Continuous Phase Transition without Gap Closing in Non-Hermitian Quantum Many-Body Systems

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Contrary to the conventional wisdom in Hermitian systems, a continuous quantum phase transition between gapped phases is shown to occur without closing the energy gap Δ in non-Hermitian quantum many-body systems. Here, the relevant length scale $\xi \simeq v_{\mathrm{LR}}/\Delta$ diverges because of the breakdown of the Lieb-Robinson bound on the velocity (i.e., unboundedness of v_{LR}) rather than vanishing of the energy gap Δ. The susceptibility to a change in the system parameter exhibits a singularity due to nonorthogonality of eigenstates. As an illustrative example, we present an exactly solvable model by generalizing Kitaev's toric-code model to a non-Hermitian regime.

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Quantum phase transitions have long been a subject of active research in quantum many-body physics. A quantum phase is characterized by the low-energy and long-distance properties of a system such as the decay behavior of correlation functions of local operators in the ground state, the ground-state degeneracy, and its stability against local perturbations [1]. At the transition point between different quantum phases, physical quantities show singularities reflecting changes in the long-distance behavior [2]. For conventional quantum many-body systems described by local and Hermitian Hamiltonians, it is widely accepted that a continuous quantum phase transition between gapped phases is accompanied by closing of an excitation gap Δ. This correspondence is among the most fundamental properties of continuous phase transitions, and two gapped ground states that are connected without gap closing are generally considered to belong to the same quantum phase [3]. This implies that long-distance properties of a ground state are preserved under continuous deformation of a local and gapped Hamiltonian. In fact, a change in the ground state under such deformation can be represented as a finite-time evolution generated by a local effective Hamiltonian, which preserves the long-distance structure of the ground state [1,3,4].

Meanwhile, non-Hermitian physics [5–7] has recently attracted widespread attention [8–12]. Non-Hermiticity originates from gain and loss of energy or particles in classical systems [13–19], and non-Hermitian quantum dynamics is realized under continuous observation without quantum jumps [20–32]. Some fundamental principles in Hermitian systems break down in non-Hermitian systems. Even in single-particle problems, unique topological phases [33–42] and an unconventional bulk-boundary correspondence due to anomalous sensitivity to boundary conditions [43–50] have been found with no counterparts in Hermitian systems. In many-body systems [24–27,51–56], non-Hermiticity can induce quasi-long-range ordered phases with power-law decaying correlations even without continuous symmetry in the Hamiltonian [24]. Non-Hermiticity also leads to unconventional renormalization-group flows that are forbidden in Hermitian systems [27,51–53]. However, the crucial role of an energy gap in quantum phase transitions has yet to be fully understood in non-Hermitian many-body systems.

In this Letter, we show that a continuous quantum phase transition can occur even without gap closing in non-Hermitian quantum many-body systems. In such a transition, the susceptibility, which is related to the spatial correlation and fluctuations of a local physical quantity, develops a singularity because of the nonorthogonality of eigenstates. This makes a sharp contrast with the Hermitian case, in which the singularity of the susceptibility originates from gap closing [57–59]. These facts imply that the relationship between the correlation length and the energy gap is fundamentally altered and the framework of continuous quantum phase transitions should be reconsidered in non-Hermitian systems. By way of illustration, we construct an exactly solvable non-Hermitian model by introducing non-Hermiticity to Kitaev's toric-code model [60].

Breakdown of the Lieb-Robinson bound.—Under continuous deformation of a local and gapped Hermitian Hamiltonian $H(s)$, a change in the ground state $|\psi_0(s)\rangle$ can be described by a local unitary transformation $U(s)$, or a finite-time evolution generated by a local effective Hermitian Hamiltonian $\mathcal{D}(s)$ [1,3,4]. For a unique ground state, such a transformation is given by

$$|\psi_0(s)\rangle = U(s)|\psi_0(0)\rangle, \tag{1}$$

$$U(s) := S' \exp\left(\int_0^s i\mathcal{D}(s')ds'\right), \tag{2}$$

where $S'\exp$ denotes the s'-ordered exponential and $\mathcal{D}(s)$ is obtained from $H(s)$ as

$$i\mathcal{D}(s) = \int_{-\infty}^{\infty} F(t)e^{iH(s)t}\left(\frac{d}{ds}H(s)\right)e^{-iH(s)t}dt. \tag{3}$$

The s'-ordered exponential in Eq. (2) is defined by $\mathcal{S}'\exp[\int_0^s i\mathcal{D}(s')ds'] := \sum_{n=0}^{\infty}(1/n!)\mathcal{S}'[\int_0^s i\mathcal{D}(s')ds']^n$, where $\mathcal{S}'[\mathcal{D}(s'_1)\cdots\mathcal{D}(s'_n)]$ is defined by $\sum_{p\in S_n}\theta(s'_{p(1)}-s'_{p(2)})\cdots\theta(s'_{p(n-1)}-s'_{p(n)})\mathcal{D}(s'_{p(1)})\cdots\mathcal{D}(s'_{p(n)})$ in terms of the Heaviside unit-step function θ. In Eq. (3), $F(t)$ is an odd function that decays faster than any negative power of t for large $|t|$ and whose Fourier transform $\tilde{F}(\omega)$ is equal to $-1/\omega$ for $|\omega| > \Delta$ and infinitely differentiable. The presence of a finite gap $\Delta > 0$ guarantees that only ω with $|\omega| > \Delta$ matters, where $\tilde{F}(\omega)$ is smooth and $F(t)$ decays sufficiently fast [3,4].

The locality of $\mathcal{D}(s)$ is guaranteed by the presence of a finite gap and the Lieb-Robinson bound [61–63]—the latter determines the speed limit $v_{\rm LR}$ with which an effective range of the support of a local operator $(d/ds)H(s)$ expands under a finite-time evolution generated by $H(s)$. One can restrict the action of the time-evolved operator to this effective range since the operator distance (i.e., the operator norm of the difference) between the original and restricted operators is negligibly small [64]. The integrand in Eq. (3) thus remains local for finite t, and only the integral over small $|t|$ is relevant because of the fast decay of $F(t)$, which is guaranteed by the presence of a finite gap as mentioned above. Owing to the locality of $\mathcal{D}(s)$, properties of $|\psi_0(0)\rangle$ with respect to a local operator O are preserved under the local unitary transformation in Eq. (1). The operator $U^\dagger(s)OU(s)$ in the expectation value $\langle\psi_0(s)|O|\psi_0(s)\rangle = \langle\psi_0(0)|U^\dagger(s)OU(s)|\psi_0(0)\rangle$ remains local because of the Lieb-Robinson bound [3,4]. Here an effective range of each local term in $\mathcal{D}(s)$ is estimated to be $\xi_0 + v_{\rm LR}/\Delta$, where ξ_0 denotes the supremum of the interaction range (i.e., the diameter of the support of a local term in the Hamiltonian) of $H(s)$. Because of finite $v_{\rm LR}$, the locality of $\mathcal{D}(s)$ breaks down and a change in $|\psi_0(s)\rangle$ can be nonlocal only for $\Delta = 0$, which corresponds to a continuous phase transition.

In contrast, the Lieb-Robinson bound can, in general, break down in open-system dynamics conditioned on measurement outcomes [29] such as a non-Hermitian evolution corresponding to the null-jump process. Let H be a local non-Hermitian Hamiltonian $H = \sum_Z (h_Z^{\rm H} + ih_Z^{\rm AH})$, where $h_Z^{\rm H}$ and $ih_Z^{\rm AH}$ represent the Hermitian (H) and anti-Hermitian (AH) parts of the local term with support Z. We consider the time evolution of a local operator O with support X: $O(t) = \exp(iH^\dagger t)O\exp(-iHt)$. Then we have

$$\left.\frac{d}{dt}O(t)\right|_{t=0} = \sum_{Z: Z\cap X\neq\varnothing} i[h_Z^{\rm H}, O] + \sum_Z \{h_Z^{\rm AH}, O\}. \tag{4}$$

For the Hermitian parts $h_Z^{\rm H}$'s, commutators, which are taken with O, vanish for those Z's that satisfy $Z\cap X = \varnothing$. For the anti-Hermitian parts, in contrast, anticommutators are taken with O; then, contributions from $h_Z^{\rm AH}$'s with $Z\cap X = \varnothing$ remain nonvanishing and affect the dynamics of O directly, which indicates the breakdown of locality. To understand the physical origin, we consider the dissipative dynamics generated by a local Lindbladian $\mathcal{L}$ [65], which corresponds to the dynamics obtained after taking the ensemble average over all the possible measurement outcomes (i.e., quantum trajectories). In the Heisenberg picture, such a dissipative dynamics is described by $(d/dt)O(t)|_{t=0} = \mathcal{L}[O]$ with

$$\mathcal{L}[O] = \sum_Z \left[i[h_Z, O] + \sum_j \left(L_Z^{j\dagger}OL_Z^j - \frac{1}{2}\{L_Z^{j\dagger}L_Z^j, O\}\right)\right], \tag{5}$$

where L_Z^j's are local jump operators with support Z. In the dynamics under continuous observation without quantum jumps, the jump terms $L_Z^{j\dagger}OL_Z^j$'s play no roles and the effective non-Hermitian Hamiltonian is obtained as $h_Z^{\rm H} = h_Z$ and $h_Z^{\rm AH} = -\frac{1}{2}\sum_j L_Z^{j\dagger}L_Z^j$. We note that the sum in Eq. (5) can be restricted to Z with $Z\cap X \neq \varnothing$ since the quantum jump term $L_Z^{j\dagger}OL_Z^j$ cancels $\frac{1}{2}\{L_Z^{j\dagger}L_Z^j, O\}$ for $Z\cap X = \varnothing$; this means the preservation of the locality of the dynamics, which results in the Lieb-Robinson bound in local Lindblad equations [66–69]. In contrast, when one considers the dynamics conditioned on measurement outcomes, such as the non-Hermitian evolution, the above cancellation does not occur in general and thus the Lieb-Robinson bound can be violated. This holds true even when a finite number of quantum jumps occur as long as a subensemble of quantum trajectories is of interest for continuous observation [29].

The breakdown of the Lieb-Robinson bound demonstrated above indicates that the correspondence between quantum phase transitions and gap closing can break down in non-Hermitian systems. In fact, in the non-Hermitian case, $v_{\rm LR}$ has no general upper bound and thus the length scale $v_{\rm LR}/\Delta$ can diverge even without gap closing.

Nonorthogonality-induced singularity.—To gain further insight into the breakdown of the correspondence between quantum phase transitions and gap closing in non-Hermitian systems, we consider the fidelity susceptibility [57–59], which measures how rapidly the ground state changes under the variation of the system's parameter λ and scales superextensively (i.e., grow more than extensively as a function of the system size) at a quantum phase transition reflecting long-range correlations. We consider a non-Hermitian local Hamiltonian $H(\lambda) = H_0 + \lambda V$, where $V \coloneqq \sum_i V_i$ with V_i's being local, and let $|\psi_n^R(\lambda)\rangle$ and $|\psi_n^L(\lambda)\rangle$ denote the right and left eigenstates, respectively, with the (generally complex) eigenenergy $E_n(\lambda)$ and the normalization conditions $\langle\psi_n^R(\lambda)|\psi_n^R(\lambda)\rangle = 1$ and $\langle\psi_m^L(\lambda)|\psi_n^R(\lambda)\rangle = \delta_{m,n}$ [70]. The right (left) eigenstates with different eigenenergies can be nonorthogonal, i.e., $\langle\psi_m^{R(L)}(\lambda)|\psi_n^{R(L)}(\lambda)\rangle \neq 0$ for $m \neq n$, owing to non-Hermiticity. We assume that the ground state $|\psi_0^R(\lambda)\rangle$ is unique with an excitation gap above it. Here, we define the ground state as the state with the lowest real part of the eigenenergy and the energy gap as $\min_{n\neq 0} |E_n(\lambda) - E_0(\lambda)|$. We consider the fidelity $F(\lambda, \delta\lambda) \coloneqq |\langle\psi_0^R(\lambda)|\psi_0^R(\lambda + \delta\lambda)\rangle|$ for the right eigenstates [71]. To the second order in $\delta\lambda$, we have [72]

$$F(\lambda, \delta\lambda)^2 = 1 - \delta\lambda^2 \langle\partial_\lambda\psi_0^R(\lambda)|\partial_\lambda\psi_0^R(\lambda)\rangle. \qquad (6)$$

Hence the fidelity susceptibility is given by

$$\chi_F(\lambda) \coloneqq \lim_{\delta\lambda\to 0} \frac{-2\ln F(\lambda, \delta\lambda)}{\delta\lambda^2} = \langle\partial_\lambda\psi_0^R(\lambda)|\partial_\lambda\psi_0^R(\lambda)\rangle. \qquad (7)$$

Using the perturbation theory, we have

$$\chi_F(\lambda) = \sum_{m,n\neq 0} \frac{\langle\psi_0^R|V^\dagger|\psi_m^L\rangle}{(E_0 - E_m)^*} \frac{\langle\psi_n^L|V|\psi_0^R\rangle}{E_0 - E_n} (\langle\psi_m^R|\psi_n^R\rangle - \langle\psi_m^R|\psi_0^R\rangle\langle\psi_0^R|\psi_n^R\rangle). \qquad (8)$$

If the Hamiltonian is Hermitian, owing to the orthogonality of eigenstates, we have [57–59]

$$\chi_F(\lambda) = \sum_{n\neq 0} \frac{|\langle\psi_n(\lambda)|V|\psi_0(\lambda)\rangle|^2}{|E_0(\lambda) - E_n(\lambda)|^2}, \qquad (9)$$

which can be rewritten as

$$\int_0^\infty d\tau\tau \sum_{i,j} [\langle V_i(\tau)V_j(0)\rangle - \langle V_i(\tau)\rangle\langle V_j(0)\rangle], \qquad (10)$$

where $V_i(\tau) \coloneqq e^{H(\lambda)\tau} V_i e^{-H(\lambda)\tau}$. Here, the superscripts L and R are omitted since the left and right eigenstates are equivalent. When the excitation gap closes, the denominator of the right-hand side of Eq. (9) vanishes for some n in the thermodynamic limit, which results in a superextensive scaling of χ_F and signals a quantum phase transition. If the gap is open, in contrast, correlations are short ranged and the summands in Eq. (10) decay rapidly with distance, which is also guaranteed by the Lieb-Robinson bound [84]; thus Eq. (10) cannot grow superextensively [58,59]. This gives an alternative explanation for the correspondence between gap closing and a quantum phase transition in Hermitian systems.

In non-Hermitian systems, however, the fidelity susceptibility can exhibit a superextensive scaling even without gap closing. This is because a large number of terms in the double sum in Eq. (8) contribute to χ_F owing to the nonorthogonality of eigenstates in sharp contrast with the Hermitian case. In fact, Eq. (8) can be rewritten as [72]

$$\int_{-\infty}^0 d\tau' \int_{-\infty}^0 d\tau \sum_{i,j} [\langle V_i(\tau')^\dagger V_j(\tau)\rangle - \langle V_i(\tau')^\dagger\rangle\langle V_j(\tau)\rangle]. \qquad (11)$$

This form looks similar to Eq. (10) but can grow superextensively even if the energy gap is nonzero owing to the long-range correlations arising from the breakdown of the Lieb-Robinson bound. Here we emphasize that this superextensive scaling without gap closing contrasts sharply with that found in a Hermitian model with a long-range coupling [85]; in the latter case, the breakdown of the Lieb-Robinson bound is caused by the long-range coupling. We also note that the breakdown of the Lieb-Robinson bound and the nonorthogonality of eigenstates are, in general, neither necessary nor sufficient to each other.

Non-Hermitian toric-code model.—As an illustrative example, we consider the following non-Hermitian extension of Kitaev's toric-code model [60]:

$$H(\beta) = -\sum_{v\in\{\text{vertex}\}} A_v(\beta) - \sum_{p\in\{\text{plaquette}\}} B_p, \qquad (12)$$

where $A_v(\beta) \coloneqq \prod_{i=1}^4 \sigma_{v,i}^\beta$ and $B_p \coloneqq \prod_{i=1}^4 \sigma_{p,i}^z$ are defined on four edges around a vertex v and on a plaquette p of a square lattice [Fig. 1(a)]. Here σ_i^x, σ_i^y, σ_i^z are the Pauli matrices on the edge i, and the non-Hermitian operator σ_i^β is defined as

$$\sigma_i^\beta \coloneqq \cosh(\beta)\sigma_i^x + i\sinh(\beta)\sigma_i^y = \begin{pmatrix} 0 & e^\beta \\ e^{-\beta} & 0 \end{pmatrix}, \qquad (13)$$

where $\beta \geq 0$ parametrizes non-Hermiticity. This non-Hermitian operator physically represents an asymmetric spin flip. The original Hermitian model $H(0)$ is a prototypical solvable model that exhibits $\mathbb{Z}_2$ topological order [60]. Since all the terms appearing in the non-Hermitian Hamiltonian (12) commute with one another, the exact solvability of the original model is maintained under the non-Hermitian extension.

The original Hermitian model $H(0)$ exhibits fourfold degenerate ground states below an excitation gap under the periodic boundary conditions (i.e., on a torus) [60]. Importantly, the energy gap remains open in the presence of non-Hermiticity $\beta > 0$ as we explain in the following. Here $H(\beta)$ is related to the original Hermitian model $H(0)$ by a similarity transformation $H(\beta) = S(\beta)H(0)S(\beta)^{-1}$, where $S(\beta) \coloneqq \exp[(\beta/2)\sum_i \sigma_i^z]$. Thus, regardless of β, the

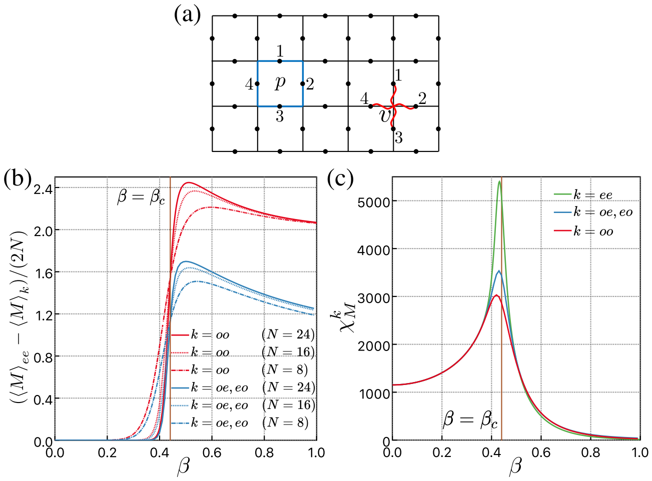

FIG. 1. Non-Hermitian toric-code model. (a) Spin-1/2 magnetic moments placed at the edges of a square lattice with $N \times N$ vertices. (b) Difference (scaled by $2N$) in the magnetization $\langle M\rangle_k(\beta)$ between different ground states for $N = 8$, 16, 24. Here $k \in \{ee, eo, oe, oo\}$ labels different topological sectors, where the first (second) letter represents the parity of the number of noncontractible loops of down spins on the dual lattice winding around the torus in the x (y) direction. For $\beta < \beta_c$ ($\beta > \beta_c$), the difference in $\langle M\rangle_k(\beta)$ tends to vanish (becomes of the order of N) with an increase in N, which suggests a topological (trivial) phase. Moreover, the different curves cross at the transition point $\beta = \beta_c$. (c) Magnetic susceptibility $\chi_M^k(\beta) = (d/d\beta)\langle M\rangle_k(\beta)$ for $N = 24$, which exhibits a singularity at $\beta = \beta_c$.

energy spectrum remains unchanged in comparison with the Hermitian case, and there are fourfold degenerate ground states below the energy gap. The right (left) eigenstates with the eigenenergy E_n are $|\psi_{n,k}^R(\beta)\rangle \propto S(\beta)|\psi_{n,k}(0)\rangle$ $[\langle\psi_{n,k}^L(\beta)| \propto \langle\psi_{n,k}(0)|S^{-1}(\beta)]$, where k is the index labeling degenerate eigenstates and the superscripts L, R are omitted for the Hermitian case ($\beta = 0$). The fourfold ground states are superposition states of spin configurations $\{\sigma_i^z\}$ in which down spins form closed loops on the dual lattice. For $\beta = 0$, such spin configurations are superposed with an equal weight within each topological sector characterized by the parities (p_x, p_y) of the numbers of noncontractible loops that wind around the torus in the x and y directions. As β increases, the weight of a configuration with a larger magnetization (i.e., a smaller total length of loops) becomes exponentially larger. For $\beta \to \infty$, one of the ground states becomes fully polarized, and the topological feature is entirely lost. In fact, a topological phase transition takes place at $\beta = \beta_c := (1/2)\ln(\sqrt{2} + 1) \simeq 0.4407$ [86,87], as shown below.

Topological phase transition.—A signature of topological order is given by topological entanglement entropy [88,89], which is a subleading constant term γ following the area-law term αL in the entanglement entropy S for a subregion of the ground state: $S = \alpha L - \gamma + o(L^0)$, where L denotes the perimeter of the subregion and α is a constant. In particular, the original Hermitian toric-code model has $\gamma = \ln 2$ [90,91], which is a universal value for $\mathbb{Z}_2$ topological order. Our non-Hermitian model possesses $\gamma = \ln 2$ ($\gamma = 0$) for $\beta < \beta_c$ ($\beta > \beta_c$), which indicates a topological (trivial) phase. To show this, we note that $H(\beta)$ shares the same ground states with the following Hermitian model

$$H = -\sum_p B_p - \sum_v A_v + \sum_v \exp\left(-\beta \sum_{i\in v} \sigma_i^z\right), \tag{14}$$

with $A_v := A_v(0)$. This Hermitian model was introduced in Ref. [86] and γ is analytically obtained [92] (see also Ref. [93]). Here, β physically represents an external magnetic field for $|\beta| \ll 1$ in the Hermitian model (14) while β represents the degree of the asymmetric spin flips in our non-Hermitian model (12). We note that gap closing at the transition point was numerically demonstrated in the former model [94], while the gap is constant regardless of β in the latter one as similarity transformations do not alter the spectrum.

Another important property of topological order is that the projection of any local operator onto the ground-state manifold is proportional to the identity: $\langle\psi_{0,k'}|O|\psi_{0,k}\rangle = c_O \delta_{k',k}$ [64,95–100], which indicates that the degenerate ground states cannot be distinguished by any local observable. We examine this property for the total magnetization $M := \sum_i \sigma_i^z$. Figure 1(b) shows the difference in the magnetization $\langle M\rangle_k(\beta)$ between different ground states, where $\langle O\rangle_k(\beta) := \langle\psi_{0,k}^R(\beta)|O|\psi_{0,k}^R(\beta)\rangle$ [72]. For $\beta < \beta_c$ ($\beta > \beta_c$), the difference in $\langle M\rangle_k(\beta)$ tends to vanish (becomes of the order of N) with an increase in N, which indicates a topological (trivial) phase.

The magnetic susceptibility $\chi_M^k(\beta) := (d/d\beta)\langle M\rangle_k(\beta)$ exhibits a superextensive scaling at $\beta = \beta_c$ [Fig. 1(c)] [72]. For our model, this also indicates a superextensive scaling of the fidelity susceptibility. In fact, using the perturbation theory developed above with $V = (d/d\beta)H(\beta)$ and the fact that each excited state with a nonzero contribution to the sum in Eq. (8) can be created by acting local operators as $\sigma_i^z|\psi_{0,k}^R(\beta)\rangle$, we can show $\chi_F^k(\beta) = \frac{1}{4}\chi_M^k(\beta)$ [72], where $\chi_F^k(\beta)$ denotes the fidelity susceptibility for $|\psi_{0,k}^R(\beta)\rangle$ [101]. Our model illustrates the superextensive scaling of the (fidelity) susceptibility due to the nonorthogonality of eigenstates as the energy gap remains nonvanishing for any β. We note that such a transition cannot occur under similar transformations in one-dimensional non-Hermitian systems [72].

Experimental situation.—The non-Hermitian model in Eq. (12) can be simulated experimentally with ultracold atoms. The dynamics by $H(\beta)$ can be decomposed as

$$e^{-iH(\beta)t} = \left(\prod_p e^{iB_p t}\right) S(\beta) \left(\prod_v e^{iA_v t}\right) S^{-1}(\beta). \tag{15}$$

Schemes for simulating the unitary dynamics by A_v or B_p with ultracold atoms have been proposed in Refs. [102–104], where the four-body interactions are simulated using the controlled-NOT gates [105] which can be implemented with Rydberg atoms [106–108] and electromagnetically induced transparency [109–111]. Moreover, the nonunitary dynamics $S(\beta)$ and $S^{-1}(\beta)$ can be implemented by postselection of events without spontaneous decay of one of the spin components under continuous measurement [24,25]. All of these elements can be implemented, for example, with ^{87}Rb atoms [25,72,102].

Physically, the ground state of $H(\beta)$ is a stationary state of the conditional dynamics, and can be prepared by starting with the ground state of the Hermitian counterpart at zero temperature and then adiabatically ramping up β [27]. A signature of the proposed transition can be detected through a singularity in the magnetic susceptibility of the ground state.

In summary, we have demonstrated that continuous quantum phase transitions can occur without gap closing in non-Hermitian quantum many-body systems. In such a transition, the singularity of the fidelity susceptibility arises from nonorthogonality of eigenstates. Possible applications of our theory include adiabatic preparation of a topological phase from a trivial phase [112] and an improved efficiency of quantum annealing [113–115]. The former can be realized by continuously changing the Hamiltonian without gap closing in a finite time via a process converse to that presented in this Letter—the presence of a gap helps to suppress nonadiabatic excitations. For the latter, the annealing time, which is inversely proportional to the excitation gap by the adiabatic theorem, may be short even for a large non-Hermitian system. In Hermitian systems, by contrast, the energy gap is smaller and hence the operation time is longer for a larger system size. In both examples, a short operation time is also important in practice, since the probability of successful postselection under continuous measurement decays with time. It is worthwhile to explore concrete applications in these directions.

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