QED$_3$-Inspired Three-Dimensional Conformal Lattice Gauge Theory without Fine-Tuning

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We construct a conformal lattice theory with only gauge degrees of freedom based on the induced nonlocal gauge action in QED$_3$ coupled to large number of flavors N of massless two-component Dirac fermions. This lattice system displays signatures of criticality in gauge observables, without any fine-tuning of couplings and can be studied without Monte Carlo critical slowdown. By coupling exactly massless fermion sources to the lattice gauge model, we demonstrate that nontrivial anomalous dimensions are induced in fermion bilinears depending on the dimensionless electric charge of the fermion. We present a proof-of-principle lattice computation of the Wilson-coefficients of various fermion bilinear three-point functions. Finally, by mapping the charge q of fermion in the model to a flavor N in massless QED$_3$, we point to a universality in low-lying Dirac spectrum and an evidence of self-duality of $N = 2$ QED$_3$.

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Introduction.—Extraction of conformal field theory (CFT) data plays an important role in our understanding of critical phenomena. An important set of conformal data are the scaling dimensions of operators that classify the relevant and irrelevant operators in a CFT. These data can be used to abstract the source of dynamical scale breaking in the long-distance limit of quantum field theories in terms of few symmetry-breaking operators that turn relevant. The operator product expansion (OPE) coefficients in the CFT correlation functions are another set of highly constrained conformal data. The formal structure of CFT and its data have been explored over decades and one can refer to [1] for a survey of the subject, [2] for a discussion not restricted to two dimensions, and [3–5] for recent developments in dimensions greater than two. Monte Carlo (MC) studies of strongly interacting CFTs are difficult owing to a combined effect of the required precise tuning of couplings, an increase in MC autocorrelation time closer to a critical point and the need for large system sizes. Notwithstanding such difficulties, the CFT data in many bosonic spin systems have been extracted from traditional MC [e.g., [6,7] for recent determinations in 3D O(N) models] as well as using radial lattice quantization [8–10]. At present, however, three-dimensional fermionic CFTs have been of great interest, particularly owing to recent works related to dualities [11–13], and therefore, MC based search for three-dimensional fermionic CFTs (such as [14–19]) is of paramount importance.

One such three-dimensional interacting fermionic CFT is approached in the infrared limit of the parity-invariant noncompact quantum electrodynamics (QED$_3$) with N (even) flavors of massless two-component Dirac fermions in the limit of large N; to leading order, the effect of fermion is to convert the p^{-2} Maxwell photon propagator into a conformal $16(Ng^2p)^{-1}$ photon propagator [20] in the limit of small momentum p, where g^2 is the dimensionful Maxwell coupling. This suggests replacing the usual Maxwell action for the gauge field A_μ by a conformal gauge action [21]

$$S_g = \frac{1}{q^2}\int \frac{d^3p}{(2\pi)^3} A_\mu(p)\left(\frac{p^2\delta_{\mu\nu} - p_\mu p_\nu}{p}\right)A_\nu(-p), \quad (1)$$

with a dimensionless coupling $q^2(N) = 32/N$ for large N, thereby obtaining results consistent with an interacting conformal field theory in a $1/N$ expansion. The conformal nature of the above quadratic action can be seen in the tensorial structure of n-point functions of field strength $F_{\mu\nu}$ that is consistent with conformal symmetry [21,22]. Since the dimension of $F_{\mu\nu}$ is fixed by gauge invariance, it is only for the $1/p$ kernel of the above quadratic action, the coupling becomes dimensionless in three dimensions. Both approaches in [20,21] are consistent with a scale-invariant field theory only if N is above some critical value, but recent numerical analyses [23,24] of QED$_3$ have shown that the theory likely remains scale (or conformal) invariant all the way down to the minimum $N = 2$. This suggests that the induced gauge action from the fermion is conformal for any nonzero N, and it might be possible to capture many aspects of the infrared physics of QED_3 by appropriately modeling this induced nonlocal action—we do so by using the quadratic conformal gauge action Eq. (1), however, with an otherwise unknown q-N relation, $q^2(N)$, which for general N needs to be determined from first principles, and assuming that effect of terms in the induced-action which are higher order in gauge field are negligible. This motivated us to consider the action in Eq. (1) in its own right as an interacting CFT for any q^2 obtained without tuning any couplings, and probed by massless spectator fermions. It is the primary aim of this Letter to use a lattice regularization of Eq. (1) and show that this CFT induces nontrivial conformal data in fermionic observables depending on the value of q, thereby making it a powerful model system for lattice studies of fermion CFTs. Finally, we will close the loop and demonstrate numerically that this conformal gauge theory for arbitrary q^2 probed by spectator fermions can describe universal features in a corresponding N-flavor QED_3.

The model and signatures of its criticality in pure-gauge observables.—The noncompact U(1) lattice gauge model we consider is the regularized version of Eq. (1) on L^3 periodic lattice, given by

$$Z = \left(\prod_{x,\mu}\int_{-\infty}^{\infty} d\theta_\mu(x)\right) e^{-S_g(\theta)}, \quad \text{with}$$

$$S_g = \frac{1}{2}\sum_{\mu,\nu=1}^{3}\sum_{x,y} F_{\mu\nu}(x)\Box^{-1/2}(x,y)F_{\mu\nu}(y), \tag{2}$$

where $\theta_\mu(x)$ are real-valued gauge fields that reside on the links connecting site x to $x+\hat{\mu}$, with a field strength $F_{\mu\nu} = \Delta_\mu\theta_\nu(x) - \Delta_\nu\theta_\mu(x)$, where Δ_μ is the discrete forward derivative. The three-dimensional discrete Laplacian is $\Box = \sum_\mu \Delta_\mu\Delta_\mu^\dagger$. The model lacks any tunable dimensionful parameter at the cost of being nonlocal, which is not a hindrance for a numerical study; a MC sampling of the gauge fields weighted by Eq. (2) becomes simple in the Fourier basis where the Laplacian is diagonalized and the modes are decoupled. We absorbed the fundamental real-valued charge q in Eq. (1) in a redefinition of gauge fields when defining the parameterless lattice model, and hence the observables will couple to gauge fields as $q\theta_\mu(x)$, or integer multiples thereof. We discuss further details of the model and the algorithm in the Supplemental Material [25].

The absence of tunable parameters in the lattice action by itself is not an indication of it being critical. Strong evidence of the scale-invariant behavior was seen in the sole dependence on aspect ratio $\zeta = l/t$ of all $l\times t$ Wilson loops, $\mathcal{W}(q\theta)$, after a simple perimeter term is removed. The asymptotic behavior [33] is characterized by $\nu\zeta$ as $\zeta\to\infty$ and ν/ζ for $\zeta\to 0$ with the coefficient $\nu = -0.0820(8)q^2$ that should be universal for all theories approaching this CFT, such as QED_3 (see Supplemental Material [25]). Another interesting pure-gauge observable is the topological current, $V_\mu^{\mathrm{top}} \equiv (q/4\pi)\sum_{\nu\rho}\epsilon_{\mu\nu\rho}F_{\nu\rho}$, which is trivially conserved in this noncompact U(1) theory. We also checked that its two-point function for $1 \ll |x| \ll L/2$ behaves like a conserved vector correlator $\sum_\mu \langle V_\mu^{\mathrm{top}}(0)V_\mu^{\mathrm{top}}(x)\rangle = C_V^{\mathrm{top}}|x|^{-4}$, with the coefficient $C_V^{\mathrm{top}} = (q^2/4\pi^4)$ as expected from the continuum regulated calculation [34–36]. The trivial q^2 dependence of conformal data in pure-gauge observables becomes nontrivial in gauge-invariant observables formed out of spectator massless fermions.

Conformal data in fermionic observables.—The lattice model *per se* does not have dynamical fermions. But, one can couple spectator massless fermion sources to the model in order to construct a variety of gauge-invariant hadronic correlation functions. Formally, the source term for a pair of parity-conjugate Dirac fermions is $\bar{\psi}_q^+\mathcal{G}_q\psi_q^+ - \bar{\psi}_q^-\mathcal{G}_q\psi_q^-$, where $\mathcal{G}_q$ is the exactly massless overlap lattice fermion propagator [24,37,38] coupled to the gauge fields through the gauge links $e^{iq\theta_\mu(x)}$ (see Supplemental Material [25] for the implementation of overlap Dirac operator, which includes Refs. [39,40]). The flavor-triplet fermion bilinears are *defined* by taking appropriate derivatives

$$O^\pm(x;q) = \left(\frac{\partial}{\partial\bar{\psi}_q^\pm}\Gamma\frac{\partial}{\partial\psi_q^\mp}\right)(x);$$

$$O^0(x;q) = \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial\bar{\psi}_q^+}\Gamma\frac{\partial}{\partial\psi_q^+} + \frac{\partial}{\partial\bar{\psi}_q^-}\Gamma\frac{\partial}{\partial\psi_q^-}\right)(x), \tag{3}$$

of the effective action; $\Gamma = 1$ for scalar bilinear $S^{\pm,0}$, and Pauli matrices $\Gamma = \sigma_\mu$ for the conserved vector bilinears $V_\mu^{\pm,0}$. Practically, this procedure is equivalent to a prescription of replacing fermion lines with massless fermion propagators to form gauge-invariant observables. We also imposed antiperiodic boundary conditions on fermion sources in all three directions which is symmetric under both lattice rotation and charge conjugation while removing the issue of trivial Dirac zero modes present even in the free-field $q=0$ limit. We will denote the n-point functions formed out of these fermion bilinears by $G^{(n)}(x_{ij};q)$ and the dependence on the x_{ij}, the separation between the location of the ith and jth bilinears should match the structure deduced from conformal symmetry. Since we are only interested in changes to observables from free-field theory, we form the ratios $\tilde{G}^{(n)}(x_{ij};q) = G^{(n)}(x_{ij};q)/G^{(n)}(x_{ij};0)$, which we henceforth refer to as *reduced* n-point functions; this also helps decrease any finite-size and short-distance lattice effects that are already present in the free-field case.

We define scaling dimensions $\Delta_i = 2-\gamma_i$ governing the scaling $\tilde{G}^{(2)}_{O_iO_i}(x_{12}) = C_i|x_{12}|^{2\gamma_i}$ for distances larger than few lattice spacings. The scaling dimension $\Delta_S(q) = 2-\gamma_S(q)$ of $S^{\pm,0}$ is an example of nontrivial conformal data that is induced in this model. The q-dependent nonzero γ_S can be obtained from the finite-size scaling

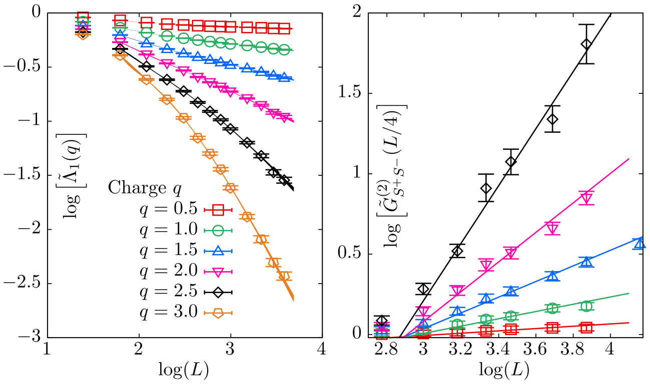

FIG. 1. Mass anomalous dimension as computed at different charges q. Left: the dependence of smallest Dirac eigenvalue $\tilde{\Lambda}_1(q)$, normalized by free theory value, on L. The curves are the fits to extract the leading $L^{-\gamma_S}$ dependence. Right: the finite size scaling of the scalar two-point function $\tilde{G}(|x_{12}|)$ at separations $|x_{12}| = L/4$. The lines are the expected asymptotic dependence $\tilde{G}(|x_{12}| = L/4) \sim L^{2\gamma_S}$ at different q, with γ_S determined from $\tilde{\Lambda}_n$.

(FSS) of the scalar two-point function, $\tilde{G}^{(2)}_{S^+S^-}(|x| = \rho L) = L^{2\gamma_S}[g(\rho) + \mathcal{O}(1/L)]$ at fixed ρ. The data for $\log[\tilde{G}^{(2)}_{S^+S^-}]$ at $\rho = 1/4$ are shown as a function of $\log(L)$ using values of q ranging from $q = 0.5$ to 2.5 in the right panel of Fig. 1, and one sees that the slope of $\log(L)$ dependence (which is $2\gamma_S$) increases monotonically from 0 when q is increased. Better estimates of $\gamma_S(q)$ were obtained by studying the FSS of the low-lying discrete overlap-Dirac eigenvalues $\Lambda_j(L;q)$, satisfying $\mathcal{G}_q^{-2} v_j = -\Lambda_j^2 v_j$; the FSS, $\Lambda_j(L;q) \propto L^{-1-\gamma_S(q)}$, is a consequence of the FSS of the scalar susceptibility. In the left panel of Fig. 1, we show the reduced eigenvalues, $\tilde{\Lambda}_j(L;q) \equiv \Lambda_j(L;q)/\Lambda_j(L;0)$ for $j = 1$ as a function of L along with curves from combined fits using a functional form $\tilde{\Lambda}_j(L;q) = a_j L^{-\gamma_S}(1 + \sum_k^4 b_{jk} L^{-k})$ to first five $\tilde{\Lambda}_j$ using data from $L = 6$ up to $L = 36$ (see Supplemental Material [25]). Such a functional form with leading scaling behavior and subleading scaling corrections nicely describes the data and leads to precise estimates of $\gamma_S(q)$ that increase continuously from $\gamma_S = 0$ to $\mathcal{O}(1)$ in the vicinity of $q \approx 2$; this dependence is captured to a good accuracy by $\gamma_S(q) = 0.076(11)q^2 + 0.0117(15)q^4 + \mathcal{O}(q^6)$, over this entire range of q. For some charge $q = q_c \approx 2.9$, the value of γ_S becomes greater than 1.5, which is the unitarity bound on scalars in a three-dimensional CFTs (cf. [4]); therefore, within the framework of constructing fermionic observables in this pure-gauge theory, we need to restrict ourselves to values of $q < q_c$ to be consistent with being an observable in a CFT. Unlike the scalar bilinear, V^a_μ is conserved current and hence, does not acquire an anomalous dimension. Therefore, the only nontrivial conformal data are the two-point function amplitude, $C_V(q) = \sum_{\mu=1}^3 \tilde{G}^{(2)}_{V^a_\mu V^a_\mu}(|x|;q)$ that we were able to obtain from the plateau in the reduced vector two-point correlator as a function of separations, $0 \ll |x| \ll L/2$ (see Supplemental Material [25]). Its q dependence can be parametrized as $4\pi^2 C_V(q) = 1 - 0.0478(7)q^2 + 0.0011(2)q^4 + \mathcal{O}(q^6)$.

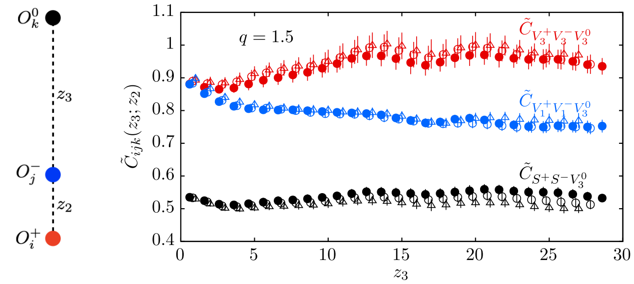

FIG. 2. Left: a configuration of collinearly placed operators. Right: the effective OPE coefficients $\tilde{C}_{ijk}(z_2, z_3; q)$ of three different collinear three-point functions (distinguished by colors and slightly displaced) are shown as a function of z_3 at three different fixed $z_2 = 6$ (open triangles), 8 (open circles), 10 (filled circles).

In order to demonstrate further the efficacy of the model as a CFT with nontrivial conformal data in the massless spectator fermion observables that is tractable numerically on the lattice, we also present a proof-of-principle computation of the OPE coefficients $\tilde{C}_{ijk}(q)$ of the reduced three-point functions $\tilde{G}^{(3)}_{O_1O_2O_3}(x_{12}, x_{23}, x_{31}; q)$ when three operators lie collinearly, that is, $x_1 = (0,0,0)$, $x_2 = (0,0,z_2)$, and $x_3 = (0,0,z_2 + z_3)$ as described in the left panel of Fig. 2. We looked at three distinct three-point functions, chosen so as to reduce finite size effects, and whose dependences are fixed by conformal invariance [2] to be

$$\tilde{G}^{(3)}_{V^+_\mu V^-_\mu V^0_3}(z_2, z_3) = \tilde{C}_{V^+_\mu V^-_\mu V^0_3}; \qquad \mu = \mu_\perp (= 1, 2) \quad \text{or} \quad 3,$$
$$\tilde{G}^{(3)}_{S^+S^-V^0_3}(z_2, z_3) = \tilde{C}_{S^+S^-V^0_3} z_2^{2\gamma_S}, \tag{4}$$

when $0 \ll z_2, z_3, z_2 + z_3 \ll L/2$ on a periodic lattice. For any other separations, we use these expressions to define the effective z_2 and z_3 dependent OPE coefficients which will display a plateau as a function of z_2, z_3 provided the theory is a CFT. In the right part of Fig. 2, we show the three effective OPE coefficients as a function of z_3 at three different fixed $z_2 (= 6, 8, 10)$ as determined on the 64^3 lattice using $q = 1.5$. The plot demonstrates the independence of the three coefficients on z_3 by a plateau over a wide range of z_3 that is not too small or too large. It also demonstrates their independence on z_2 since the data from three different intermediate values of z_2 are consistent, with this being quite nontrivial especially for $\tilde{C}_{S^+S^-V^0_3}$ as it comes from a cancellation with a factor $z_2^{2\gamma_S}$. The conformal symmetry in general allows nondegenerate OPE coefficients $\tilde{C}_{V^+_3 V^-_3 V^0_3} = [(a+b)/b_0]$ and $\tilde{C}_{V^+_{\mu_\perp} V^-_{\mu_\perp} V^0_3} = (b/b_0)$, with $a = 0$, $b = b_0$ in free theory. From Fig. 2, it is evident that $a \neq 0$ and $b \neq b_0$, clearly indicating that the result is for an interacting CFT.

Relevance of the model to QED_3.—We will show a correspondence between the behavior of the CFT at one particular q and QED_3 with N flavors of massless two component fermions. Our surprising observation for which we will present empirical evidence is that, for any finite N, as long as QED_3 flows to an infrared fixed point, the dominant effect of fermion determinant in QED_3 path integral is to induce a nonlocal quadratic conformal action for the gauge fields with a coupling $q = \mathcal{Q}(N)$ for some function $\mathcal{Q}$ that has to be determined *ab initio*, with the only condition being $\mathcal{Q}(N) \sim \sqrt{32/N}$ for *large* values of N. That is, if the map $\mathcal{Q}(N)$ is known for all N, then one can study universal features of the N-flavor QED_3 by studying the same properties in the conformal lattice model at the corresponding $q = \mathcal{Q}(N)$ with nondynamical massless fermion sources, whose purpose is simply to aid the construction of fermionic n-point functions. In order to find $\mathcal{Q}(N)$, we propose to map values of q in the lattice model to N in QED_3 such that the values of scalar anomalous dimensions γ_S, determined nonperturbatively in both theories, are the same. Such an identification of q and N is made in the bottom panel of Fig. 3, where we have plotted $\gamma_S(q)$ as a function of q, and determined expected 1–σ ranges of q that correspond to $N = 2, 4, 6, 8$ flavor QED_3 based on estimates of γ_S from our previous lattice studies of QED_3 [23,24]; namely, we find the expected ranges $q \in [2.32, 2.76], [1.88, 2.49], [1.57, 2.03], [1.33, 1.88]$ for $N = 2$, 4, 6, 8, respectively. Below, we discuss two consequences of this connection.

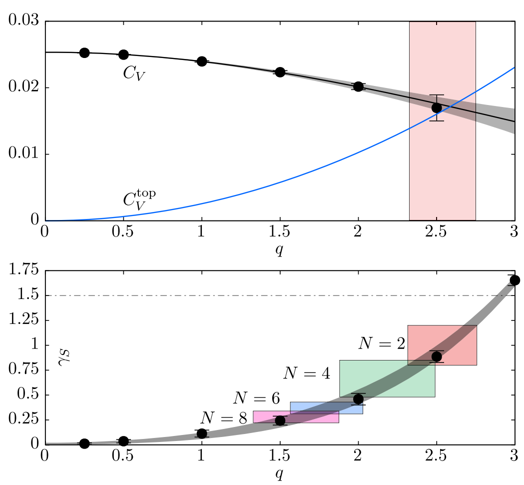

FIG. 3. Bottom panel: mass anomalous dimension γ_S is shown as a function of charge q. The filled circles are numerical determinations in the lattice model and the black band is the resulting spline interpolation of the data. The expected region corresponding to $N = 2$, 4, 6, 8 flavor QED_3 are shown by the rectangular boxes, so as to match the values of γ_S. The dashed line is the unitarity bound on γ_S. Top panel: the C_V in the lattice model and $C_V^{\mathrm{top}} = q^2/(4\pi^4)$ are shown as a function of q. The two intersect in the region of q corresponding to $N = 2$ QED_3, as inferred from the bottom panel.

In the lattice model, the two-point functions of both V_μ^a and V_μ^{top} behave as $|x|^{-4}$ with amplitudes $C_V(q)$ having a nontrivial dependence on q and $C_V^{\mathrm{top}}(q)$ being quadratic in q. In the top panel of Fig. 3, we have shown these q dependences of the two amplitudes, wherein one finds C_V^{top} increases as $q^2/(4\pi^4)$ whereas C_V decreases from the free-field value $1/(4\pi^2)$ as a function of q, and the two curves intersect around $q = 2.6$; at this intersecting point, $(V_\mu^+, V_\mu^0, V_\mu^-, V_\mu^{\mathrm{top}})$ form an enlarged set of degenerate conserved vector currents in the lattice model. It is fascinating that this value of $q \approx 2.6$ lies in the probable range corresponding to $N = 2$ QED_3, where such a degeneracy is expected from a conjectured self-duality of $N = 2$ QED_3 [41–43] (conditional to the theory being conformal), and the $q - N$ mapping presented here suggests that such a degeneracy could occur in $N = 2$ QED_3 (and also numerically observed in [44]).

Quite strikingly, we also find evidence for microscopic matching between QED_3 and the conformal model studied in this Letter. The probability distribution $P(z_i)$ of the scaled low-lying discrete Dirac eigenvalues $z_i = \Lambda_i/\langle\Lambda_i\rangle$ are universal to QED_3 in the infrared limit and the lattice model at the matched point $\mathcal{Q}(N)$. In the top panels of Fig. 4, we show the nice agreement between $P(z_i)$ for the lowest three eigenvalues from $N = 2$ QED_3 at two different large box sizes ℓ (measured in units of Maxwell coupling g^2) [23,24] which are in the infrared regime, and the distributions $P(z_i)$ from the lattice model discussed here at $q = 2.5$ which lies in the expected range of q for $N = 2$. Such an agreement is again seen between $P(z_i)$ in the lattice model at $q = 2.0$ (which lies near the upper edge of the expected range of q for $N = 8$) and in $N = 8$ QED_3

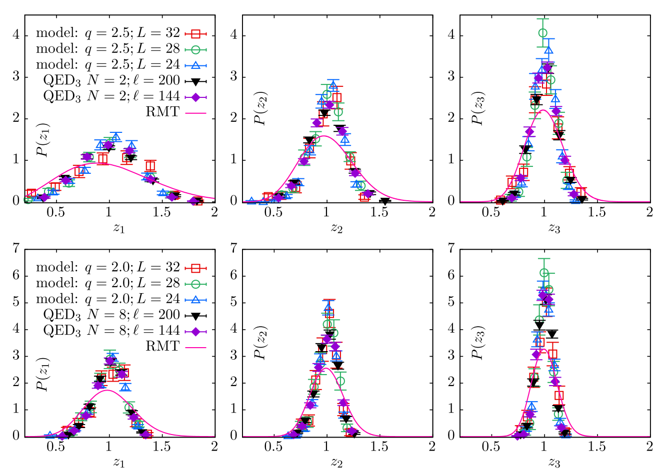

FIG. 4. Distribution of scaled eigenvalues $z_i = (\Lambda_i/\langle\Lambda_i\rangle)$ for the three lowest eigenvalues (left to right) from the conformal lattice model at $q = 2.5$ (top) and $q = 2.0$ (bottom) are compared with those from $N = 2$ and $N = 8$ QED_3. For the lattice model, results from $L = 24$, 28, 32 are shown, where as for QED_3, results from two large box sizes ℓ (measured in units of coupling g^2) are shown.

shown in the bottom panels. To contrast, such universality in low-lying eigenvalue distribution has previously been studied only between fermionic theories with a condensate and random matrix theories (RMT) with same global symmetries [45]. The results for $P(z_i)$ from nonchiral RMT [45] corresponding to $N = 2$ and 8 flavor theories are also shown for comparison in top and bottom panels of Fig. 4, using analytical results in [46,47]; the observed disagreement between $P(z_i)$ in $N \geq 2$ QED$_3$ and the corresponding RMTs is evidence for the absence of condensate in parity-invariant QED$_3$ with any nonzero number of massless fermions (as previously observed by us in [23]), and instead, the striking compatibility of the QED$_3$ distributions with those from a CFT studied here is a remarkable counterpoint.

Discussion.—We have presented a three-dimensional interacting conformal field theory where one can compute conformal data by a lattice regularization without fine-tuning. We showed that by probing this CFT with massless spectator fermions, one is able to obtain a more elaborate set of conformal data that is tunable based on the charge of the fermions. For the sake of demonstration, we computed only two- and three-point functions of fermion bilinear that have the same charge. A simple extension for the near future is a computation of n-point functions of four-Fermi operators $\bar{\psi}_{n_1 q}\bar{\psi}_{n_2 q}\psi_{n_3 q}\psi_{(n_1+n_2-n_3)q}$ that is gauge invariant nontrivially and has only connected diagrams. We demonstrated a direct correspondence between the model with charge-q fermions and an N-flavor QED$_3$; by tuning q so as to match a scaling exponent (we chose γ_S), one is able to observe many other universal features between the two corresponding theories. We stress that we did not perform an all-order calculation in $1/N$ for QED$_3$ [34,48,49] via a lattice simulation of the model; rather, the lattice calculation is an all-order computation in charge-q which might or might not be expandable in $1/N$ via a mapping $q = \mathcal{Q}(N)$ that we determined by a nonperturbative matching condition. However, a lattice perturbation theory approach to the results presented here would be interesting. It would also be interesting to use this model to test for robust predictions of infrared fermion-fermion dualities [12,13] by tuning the value of $q = \mathcal{Q}(N)$ and adding required level-k lattice Chern-Simons term $\det[(1-\mathcal{G})/(1+\mathcal{G})]^k$ [50].

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