Superradiant Phase Transition in Electronic Systems and Emergent Topological Phases

Daniele Guerci,[1,2] Pascal Simon,[2] and Christophe Mora[1]

[1]*Université de Paris, Laboratoire Matériaux et Phénomènes Quantiques, CNRS, F-75013 Paris, France*

[2]*Université Paris-Saclay, CNRS, Laboratoire de Physique des Solides, 91405 Orsay, France*

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We derive a general criterion for determining the onset of superradiant phase transition in electronic bands coupled to a cavity field, with possibly electron-electron interactions. For longitudinal superradiance in 2D or genuine 1D systems, we prove that it is always prevented, thereby extending existing no-go theorems. Instead, a superradiant phase transition can occur to a nonuniform transverse cavity field and we give specific examples in noninteracting models, either through Fermi surface nesting or parabolic band touching. Investigating the resulting time-reversal symmetry breaking superradiant states, we find in the former case Fermi surface lifting down to four Dirac points on a square lattice model, with topologically protected zero modes, and in the latter case topological bands with nonzero Chern number on an hexagonal lattice.

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The study of the quantum-mechanical interaction between light and matter has been a driving field in physics in the past century with its application in different research fields, such as laser cooling [1,2], quantum information, and quantum computing [3–5]. Experimental advances in cavity quantum electrodynamics [6,7] have made it possible to integrate solid-state materials with optical cavities [8–12], thus paving the way for cavity quantum electrodynamics at the micrometer and even nanometer scale. This recent tremendous progress opens the door to harness electronic properties of solid-state materials [13–15] and eventually to emulate new exotic collective phases [16,17].

In this context, the phenomenon of superradiance plays a pivotal role [18]. It was originally predicted in the Dicke model [19–21], where a single cavity mode is coupled to an ensemble of two-levels systems (dipoles). The collective and coherent interaction can lead to a so-called superradiant phase in which the dipoles emit light at high intensity, i.e., macroscopically populate the cavity. This phase transition has been observed first in optically pumped gas [22], in photoexcited semiconducting quantum dots [23,24], and in pumped ultracold gases trapped in an ultrahigh-finesse optical cavity [25]. These experiments involve, however, an external drive and no equilibrium version of superradiance has yet been experimentally demonstrated.

Indeed, in realistic systems, the linear light-matter coupling of the Dicke model is supplemented by a diamagnetic term, quadratic in the potential vector and detrimental to a superradiant phase transition. The relative balance between the two competing terms is generally fixed by the Thomas-Reiche-Kuhn (TRK) sum rule and prevents a superradiant state to occur through no-go theorems [26–28] in most systems. Suggestions to bypass no-go theorems have been made, involving for instance magnetic dipolar interactions [29–31] or electron-electron interactions [32], but the proper account or not of the TRK sum rule have led to mistakes and controversies in past studies [16,33–39]. In the case of electronic systems, a no-go theorem for photon condensation (or equilibrium superradiance) has been recently proven [40], seemingly closing the door to equilibrium exotic polaritonic phases. It holds even in the presence of (strong) electron-electron interactions but requires a uniform cavity field. Incidentally, a crossing of Landau levels in a 2D electron gas has been predicted to induce a superradiant instability in a spatially varying cavity field [41].

The purpose of this Letter is to provide a general framework for predicting superradiant phase transitions in electronic systems, thereby connecting the above studies. Building on a lattice model, which automatically exhibits gauge invariance and the associated TRK sum rule, we derive a general criterion for the occurrence of superradiance. The no-go theorem of Ref. [40] is circumvented by taking into account the finite momentum exchanged between the photon mode and the electron gas, extending the findings of Ref. [41]. We find that superradiance can occur only for a transverse cavity field. For a longitudinal field [42], we derive an extended TRK sum rule at finite momentum which definitely prevents photon condensation and superradiance in one-dimensional settings. We remark that the aforementioned sum rule does not forbid ferroelectric transitions driven by the electron-electron interaction in one-dimensional systems [30], or the photon-mediated charge density wave transition predicted theoretically in 1D optical gases [43]. We explore several explicit noninteracting models in which a superradiant phase transition takes place, either through nesting or quadratic band touching. We detail the resulting superradiant phases, where the magnetic flux gives rise to a spatially modulated charge current order. Interestingly, the symmetry broken phases bear nontrivial topological properties.

The model.—Without loss of generality, we consider a lattice, or tight-binding, model to describe the crystalline band structure of a solid-state material. The Hamiltonian $H_{el} = H_0 + H_{\rm int}$ includes a kinetic term

$$H_0 = -\sum_{\boldsymbol{j},\boldsymbol{\delta}} \sum_{\alpha\beta} t^{\boldsymbol{\delta}}_{\alpha\beta} c^\dagger_{\boldsymbol{R}_{j,\alpha}} c_{\boldsymbol{R}_{j,\beta}+\boldsymbol{\delta}} \tag{1}$$

with the cell index $\boldsymbol{j}$, the orbital indices α/β and $\boldsymbol{R}_{\boldsymbol{j},\alpha}$ the corresponding site positions. The resulting Bloch Hamiltonian $h_{\alpha\beta}(\mathbf{k}) = -\sum_{\boldsymbol{\delta}} t^{\boldsymbol{\delta}}_{\alpha\beta} e^{i\mathbf{k}\cdot(\boldsymbol{R}_{j,\beta}+\boldsymbol{\delta}-\boldsymbol{R}_{j,\alpha})}$ can be written in terms of the hopping amplitudes $t^{\boldsymbol{\delta}}_{\alpha\beta}$. $H_{\rm int}$ is assumed to contain only density-density interactions. The electronic system is either embedded into a three-dimensional cavity or coupled to free space photons described by the quantum potential vector $\hat{\boldsymbol{A}}(\boldsymbol{r})$. The light-matter coupling is performed in the Coulomb gauge through Peierls substitution in Eq. (1), $t^{\boldsymbol{\delta}}_{\alpha\beta} \to t^{\boldsymbol{\delta}}_{\alpha\beta} e^{-ie\lambda/c}$ with

$$\lambda = \int_{\boldsymbol{R}_{j,\alpha}}^{\boldsymbol{R}_{j,\beta}+\boldsymbol{\delta}} d\boldsymbol{r}\cdot\hat{\boldsymbol{A}}(\boldsymbol{r}), \tag{2}$$

changing the hopping terms but leaving the interaction part $H_{\rm int}$ invariant. $-e$ is the electron charge and c the speed of light.

Inherited from the original minimal coupling, the Peierls substitution entails an associated gauge invariance. It is best described by replacing $\hat{\boldsymbol{A}}(\boldsymbol{r})$ with a classical uniform and time-independent vector potential $\boldsymbol{A}_0$. Equation (2) becomes $\lambda = \boldsymbol{A}_0\cdot(\boldsymbol{R}_{\boldsymbol{j},\beta}+\boldsymbol{\delta}-\boldsymbol{R}_{\boldsymbol{j},\alpha})$. The resulting phase factor is readily absorbed by the gauge transform $c_{\boldsymbol{R}_{j,\alpha}} \to e^{ie\boldsymbol{A}_0\cdot\boldsymbol{R}_{j,\alpha}/c} c_{\boldsymbol{R}_{j,\alpha}}$ and Eq. (1) is recovered. This is expected on physical ground as a constant vector potential is associated with vanishing electric and magnetic fields. As discussed below, this gauge invariance ensures the TRK sum rule. The Bloch Hamiltonian is modified as $h_{\alpha\beta}(\mathbf{k} - e\boldsymbol{A}_0/c)$, i.e., a simple momentum shift removes $\boldsymbol{A}_0$. The momentum shift is harmless as the Brillouin zone is a compact space, in contrast with continuous or $\mathbf{k}\cdot\mathbf{p}$ approximations which often violate sum rules and incorrectly predict superradiance. The generic lattice model (1) offers a powerful antidote to enforce gauge invariance and protect sum rules.

We are interested at the onset of superradiance and therefore expand the phase factors (2) to second order to obtain the Hamiltonian $H_0(\boldsymbol{A}) = H_0 + H_A + H_{A^2}$ with

$$H_A = \frac{e}{c}\sum_{\boldsymbol{q}} \hat{\boldsymbol{A}}(\boldsymbol{q})\cdot\boldsymbol{J}_p(-\boldsymbol{q}), \tag{3a}$$

$$H_{A^2} = -\frac{e^2}{2c^2}\sum_{\boldsymbol{q}_1,\boldsymbol{q}_2} \hat{A}_i(\boldsymbol{q}_1)\mathcal{T}^{i,j}(-\boldsymbol{q}_1,-\boldsymbol{q}_2)\hat{A}_j(\boldsymbol{q}_2). \tag{3b}$$

We thereby introduce the paramagnetic current $\boldsymbol{J}_p(\mathbf{q})$ and the diamagnetic tensor $\mathcal{T}^{i,j}(\mathbf{q}_1,\mathbf{q}_2)$. As detailed in the Supplemental Material (SM) [44], they can be written solely in terms of the Bloch Hamiltonian $h_{\alpha\beta}(\mathbf{k})$. Together, they define the current operator

$$J_i(\mathbf{q}) = J_{p,i}(\mathbf{q}) - \frac{e}{c}\sum_{\mathbf{q}',j} \mathcal{T}^{i,j}(\mathbf{q},-\mathbf{q}')\hat{A}_j(\mathbf{q}'), \tag{4}$$

where $i = x, y, z$ (x, y) in three (two) dimensions. For a classical potential vector $\boldsymbol{A}(\boldsymbol{r})$, the average current $\boldsymbol{j} = \langle\boldsymbol{J}\rangle/V$ follows from linear response theory

$$j_i(\omega,\mathbf{q}) = \frac{e}{c}\sum_j Q^{i,j}(\omega,\mathbf{q})A_j(\omega,\mathbf{q}), \tag{5}$$

with the current susceptibility $Q^{i,j}(\omega,\mathbf{q})$. The above-mentioned gauge invariance implies that the current response Eq. (5) to the uniform field $\boldsymbol{A}_0$ must vanish in the static limit, and therefore

$$\lim_{\boldsymbol{q}\to 0} Q^{i,j}(0,\boldsymbol{q}) = 0. \tag{6}$$

This is the TRK sum rule expressing the cancellation of paramagnetic and diamagnetic responses at long wavelength.

Condition for superradiance.—We turn to the electromagnetic cavity in which, for the sake of simplicity, we keep only two modes with wave vectors $\boldsymbol{q}$ and $-\boldsymbol{q}$. The potential vector takes the form

$$\hat{\boldsymbol{A}}(\boldsymbol{r}) = \bar{A}\boldsymbol{u}e^{i\boldsymbol{q}\cdot\boldsymbol{r}}(a_{\boldsymbol{q}} + a^\dagger_{-\boldsymbol{q}}) + \text{H.c.}, \tag{7}$$

where the direction is determined by the unit vector $\boldsymbol{u}$ and $\bar{A}$ sets the strength of light-matter interaction. The light-matter Hamiltonian is then $H_{el} + H_A + H_{A^2} + H_{\rm cav}$ with the cavity energy $H_{\rm cav} = \hbar\omega_q(a^\dagger_{\boldsymbol{q}}a_{\boldsymbol{q}} + a^\dagger_{-\boldsymbol{q}}a_{-\boldsymbol{q}})$. In the thermodynamic limit, the light-matter ground state factorizes and one can show that the photon state is a coherent state. This justifies the replacement of bosonic operators $a_{\pm\boldsymbol{q}} \to \alpha_{\pm\boldsymbol{q}}$ by classical fields which must be chosen to minimize the ground state energy. We use linear response theory and the stiffness theorem to arrive at the ground state energy to leading order in $\alpha_{\pm\boldsymbol{q}}$

$$E(\alpha_{\boldsymbol{q}}) - E(0) = \mathcal{N}_q[\hbar\omega_q + 2\gamma^2 Q_{T/L}(0,\boldsymbol{q})]|\alpha_{\boldsymbol{q}}|^2, \tag{8}$$

with $\alpha_{-\boldsymbol{q}} = \alpha^*_{\boldsymbol{q}}$, $\gamma = e|\bar{A}|/c$ and T/L depends on whether Eq. (7) is a transverse ($\boldsymbol{q}\cdot\boldsymbol{u} = 0$) or longitudinal field. $\mathcal{N}_q$ is an intensive positive factor given in the SM [44]. $E(\alpha_{\boldsymbol{q}})$ is the ground state energy with a coherent state of photons of amplitude $\alpha_{\boldsymbol{q}}$ (and $\alpha_{-\boldsymbol{q}}$), where $|\alpha_{\boldsymbol{q}}|^2$ scales linearly with the system size. When $\alpha_{\boldsymbol{q}} \neq 0$, it describes the superradiant state and the phase transition occurs when the term inside the bracket in Eq. (8) changes sign. The derivation leading to Eq. (8), detailed in the SM [44], follows from Ref. [40] but extends it to finite $\boldsymbol{q}$. In the uniform case $\boldsymbol{q} = 0$, the TRK sum (6) and Eq. (8) prove the so-called no-go theorem [40] which prevents any photon condensation (superradiance) to a uniform cavity field.

However, Eq. (8) at finite $\boldsymbol{q}$ goes beyond the TRK sum rule and predicts a superradiant state if the following condition is achieved

$$Q_T(0, \boldsymbol{q}) < -\frac{\hbar\omega_q c^2}{2e^2|\bar{A}|^2}. \tag{9}$$

This criterion can alternatively be obtained from computing the pole of the photon Green's function at vanishing frequency. The longitudinal response function behaves quite differently from the transverse one at finite $\boldsymbol{q}$. We identify a second sum rule, called the f-sum rule (see SM [44]),

$$Q_L(0, \boldsymbol{q}) = 0, \tag{10}$$

stemming from charge conservation. Inserting Eq. (10) into the ground state energy (8), we find no phase transition to a longitudinal potential vector, fully excluding superradiance in one-dimensional electron lattice systems, ladder models aside.

Our analysis has shown that the transverse current susceptibility Q_T determines the onset of superradiance. For noninteracting electrons, we consider the eigenstates $|\boldsymbol{k}, n\rangle$ of the Bloch Hamiltonian $h_{\alpha\beta}(\mathbf{k})$ with energies $\epsilon_{\mathbf{k},n}$. For convenience, we label the states with $n < 0$ (> 0) for negative (positive) energies. We introduce the notation $|\boldsymbol{k}, \boldsymbol{q}\rangle_{n,m}$ for an electron-hole excitation on top of the ground state $|0\rangle$, where the hole (electron) is in state $|\boldsymbol{k}, n\rangle$ ($|\boldsymbol{k}+\boldsymbol{q}, m\rangle$). At zero temperature, the susceptibility is given by $Q_T(\boldsymbol{q}) = K_T(\boldsymbol{q}) - u_i\langle \mathcal{T}^{i,j}_{\boldsymbol{q},-\boldsymbol{q}}\rangle u_j/V$ with the paramagnetic response (d is the space dimension)

$$K_T(0, \boldsymbol{q}) = \int_{BZ} \frac{d^d\mathbf{k}}{(2\pi)^d} \sum_{n<0<m} \sum_{\pm} \frac{|g^{n,m}_{\mathbf{k},\pm\boldsymbol{q}}|^2}{\epsilon_{\mathbf{k},n} - \epsilon_{\mathbf{k}\pm\mathbf{q},m}}, \tag{11}$$

where the denominator is the energy of the electron-hole excitation. The numerator depends on the dipole couplings $g^{n,m}_{\mathbf{k},\boldsymbol{q}} = \langle 0|J^T_{\boldsymbol{q}}|\boldsymbol{k}, \boldsymbol{q}\rangle_{n,m}$, which selects the symmetry of the electronic states coupled by the light-matter interaction. Interestingly, the corresponding dipole for the longitudinal response vanishes with $\epsilon_{\mathbf{k},n} - \epsilon_{\mathbf{k}+\mathbf{q},m}$, and the paramagnetic contribution to the current-current susceptibility cancels the diamagnetic term. The absence of such cancellation for the transverse part is crucial and opens the way for a diverging paramagnetic response (11). There are various ways to obtain a singularity, either by having two lines of points in the Fermi surface connected by a single momentum $\boldsymbol{q}$ (nesting) in two dimensions, or if the density of states at a Fermi point becomes infinite, such as quadratic band touching or Landau level crossing [41]. Since the paramagnetic susceptibility K_T is negative, its divergence signals a superradiant phase transition, no matter how weak the light-matter interaction is, since the criterion (9) is always satisfied. But this singularity is by no means necessary to the superradiance. Indeed, Eq. (8) implies that the superradiant ground state is energetically favored as soon as Eq. (9) is satisfied. In other words, the normal ground state is unstable when the light-matter energy gain is larger than the energy cost of generating a finite density of photons with wave vector $\boldsymbol{q}$.

Superradiant phase.—We illustrate the above criterion (9) for superradiance with concrete examples of tight-binding models. In the following, we proceed in two steps. First, we apply the linear response theory to look for the superradiant instability. Then, in a second stage, we characterize the properties of the electronic model in the superradiant phase.

The first model that we consider is the textbook two-dimensional square lattice with nearest-neighbor hopping. The Bloch Hamiltonian is $h(\mathbf{k}) = -t(\cos k_x + \cos k_y)$, with unit lattice spacing for simplicity. At half-filling, the electronic ground state exhibits a square Fermi surface shown as a solid line in Fig. 1, and a nesting between two segments of the Fermi surface by the wave vector $\boldsymbol{q}^* = (\pi, \pi)$. The whole band structure can be arbitrarily separated into a valence band and a conduction band depending on the sign of $\epsilon_{\mathbf{k}}$. Following the above steps

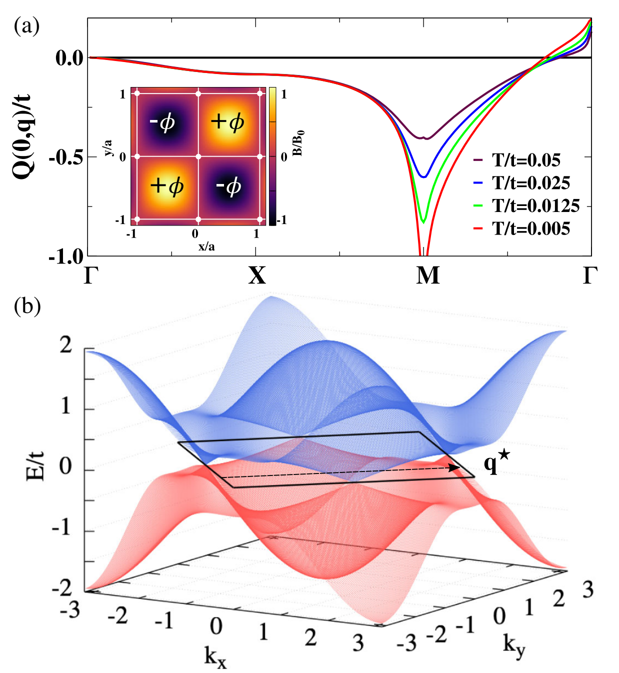

FIG. 1. (a) Current susceptibilities for $\boldsymbol{q}$ along high-symmetry lines. The longitudinal (black line) satisfies Eq. (10), the transverse develops a peak as the temperature is lowered. Inset: alternating cavity field configuration in the superradiant phase. (b) Representative spectrum in the superradiant phase with the four Dirac points and the nesting vector $\boldsymbol{q}^* = (\pi, \pi)$. The original Fermi surface is indicated by a solid black line.

starting with the Peierls substitution, one arrives at Eq. (11) for the transverse and longitudinal responses, with the dipole elements

$$g^{T/L}_{\mathbf{k},\mathbf{q}^*} = -i\frac{\sqrt{2}t}{\pi}(\cos k_x \mp \cos k_y), \tag{12}$$

coupling the valence and conduction bands. The nesting by $\mathbf{q}^*$, represented in Fig. 1(b), implies a divergence of Eq. (11) when $\mathbf{k}$ approaches one side of the Fermi surface while the transverse dipole (12) remains finite. To the contrary, the longitudinal dipole vanishes ensuring a finite paramagnetic response. This is illustrated in Fig. 1(a) where the divergence in the transverse response develops at low temperature at the nesting vector $\mathbf{q}^*$ (M point). Extracting the divergence, we obtain the critical temperature $T_c \sim te^{-(\pi^2/4)\sqrt{\hbar\omega_q/t}(\hbar/a\gamma)}$ (a is the lattice spacing) below which the superradiant phase is energetically favorable. The occupied superradiant bosonic mode is not uniform in space but spatially modulated at the nesting wave vector $\mathbf{q}^*$.

Next, we investigate the superradiant phase for the square lattice model. There is an absence of light-matter entanglement in the thermodynamic limit (see SM [44]) and we assume a photon coherent state. Viewed from the electrons, we obtain an effective Hamiltonian (1) with classical phases dressing the hoppings, similar to piercing a nonuniform magnetic flux through the lattice. For $\boldsymbol{q}^* = (\pi, \pi)$, the corresponding flux configuration alternates between plaquettes as $\pm\phi$, see Fig. 1, thus breaking time-reversal symmetry. With this flux, the new unit cell has length $\sqrt{2}$, along the diagonals of the original square lattice, and contains two inequivalent sites A and B [53]. The new Bloch Hamiltonian takes the form

$$h(\mathbf{k},\boldsymbol{A}) = -t\begin{pmatrix} 0 & d_{AB} \\ d^*_{AB} & 0 \end{pmatrix}, \tag{13}$$

with the matrix element $d_{AB} = e^{i\phi'}\cos k_x + e^{-i\phi'}\cos k_y$, $\phi' = \phi/4$. The alternating flux $\pm\phi$ hence opens a gap almost everywhere on the Fermi surface except at four C_{4z}-related points, $k_x = \pm\pi/2$, $k_y = \pm\pi/2$, from which four Dirac cones emerge. The band spectrum is represented in Fig. 1. Like in graphene, the $C_{2z}T$ symmetry, $\sigma_x h^*(\mathbf{k})\sigma_x = h(\mathbf{k})$, imposes a vanishing Berry curvature and protects the Dirac points [54–56] characterized by the Berry phases $\pm\pi$ (C_{4z} reversing the Berry phase). The similarities with graphene extend to zero-energy boundary modes [57,58], which develop in graphene for zigzag edges while they are absent at armchair termination [59]. Here, we find a collection of zero-energy states when the square lattice has a termination along the diagonals of the original lattice, but not for edges parallel to the x or y direction.

The second model we discuss consists of electrons moving on a honeycomb with a quadratic band touching dispersion. The Bloch Hamiltonian incorporates nearest- and third-nearest-neighbor hoppings [60,61]. It takes the form of Eq. (13) with $d_{AB} = \sum_{j=1}^{3} e^{i\boldsymbol{k}\cdot\boldsymbol{\Delta}_j} + r\sum_{j=1}^{3} e^{-2i\boldsymbol{k}\cdot\boldsymbol{\Delta}_j}$, where the three vectors Δ_j connect nearest neighbors on the lattice. $r = 0$ is the standard model describing electronic bands in graphene. It possesses two inequivalent Dirac cones centered at the K and K' points with Berry phases $\pm\pi$. Additional Dirac cones enter the Brillouin zone for nonzero r and fuse with the original ones at $r = 1/2$ resulting in $d_{AB} = -\frac{9}{8}(\delta k_x \pm i\delta k_y)^2$ in the vicinity of the K (K') point. They give rise to two parabolic band contacts with Berry phases $\mp 2\pi$ at K and K'. At half filling, setting $\mathbf{q}^*$ to be the vector connecting K and K', we find a finite transverse dipole element

$$|g^T_{\boldsymbol{K},\boldsymbol{q}^*}| = \frac{9\sqrt{3}}{8\pi}t, \tag{14}$$

in Eq. (11), resulting in a divergence in the zero-temperature current susceptibility Q_T, for details we refer to the SM [44], and therefore to a superradiant phase at arbitrary weak light-matter coupling. In this case, the divergence is produced by the parabolic form of the energy difference in Eq. (11) (and not by line nesting) while the Fermi surface reduces to the two points K and K'. As expected, the longitudinal dipole element vanishes at $(\boldsymbol{K}, \boldsymbol{q}^*)$, protected by the f-sum rule (10).

The superradiant state is again described by a classical photon field modulated at $\boldsymbol{q}^*$, with six sites per unit cell. As shown in Fig. 2(a), the photonic condensate opens a gap at

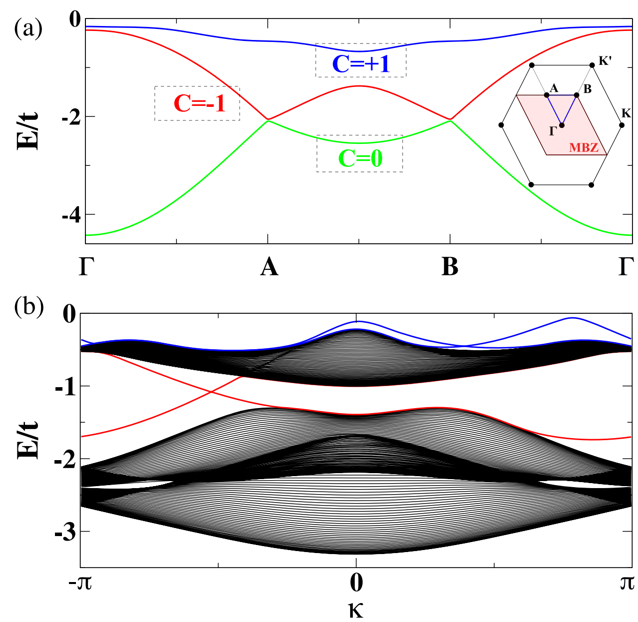

FIG. 2. (a) Band spectrum (three lowest) of the superradiant state with nonzero Chern numbers obtained for $\bar{A} = 0.3$, inset: reciprocal space. (b) Spectrum obtained in a ribbon geometry for $\bar{A} = 1.5$. Black lines correspond to bulk states while the (non-) topologically protected edge states are shown in (blue) red.

the K and K' points and time-reversal symmetry breaking results in topological bands with nonzero Chern numbers and therefore in a topological superradiant phase. The model also displays a chiral symmetry anticommuting with the Bloch Hamiltonian [62] which imposes bands of opposite energies to have the same Chern number. For ribbon boundary conditions (periodic boundary conditions along $\boldsymbol{y}$, while open along the other principal direction) the superradiant phase presents topologically protected 1D edge states, displayed in Fig. 2(b), crossing the band gap between bands 2 and 3 characterized by opposite Chern numbers. Such "superradiant edge state" could be directly measured by light probe [63,64].

Conclusions and outlook.—We established a framework for finding superradiant phase transitions in electronic systems. The divergence of the transverse current susceptibility is sufficient but not necessary to obtain superradiance and a strong enough light-matter coupling also works provided it is simply negative. This condition is verified at finite wave vector $\boldsymbol{q}$ in solid-state systems where the paramagnetic contribution to the current-current susceptibility prevails over the diamagnetic one that always disfavors photon condensation. The resulting superradiant ground state is characterized by a spatially modulated orbital ferromagnetic order, and possesses nontrivial topological properties. We also envision a superradiant phase transition close to magic angles [65–67] in twisted bilayer graphene, where the scenario of parabolic band touching is very similar to one discussed here [68,69].

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Note added.—Recently, we learned about the theoretical work of Andolina *et al.* [70] which overlaps with the first part of our work and reaches a similar conclusion concerning the occurrence of a superradiant phase transition.

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