)]$, whose spectrum is $-\ln \lambda_i$.
We can define the gap in the many-body spectrum as We can define the gap in the many-body spectrum as $\Delta_{\rho} = \ln(\lambda_0/\lambda_1)$. This is analogous to the spectrum of the reduced density matrix and the corresponding entanglement Hamiltonian and entanglement gap in closed quantum systems [\[51,52\]](#page-5-15). Since the initial state is pure, the $t = 0$ spectrum is trivial [\[53\].](#page-5-16) During the time evolution, the density matrix is brought to diagonal form after an instantaneous Bogoliubov transformation as $\rho(t)$ ~ $\exp[-\sum_{q} \Omega_{q}(t)\tilde{b}_{q}^{+}\tilde{b}_{q}]$, and for each momentum sector, the single particle spectrum is $\Omega_q(t) = \ln\{1 + [1/N_q(t)]\}$. At $t = 0$, all $N_r(t=0) = 0$, therefore, $\Omega_r(t=0) = \infty$, and $t = 0$, all $N_q(t = 0) = 0$, therefore, $\Omega_q(t = 0) = \infty$, and the \tilde{b}_q bosons are in their vacuum state, the gap in the

FIG. 2. The early time scaling of the von Neumann entropy is shown for $N = 14$ using ED for various parameters. Hardly any finite size effects are present since the $N = 10$ data falls almost on top of this. The parameter $\delta = \alpha(J_z)/\alpha(0)$ accounts for the renormalization of α with interaction, and is expected to increase [\[50\]](#page-5-17) with J_z . Here, we used $\delta = 0.73$ and 1.15 for $J_z/J = -0.5$ and 0.3, respectively, with $\delta = 1$ for the noninteracting case.

spectrum is infinitely large. After switching on the dissipation, the gap in the many-body spectrum, which parallels closely to the entanglement gap, starts to decrease slowly for early times as

$$
\Delta_{\rho} \approx \ln \left(\frac{\pi K \alpha}{\gamma t} \right). \tag{13}
$$

The bosonization approach is valid for momenta $|q| < 1/\alpha$. Our analytical results show that these modes definitely give a singular, $t \ln(t)$ and $\ln(t)$ contribution to the entropy and to the gap in the many-body spectrum at short times, respectively. We cannot analytically determine the contribution of the high energy modes which lie outside the range of the bosonization approach. However, our numerics indicate that the contribution of these high energy modes is subleading, compared to the LL contribution.

Interacting fermions within the Lindblad equation.—To illustrate our findings and check their validity in lattice models, we have investigated one dimensional spinless fermions in an open tight-binding chain with nearest neighbor interaction at half filling using several numerical techniques. The closed system is equivalent to the 1D Heisenberg XXZ chain after a Jordan-Wigner transformation [\[26,27\]](#page-5-1). The Hamiltonian is

$$
H = \sum_{m=1}^{N} \left[\frac{J}{2} (c_{m+1}^+ c_m + c_m^+ c_{m+1}) + J_z n_{m+1} n_m \right], \quad (14)
$$

where c's are fermionic operators, $n_m = c_m^+ c_m$ and J_z
denotes the nearest neighbor repulsion N the number of denotes the nearest neighbor repulsion, N the number of lattice sites and the model hosts $N/2$ fermions. This model realizes a LL for $|J_z| < J$, and the strength of the interaction is characterized by the dimensionless LL parameter [\[26\]](#page-5-1) $K = \pi/2[\pi - \arccos(J_z/J)]$ from the Bethe ansatz solution
of the model. Because of the bounded spectrum of Eq. (14) of the model. Because of the bounded spectrum of Eq. [\(14\)](#page-3-0), the bosonization results are only applicable for early times, before the whole band is populated during heating.

The lattice version of the current operator in Eq. [\(3\)](#page-1-0) reads as

$$
j_m = i(c_{m+1}^+ c_m - c_m^+ c_{m+1})/2, \qquad (15)
$$

which appears in the environmental part of the Lindblad equation as $\Gamma \sum_m ([j_m, \rho j_m] + \text{H.c.})$. To make contact
with bosonization we use $\gamma/\sigma \sim \Gamma$. A similar problem with bosonization, we use $\gamma/\alpha \sim \Gamma$. A similar problem with a different jump operator [\[54\]](#page-5-18) was considered in Refs. [\[17](#page-4-5)–19].

The Lindblad equation for this dissipative many-body system is attacked by three different methods. By vecing [\[55\]](#page-5-19), i.e., rearranging the square density matrix as a vector, one can use standard exact diagonalization (ED) and Krylov-space time evolution, reaching $N = 14$. Second, using the quantum jump method [\[5,32,33\]](#page-4-7) for the same system, we can reach $N = 22$ at the expense of having to average over the quantum trajectories. For these two methods, periodic boundary condition (PBC) is used to minimize finite size effects. Finally, we use the time dependent variational principle (TDVP) with open boundary condition (OBC) [\[56](#page-5-20)–58] within the matrix product states framework to directly simulate the density matrix. Initially, we prepare the system in the ground state by using the density matrix renormalization group [\[59\],](#page-5-21) and use the ground state $|\Psi_0\rangle$ to build the density matrix $\rho_0 = |\Psi_0\rangle \langle \Psi_0|$ in the form of a matrix product operator. Next, by vecing the density matrix to $|\rho\rangle_{\#}$ the Lindblad equation [\(5\)](#page-1-1) is rewritten as $\partial_t |\rho(t)\rangle_{\#} = \mathcal{L} |\rho(t)\rangle_{\#}$, with $\mathcal L$ the Lindbladian organized now as a matrix product operator.

Using these techniques, we determine the equal time Green's function, i.e., $G(x, t) = Tr[\rho(t)c_{m+x}^{\dagger}c_m]$. For PBC,
this becomes independent of m due to translational invarithis becomes independent of m due to translational invariance, while for OBC, m and $m + x$ are chosen symmetrically to the chain center to reduce the effects from the boundary condition. As expected, $G(0, t) = 1/2$ is recovered in all numerics (not shown). The spatiotemporal dynamics of the single particle density matrix is plotted in Fig. [1,](#page-2-0) confirming the results of bosonization: the spatial and temporal dynamics practically decouples, the former preserves the LL correlation encoded in the initial state, while the latter displays pure dephasing for short times, analogously to Ref. [\[35\]](#page-5-11). However, the temporal decay rate is strongly influenced by the LL parameter K and decreases monotonically with the interaction. The curves for different J_z 's are not *a priori* expected to fall on top of each other as α in Eq. [\(9\)](#page-1-6) can follow a weak J_z dependence. For longer times, deviations from the bosonization results are expected when the explicit nature of the high energy degrees come into play. These induce model dependent [\[17\]](#page-4-5), nonuniversal features, whose study is beyond the scope of our current work.

With the knowledge of the time dependent density matrix, the dynamics of the von Neumann entropy is evaluated. For early times, it follows the expected $-\Gamma t \ln(\Gamma t)$ early time growth and obeys the scaling form predicted by bosonization, as shown in Fig. [2.](#page-2-1) Here, we had to account for the mild interaction dependence of the cutoff by slightly renormalizing the value of the rate $\Gamma \rightarrow \Gamma/\delta$ [\[50\]](#page-5-17). Distinct cutoff dependent physical quantities, i.e., the single particle density matrix vs entropy, may require slightly different interaction dependence of the cutoff. The explicit value of the decay rate for a given microscopic model can be determined similarly to the gap in sine-Gordon related models [\[26\]](#page-5-1) by comparing the analytical results to numerics for the time dependent entropy and correlation functions. For late times, the entropy converges fast to its maximal value on the lattice $\sim N \ln(2)$ and the $ln(t)$ late time growth of the LL is not reproduced due to the small local Hilbert space dimension (i.e., 2) for fermions. We speculate that this late time growth could possibly show up in bosonic realization of LLs [\[29\],](#page-5-3) where the local Hilbert space is much bigger [\[60\]](#page-5-22).

FIG. 3. The early time scaling of the gap in the spectrum of the instantaneous Hamiltonian of the time evolved density matrix is plotted for several parameters. It agrees with Eq. [\(13\)](#page-3-1) and is free from finite size effects.

Finally, we evaluate the gap in the spectrum of the time evolved density matrix, as discussed above. Its numerically obtained value is shown in Fig. [3](#page-4-8), which, in spite of its cutoff dependence, still follows the $-\ln(\Gamma t)$ prediction of bosonization.

Summary.—We have studied the vaporization dynamics of Luttinger liquids after coupling to dissipative environment through the local currents. Unlike unitary quantum quenches, where the dynamical Luttinger liquid exponents are different from the equilibrium ones [\[61\],](#page-5-23) in our case, the single particle density matrix reveals the persistence of fractionalization of fermionic excitations in spatial correlations with the equilibrium exponents, but with an amplitude exponentially suppressed in time.

The von Neumann entropy crosses over from an early time $-t \ln(t)$ growth to $\ln(t)$ growth for late times. The former is attributed to the logarithmic collapse in time of the instantaneous gap in the time evolved density matrix. The early time features are captured numerically in a dissipative interacting fermionic lattice model. Our results apply to a large variety of systems and are observable in bosonic Luttinger liquids.

This research is supported by the National Research, Development and Innovation Office—NKFIH within the Quantum Technology National Excellence Program (Project No. 2017-1.2.1-NKP-2017-00001), Projects No. K119442, No. K134437 and by the BME-Nanotechnology FIKP (Grant No. BME FIKP-NAT).

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