Novel Quantum Phases of Two-Component Bosons with Pair Hopping in Synthetic Dimension

Zhi Lin,[1,2,*] Chenrong Liu,[1] and Yan Chen[1,†]
[1]Department of Physics and State Key Laboratory of Surface Physics, Fudan University, Shanghai 200433, China
[2]School of Physics and Materials Science, Anhui University, Hefei 230601, China

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We study two-component (or pseudospin-1/2) bosons with pair hopping interactions in synthetic dimension, for which a feasible experimental scheme on a square optical lattice is also presented. Previous studies have shown that two-component bosons with on-site interspecies interaction can only generate nontrivial interspecies paired superfluid (super-counter-fluidity or pair-superfluid) states. In contrast, apart from interspecies paired superfluid, we reveal two new phases by considering this additional pair hopping interaction. These novel phases are intraspecies paired superfluid (molecular superfluid) and an exotic noninteger Mott insulator which shows a noninteger atom number at each site for each species, but an integer for total atom number.

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Ultracold quantum gases are highly controllable systems, in which various novel interaction and detection techniques can be realized, and the extreme physical parameter regimes can be reached [1–6]. Thus, ultracold quantum gas systems have been used to simulate quantum many-body systems and provide an ideal platform to discover novel quantum states. In bosonic systems, there are two kinds of boson pair condensation states with either the intraspecies pairing [7] or the interspecies pairing [8]. The interspecies paired superfluid state has been proposed in a two-component Bose-Hubbard model with on-site interspecies interaction [8–17], and in a bilayer (or two-coupled chains) dipolar boson systems [18–20]. Moreover, the intraspecies paired superfluid or molecular superfluid (MSF) has also been predicted in three different single-component bosonic systems, i.e., an atomic Bose gas with a Feshbach resonance [21–23], attractive Bose-Hubbard model with three-body on-site constraint [24,25] and the extended Bose-Hubbard model (EBHM) with pair hopping [26–28].

Unfortunately, the MSF in single-component bosonic systems has not been observed experimentally. One reason is the short lifetime of molecular condensates by using the Feshbach resonance technique [23]. Besides, it is quite difficult to realize the attractive Bose-Hubbard model with a three-body constraint. Moreover, MSF is predicted in EBHM (when $V \neq 0$) under large value of pair hopping P and nearest-neighbor interaction V [26–28], but it is hard to reach this parameter region in experiment. In a real experimental system, P and V are much smaller than normal hopping and on-site interaction by 3–4 orders of magnitude [29]. Indeed, the calculation in EBHM ignores the effect of an important term, i.e., density-induced tunneling T, which could be much larger than V and P. Thus, alternative feasible experimental schemes such as implementing a feasible scheme in the interacting two-component bosonic systems, are imperiously needed to observe this fascinating MSF state. Meanwhile, there is still a lack of study on the exotic Mott insulator (MI) phase in the interacting two-component bosons. On the whole, two-component bosons with novel interaction may provide an opportunity for discovering the novel phases.

On the other hand, by periodically shaking optical lattice [6,30–33] or modulating interaction strength [34,35], the Floquet technique has shown its ability to engineer the form and intensity of interactions in various experiments. So far, Floquet engineering is mainly focused on manipulating the single-particle hopping processes [36–44], where the hopping amplitude or hopping phase (Peierls phase) depends on the occupation numbers of the sites relevant to hopping processes. The internal atomic degrees of freedom, e.g., pseudospin, can be considered as the synthetic "dimensions" [45]. By coupling to a periodically modulating radio-frequency field, a new type of two-particle hopping process with pair hopping interaction along a synthetic dimension or synthetic pair hopping (SPH) interaction can be realized in a two-component boson system.

In this Letter, we propose a Floquet engineering scheme in a two-component boson system to generate a new two-particle hopping process with SPH interaction. Two novel quantum states of matter may emerge, including the MSF state and the noninteger Mott insulator (NMI) state. The NMI state displays that the number of the total atoms of two-component at each site is an integer, but each component is non integer. This NMI phase may provide a possible platform to host the exotic magnetic phases. Furthermore, the detection of these two novel states has been addressed. The realization of our scheme provides a basis for further

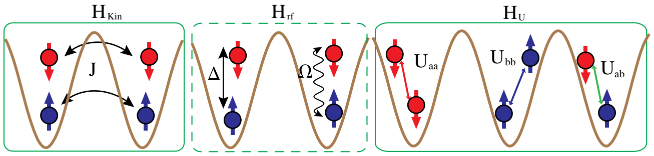

FIG. 1. The relevant physical processes of time-dependent systems. $\hat{H}_{\rm Kin}$ describes each spin hopping between the nearest neighbor site, $\hat{H}_{\rm rf}(t)$ is relevant to radio-frequency coupling of the two spin states with periodic Rabi frequency $\Omega(t)$, and $\hat{H}_{\rm U}$ represents the on-site interactions.

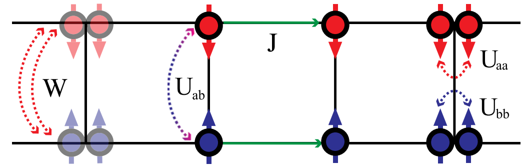

FIG. 2. This two-component boson system in a 1D chain can be mapped to a coupled two-spin chain with SPH interaction W. The green arrow indicates intraspecies normal hopping J, double both sides dashed arrow indicates SPH interaction W, on-site interactions are indicated by both sides dashed arrow.

exploration of the novel many-body physics in synthetic dimensions.

The effective Hamiltonian.—We now turn to the realization of SPH interaction for two-component bosons on square optical lattice, by using periodic modulating radio-frequency field. We first introduce the time-dependent Hamiltonian which is used to describe the physics of this periodic modulated two-component boson system. In order to illustrate conveniently and vividly, the relevant physical processes of this time-dependent systems have shown in one-dimensional (1D) systems (see Fig. 1). Then the corresponding time-dependent Hamiltonian reads

$$\begin{aligned}\hat{H}(t) = &-J\sum_s(\hat{A}_s^\dagger\hat{A}_{s-1}+\mathrm{H.c.})-\mu\sum_s\hat{A}_s^\dagger\hat{A}_s\\ &+\frac{\hbar\Delta}{2}\sum_s\hat{A}_s^\dagger\hat{\sigma}_z\hat{A}_s-\frac{\hbar\Omega(t)}{2}\sum_s\hat{A}_s^\dagger\hat{\sigma}_x\hat{A}_s\\ &+\sum_s\left[\frac{U_{\rm aa}}{2}\hat{a}_{as}^{\dagger 2}\hat{a}_{as}^2+\frac{U_{\rm bb}}{2}\hat{b}_{as}^{\dagger 2}\hat{b}_{as}^2+U_{\rm ab}\hat{n}_{as}\hat{n}_{bs}\right],\end{aligned}\tag{1}$$

where J is the spin-independent hopping amplitude, μ is spin-independent chemical potential, $\hat{A}_s=(\hat{a}_s,\hat{b}_s)^T$ are vector field with annihilation operators $\hat{a}_s$ ($\hat{b}_s$) on lattice site s for spin-down (spin-up) component, $\Delta=\omega_{\rm res}-\omega_{\rm rf}$ is the detuning of the radio wave ($\omega_{\rm rf}$) from the atomic resonance ($\omega_{\rm res}$), $\Omega(t)=\Omega\sin(\omega t)$ is Rabi frequency, $\hat{\sigma}_{x,z}$ are pauli matrices [46], and $U_{\rm aa}$, $U_{\rm bb}$, and $U_{\rm ab}$ labels the strength of the on-site repulsive interactions. In Eq. (1), the first term describes normal hopping terms between nearest neighbor site for each spin, the terms in the second line describe two spin states coupled by the periodic radio-frequency field, and the last three terms describe intra-species and interspecies on-site interactions.

Then we obtain the effective Hamiltonian [47,48] (see a derivation in the Supplemental Material [49])

$$\begin{aligned}\hat{H}_{\rm eff}= &-J\sum_s(\hat{a}_s^\dagger\hat{a}_{s-1}+\hat{b}_s^\dagger\hat{b}_{s-1}+\mathrm{H.c.})-\mu\sum_s(\hat{n}_{as}+\hat{n}_{bs})\\ &+\frac{U_{\rm aa}^{\rm eff}}{2}\sum_s\hat{n}_{as}(\hat{n}_{as}-1)+\frac{U_{\rm bb}^{\rm eff}}{2}\sum_s\hat{n}_{bs}(\hat{n}_{bs}-1)\\ &+U_{\rm ab}^{\rm eff}\sum_s\hat{n}_{as}\hat{n}_{bs}+W\sum_s(\hat{a}_s^\dagger\hat{b}_s\hat{a}_s^\dagger\hat{b}_s+\hat{b}_s^\dagger\hat{a}_s\hat{b}_s^\dagger\hat{a}_s),\end{aligned}\tag{2}$$

where the preceding five terms describe the two-component Bose-Hubbard model [8–10] and the W term represents the processes of SPH along a synthetic dimension. Here, the effective on-site interaction strength and SPH interaction in Eq. (2) are given by $U_{\rm aa}^{\rm eff}=U_{\rm aa}-[\Omega/(2\omega)]^2(U_{\rm aa}-U_{\rm ab})$, $U_{\rm ab}^{\rm eff}=U_{\rm ab}+2\Delta U[\Omega/(2\omega)]^2$, $U_{\rm bb}^{\rm eff}=U_{\rm bb}-[\Omega/(2\omega)]^2(U_{\rm bb}-U_{\rm ab})$, $W=-(\Delta U/2)[\Omega/(2\omega)]^2$, and $\Delta U=(U_{\rm aa}+U_{\rm bb})/2-U_{\rm ab}$.

To reveal the relevant physical processes of this effective Hamiltonian more clearly, we choose a 1D system as an example where it can be mapped to coupled two-spin chain (synthetic chain) systems, and every single chain represents one species of boson. The relevant processes are shown in Fig. 2. Although this Hamiltonian in Eq. (2) is obtained with detuning $\Delta=0$, we can also obtain it with an effective detuning $\hbar\Delta_{\rm eff}=\hbar\Delta-(\mu_a-\mu_b)=0$ even if detuning $\Delta\neq 0$. This condition can be satisfied by tuning μ_a and μ_b via changing fillings n_a and n_b.

The phase diagram.—Below, the phase diagram will be numerically studied by the Gutzwiller method that has been successfully used to study various phenomena such as stationary states [52–54], time evolution [55–57], and excitation dynamics [58]. We will use the cluster Gutzwiller method [59–61], which can well capture the quantum fluctuations for a larger cluster to obtain the phase diagram of the two-component boson gases with SPH interaction on a square optical lattice. We can naively assume that there exists the nontrivial MSF state ($\langle\hat{a}_i\rangle=0$ but $\phi_{\rm Da}=\langle\hat{a}_i\hat{a}_i\rangle\neq 0$) apart from the phases that have been found in the two-component Bose systems with $W=0$. The previous research on the two-component boson system with zero SPH interaction reveals that the asymmetric case ($U_{\rm aa}\neq U_{\rm bb}$) shows richer phases than the symmetric one ($U_{\rm aa}=U_{\rm bb}$) [10]. Thus, we study the phase diagram for the asymmetric case of two-component boson system with finite SPH interaction. We have chosen a typical asymmetric case $U_{\rm aa}^{\rm eff}=1.0$, $U_{\rm bb}^{\rm eff}=0.7$, $U_{\rm ab}^{\rm eff}=0.5$, and $W=-0.1$ to study the phase diagram via calculating various possible superfluid orderings. The phase diagram is presented in Fig. 3, where we choose the cluster as 1×2. A supercell cluster 1×2 includes two sites in lattice space which are equivalent to four sites in synthetic space [49].

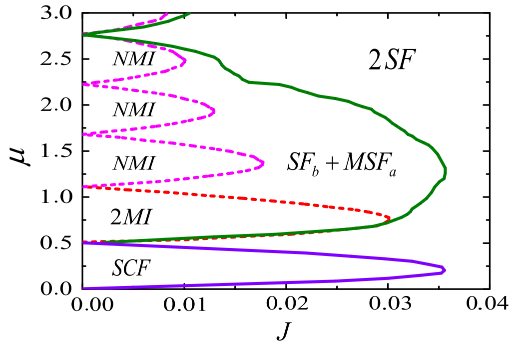

FIG. 3. The phase diagram of two species of Bose gases with SPH interaction in square optical lattice. The interaction parameters are $U_{\rm aa}^{\rm eff} = 1.0$, $U_{\rm bb}^{\rm eff} = 0.7$, $U_{\rm ab}^{\rm eff} = 0.5$, and $W = -0.1$. There are five phases; moreover, the NMI is a new phase.

There are five phases, i.e., 2MI, SCF, 2SF, NMI ($\psi_{\rm a} = \psi_{\rm b} = \phi_{\rm SCF} = \phi_{\rm Da} = \phi_{\rm Db} = 0$), and $\mathrm{SF}_b + \mathrm{MSF}_a$ ($\psi_{\rm a} = 0$, $\psi_{\rm b} \neq 0$, $\phi_{\rm Da} \neq 0$, and $\phi_{\rm Db} \neq 0$) [47]. The 2SF, 2MI, and SCF phases have been discussed [8,10], but the NMI phase and $\mathrm{SF}_b + \mathrm{MSF}_a$ are nontrivial phases which have rarely been predicted in two-component boson systems. Surprisingly, there is no $\mathrm{SF}_b + \mathrm{MI}_a$ phase which usually exists in two-component Bose-Hubbard model for the asymmetric case [10]. Transiting from the NMI phase by increasing the amplitude of tunneling strength J, the systems may enter into an intriguing $\mathrm{SF}_b + \mathrm{MSF}_a$ phase that may exist in a large parameter region of phase diagram (see Fig. 3). In this parameter region, if we switch off the SPH interaction from $|W| \neq 0$ to $|W| = 0$, the $\mathrm{SF}_b + \mathrm{MSF}_a$ phase will undergo a second order phase transition to $\mathrm{SF}_b + \mathrm{MI}_a$ phase. It is remarkable that both the NMI phase and MSF_a phase are induced by the intriguing SPH interaction.

This nontrivial NMI phase is incompressible and has a nontrivial density distribution feature, which shows an integer total atom number at each site and a noninteger atom number for each species. This distribution feature of the NMI phase is significantly different from the atom distribution of 2MI phase, and the atom distribution of each site as a function of variation μ with fixed hopping amplitude J is presented in Fig. 4(a). The reason why there exists such intriguing NMI phase is that in the limit of $J = 0$ (NMI phase), the total number $\hat{n}_i$ is a good quantum number but $\hat{n}_{ai}$ and $\hat{n}_{bi}$ are not, since the Hamiltonian $\hat{H}_{J=0}$ commutes with $\hat{n}_i$ but does not commute with $\hat{n}_{bi}$ or $\hat{n}_{ai}$. For the $J < J_{\rm critical}$ case, the property of the ground state is unchanged, but the parameter region is shrunk, thus the ground state is also the NMI phase. By mapping this new type Mott insulating system to the pseudospin system, we may realize various tunable quantum spin models. Thus, the intriguing noninteger feature of the NMI phase may provide a plausible platform to host some exotic magnetic phases.

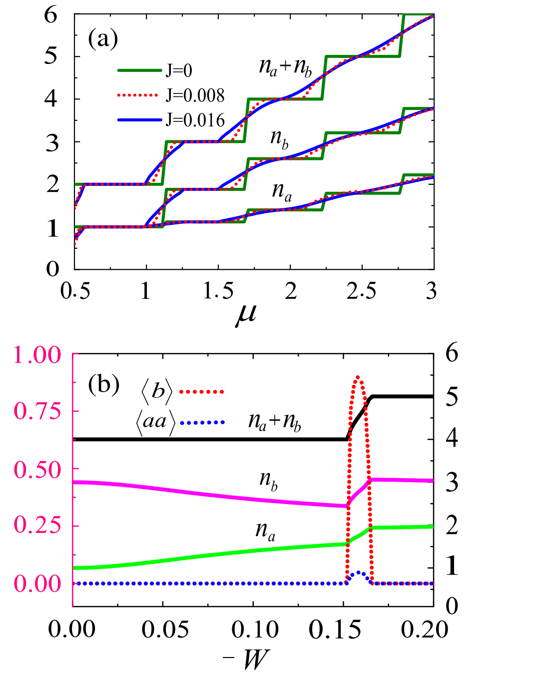

FIG. 4. (a) The total particle number $n_a + n_b$, the number of spin-down (spin-up) components n_a (n_b) as a function of chemical potential μ with $J = 0$, 0.008, 0.016. (b) $n_a + n_b$ and n_a and n_b as functions of W with $J = 0.008$ and $\mu = 2.0$; meanwhile the superfluid order parameter $\langle \hat{a}\,\hat{a} \rangle$ and $\langle \hat{b} \rangle$ as a function of W are also shown, where the pink (black) vertical axis represents the value of the superfluid order parameter (particle number). The interaction parameters are $U_{\rm aa}^{\rm eff} = 1.0$, $U_{\rm bb}^{\rm eff} = 0.7$, and $U_{\rm ab}^{\rm eff} = 0.5$.

By changing the value of W, the system may evolve from the NMI phase into the novel $\mathrm{SF}_b + \mathrm{MSF}_a$ phase [see Fig. 4(b)], where the $\mathrm{SF}_b + \mathrm{MSF}_a$ phase is characterized by normal superfluid of b (spin-up) component and nontrivial MSF of a (spin-down) component. The characteristics of the MSF_a can partly be understood via the coherent state. It is well known that the coherent state satisfies the condition $\psi_{\rm a} \neq 0$ and $\psi_{\rm Da} \neq 0$, and even or odd coherent state [62,63] satisfies the condition $\psi_{\rm a} = 0$ and $\psi_{\rm Da} \neq 0$, where even and odd coherent state read $[|0\rangle + \cdots + \alpha^{2n}|2n\rangle/\sqrt{(2n)!}]/\cosh|\alpha|^2$ and $\{[\alpha|1\rangle + \cdots + \alpha^{2n+1}|2n+1\rangle/\sqrt{(2n+1)!}]/\sinh|\alpha|^2\}$, respectively. As is well known, the perfect superfluid phase (the ground state of the Bose-Hubbard model at $U = 0$ limit) is the coherent state, but the superfluid phase (in the case of $U \neq 0$) is not the coherent state [2]. Thus, the perfect MSF_a can be considered as an odd or even coherent state, but MSF_a state is no longer an even or odd coherent state for interacting systems.

Symmetry analysis.—Here we analyze the general symmetry feature of the phases and transitions between them. It is obvious that a finite SPH interaction W breaks $U(1) \times U(1)$ symmetry of the trivial two-component boson

Hamiltonian ($W = 0$) down to $U(1) \times Z_2$ symmetry (under the phase transformations $\hat{b}_i \to \hat{b}_i e^{i\theta}$ and $\hat{a}_i \to \hat{a}_i e^{i\theta}$ [or $\hat{a}_i \to \hat{a}_i e^{i(\theta+\pi)}$], the Hamiltonian in Eq. (2) keep unchanged). Here 2MI and NMI phases break no symmetry, but the SCF, 2SF, and $SF_b + MSF_a$ phases are related to different ways that the $U(1) \times Z_2$ symmetry is broken. More specifically, the SCF phase breaks the discrete Z_2 subgroup but the $U(1)$ symmetry is remaining. The 2SF phase totally breaks the $U(1) \times Z_2$ symmetry. The $SF_b + MSF_a$ phase breaks $U(1) \times Z_2$ symmetry except for the special point $\theta = \pi$, where SF_b order changes sign ($\langle \hat{b} \rangle \to \langle \hat{b} \rangle e^{i\pi}$) and MSF_a order keeps unchanged ($\phi_{Da} \to \phi_{Da} e^{i4\pi}$ and $\phi_{Da} \to \phi_{Da} e^{i2\pi}$). This type of symmetry breaking is rarely revealed in condensed-matter systems. We need to emphasize that these two superfluids ($SF_b + MSF_a$ and the previous predicted MSF) are distinct states due to different underlying symmetry breaking physics [64].

The effective-field analysis of the possible phases.—We will qualitatively analyze the reason why such rich phases can exist in two-component bosons with SPH interaction. In a $W = 0$ case, the mean-field phase diagram can be obtained by minimizing the free energy $\mathcal{F}_0$ of two-component Bose-Hubbard model [10]. The corresponding phase diagram can be divided into two typical cases: if the interaction is symmetric, there are three phases, i.e., 2SF, 2MI, and SCF ($U_{ab} > 0$) [8]; if the interaction is asymmetric, the possible phases are 2SF, 2MI, SCF, and $SF_b + MI_a$) [10]. For the $W \neq 0$ case, we can also use the effective field theory to analyze the possible phases of this system. We assume that the free energy $\mathcal{F}$ has the form [49]

$$\begin{aligned}\mathcal{F} = \mathcal{F}_0 + \frac{1}{2}[r_{\rm Da}|\phi_{\rm Da}|^2 + r_{\rm Db}|\phi_{\rm Db}|^2 + r_{\rm DD}(\phi^*_{\rm Da}\phi_{\rm Db} + {\rm H.c.})] \\ + \frac{1}{4}[g_{\rm Da}|\phi_{\rm Da}|^4 + g_{\rm Db}|\phi_{\rm Db}|^4] - g(\phi^*_{\rm SCF}\psi^*_A\psi_B + {\rm H.c.}) \\ - g'(\phi^*_{\rm Da}\psi_B\psi_B + \phi^*_{\rm Db}\psi_A\psi_A + {\rm H.c.}), \qquad (3)\end{aligned}$$

with the condition $r_{\rm Da} > 0$, $r_{\rm Db} > 0$, $g_{\rm Da} > 0$, $g_{\rm Db} > 0$. Here the notation $\phi_{\rm Da}$ ($\phi_{\rm Db}$) is the MSF order of the spin-down (spin-up) component. For the asymmetric case ($U_{\rm aa} > U_{\rm bb}$), there are four phases, i.e., 2SF, 2MI, SCF, and $SF_b + MSF_a$ which satisfy the corresponding saddle point equations [49]. Three of them (2SF, 2MI, SCF) have been predicted in a two-component boson system without SPH interaction. Surprisingly, the phase $SF_b + MI_a$ cannot exist in such system with SPH interaction, and it is replaced by the novel phase $SF_b + MSF_a$, which has not been predicted in two-component boson system without SPH interaction. This conclusion is in good agreement with numerical calculation. Still, the reason for the existence of NMI cannot be revealed by the effective-field analysis, owing to such analysis unable to capture the information of the atom distribution.

Experimental realization and detection.—If we choose the $[\Omega/(2\omega)]^2 = 0.05 \ll 1$, the SPH interaction becomes important and the high-order terms ($\mathcal{O}[f^4(t)/\hbar^4]$) can be ignored [49], then the Hamiltonian in Eq. (2) can adequately describe all the relevant physical processes of this driving system. If we want to realize the NMI and MSF_a phase in a practical experimental system, $W \propto [\Omega/(2\omega)]^2$ must be far less than on-site interaction. By choosing the suitable values of U_{aa}, U_{bb}, U_{ab} (can be realized via a Feshbach resonance) and $[\Omega/(2\omega)]^2 = 0.05$, the effective on-site interactions have the same values as presented in the caption of Fig. 3, while the $W = -0.0117$ is far less than on-site interaction. In this feasible region, NMI and MSF_a phases can also occupy a rather large region in the phase diagram (see Fig. S3 in the Supplemental Material [49]), thus the prospects of observing NMI and MSF_a states within this Floquet driving system is quite optimistic. Moreover, the nontrivial feature of the number distribution of NMI state can be directly detected by combining the spin-removal technique [65,66] and *in situ* imaging techniques [67], which have been successfully employed to detect the bosonic MI [68–70] and fermionic MI [71,72] with single-atom and single-site resolution. The previous research has shown that the MSF and SF phases are distinguished via time-of-flight (TOF) shadow images [23], thus $SF_b + MSF_a$ can be directly detected by spin-resolved TOF images [73].

The time-periodic driving often leads to uncontrollable heating in the interacting systems. It is a hard task to calculate the effects of parametric instabilities induced by heating. According to a recent experiment [51], we can estimate roughly typical timescale τ, during which the system can stay in steady states. If we choose the interaction parameters shown in the Supplemental Material [49] with $\Omega/2 = 2\pi \times 0.5$ and $\omega = 2\pi \times 2.236(2\pi \times 5)$ KHz $\{[\Omega/(2\omega)]^2 = 0.05(0.01)\}$, the system can steadily stay in the phase $SF_b + MSF_a$, owing to $\tau \approx 41(93)$ ms $> t_c$ (see Fig. S3 in the Supplemental Material [49]). Here $t_c \approx 15$ ms [2] is the ballistic expansion time in a typical TOF experiment. If $\tau \gg t_c$, the ground state of the effective Hamiltonian is stable enough in the timescale of experiment detecting. Furthermore, we can use the two hyperfine states $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = -1\rangle$ of the $n^2S_{1/2}$ ground state of bosonic alkali atom (^{7}Li, ^{23}Na, or ^{87}Rb) to implement our driven scheme.

Discussion and conclusions.—The main focus of current research on synthetic dimension systems has remained on the construction of synthetic gauge fields. However, for the more general interaction term, e.g., SPH interaction, which can generate novel many-body states, has not been engineered in synthetic dimension. Although Floquet engineering is mainly used to manipulate the single-particle hopping processes, we have proposed to engineer a two-particle hopping process with SPH interaction in the two-component boson system. This intriguing SPH interaction can lead to two novel quantum phases, i.e., NMI and MSF_a. The NMI state is a new type of Mott insulator, in which the total number at each site is an integer, but each component is a noninteger. The MSF_a state has been proposed for some years, and not much progress has been made to host such a state in a realistic system. The region of NMI and MSF_a states are gradually shrunken with rapidly decreasing the SPH interaction (see Fig. 3 and S3 in the Supplemental Material [49]). Thus, the prospects of observing the novel NMI and MSF_a phases are rather optimistic in a realistic system. Furthermore, the detection schemes of these two novel phases are also addressed. The realization of our scheme may provide a plausible platform for further exploration of intriguing quantum many-body phases in synthetic dimensions.

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*zhilin13@fudan.edu.cn
†yanchen99@fudan.edu.cn

[1] M. Lewenstein, A. Sanpera, V. Ahufiner, B. Damski, A. Sen, and U. Sen, Adv. Phys. **56**, 243 (2007).
[2] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[3] J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, Rev. Mod. Phys. **83**, 1523 (2011).
[4] I. Bloch, J. Dalibard, and S. Nascimbène, Nat. Phys. **8**, 267 (2012).
[5] N. Goldman, G. Juzeliūnas, P. Öhberg, and I. B. Spielman, Rep. Prog. Phys. **77**, 126401 (2014).
[6] A. Eckardt, Rev. Mod. Phys. **89**, 011004 (2017).
[7] P. Nozières and D. Saint James, J. Phys. (Paris) **43**, 1133 (1982).
[8] A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett. **90**, 100401 (2003).
[9] E. Altman, W. Hofstetter, E. Demler, and M. D. Lukin, New J. Phys. **5**, 113 (2003).
[10] A. Kuklov, N. Prokofév, and B. Svistunov, Phys. Rev. Lett. **92**, 050402 (2004).
[11] A. Argüelles and L. Santos, Phys. Rev. A **75**, 053613 (2007).
[12] E. K. Dahl, E. Babaev, and A. Sudbo, Phys. Rev. Lett. **101**, 255301 (2008).
[13] S. G. Söyler, B. C. Sansone, N. V. Prokof'ev, and B. V. Svistunov, New J. Phys. **11**, 073036 (2009).
[14] A. Hubener, M. Snoek, and W. Hofstetter, Phys. Rev. B **80**, 245109 (2009).
[15] L. Mathey, I. Danshita, and C. W. Clark, Phys. Rev. A **79**, 011602(R) (2009).
[16] A. Hu, L. Mathey, I. Danshita, E. Tiesinga, C. J. Williams, and C. W. Clark, Phys. Rev. A **80**, 023619 (2009).
[17] C. Menotti and S. Stringari, Phys. Rev. A **81**, 045604 (2010).
[18] C. Trefzger, C. Menotti, and M. Lewenstein, Phys. Rev. Lett. **103**, 035304 (2009).
[19] A. Safavi-Naini, Ş. G. Söyler, G. Pupillo, H. R. Sadeghpour, and B. Capogrosso-Sansone, New J. Phys. **15**, 013036 (2013).
[20] B. Pandey, S. Sinha, and S. K. Pati, Phys. Rev. B **91**, 214432 (2015).
[21] L. Radzihovsky, J. Park, and P. B. Weichman, Phys. Rev. Lett. **92**, 160402 (2004).
[22] M. W. J. Romans, R. A. Duine, S. Sachdev, and H. T. C. Stoof, Phys. Rev. Lett. **93**, 020405 (2004).
[23] L. Radzihovsky, P. B. Weichman, and J. I. Park, Ann. Phys. (Amsterdam) **323**, 2376 (2008).
[24] A. J. Daley, J. M. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).
[25] S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **104**, 165301 (2010).
[26] Y. C. Wang, W. Z. Zhang, H. Shao, and W. A. Guo, Chin. Phys. B **22**, 096702 (2013).
[27] Q. Z. Zhu, Q. Zhang, and B. Wu, J. Phys. B **48**, 045301 (2015).
[28] O. Jürgensen, K. Sengstock, and D. S. Lühmann, Sci. Rep. **5**, 12912 (2015).
[29] O. Dutta, M. Gajda, P. Hauke, M. Lewenstein, D. S. Lühmann, B. A. Malomed, T. Sowiński, and J. Zakrzewski, Rep. Prog. Phys. **78**, 066001 (2015).
[30] A. Eckardt and E. Anisimovas, New J. Phys. **17**, 093039 (2015).
[31] M. Bukov, L. DAlessio, and A. Polkovnikov, Adv. Phys. **64**, 139 (2015).
[32] N. Goldman and J. Dalibard, Phys. Rev. X **4**, 031027 (2014).
[33] F. Mei, Jia-Bin You, Dan-Wei Zhang, X. C. Yang, R. Fazio, Shi-Liang Zhu, and L. C. Kwek, Phys. Rev. A **90**, 063638 (2014).
[34] K. Jiménez-García, L. J. LeBlanc, R. A. Williams, M. C. Beeler, C. Qu, M. Gong, C. Zhang, and I. B. Spielman, Phys. Rev. Lett. **114**, 125301 (2015).
[35] X. Luo, L. Wu, J. Chen, Q. Guan, K. Gao, Z. F. Xu, L. You, and R. Wang, Sci. Rep. **6**, 18983 (2016).
[36] T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, Nat. Commun. **2**, 361 (2011).
[37] Á. Rapp, X. L. Deng, and L. Santos, Phys. Rev. Lett. **109**, 203005 (2012).
[38] S. Greschner, L. Santos, and D. Poletti, Phys. Rev. Lett. **113**, 183002 (2014).
[39] C. Sträter, S. C. L. Srivastava, and A. Eckardt, Phys. Rev. Lett. **117**, 205303 (2016).
[40] F. Meinert, M. J. Mark, K. Lauber, A. J. Daley, and H. C. Nägerl, Phys. Rev. Lett. **116**, 205301 (2016).
[41] L. W. Clark, B. M. Anderson, L. Feng, A. Gaj, K. Levin, and C. Chin, Phys. Rev. Lett. **121**, 030402 (2018).
[42] F. Görg, M. Messer, K. Sandholzer, G. Jotzu, R. Desbuquois, and T. Esslinger, Nature (London) **553**, 481 (2018).

[43] F. Görg, K. Sandholzer, J. Minguzzi, R. Desbuquois, M. Messer, and T. Esslinger, Nat. Phys. **15**, 1161 (2019).

[44] C. Schweizer, F. Grusdt, M. Berngruber, L. Barbiero, E. Demler, N. Goldman, I. Bloch, and M. Aidelsburger, Nat. Phys. **15**, 1168 (2019).

[45] A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I. B. Spielman, G. Juzeliūnas, and M. Lewenstein, Phys. Rev. Lett. **112**, 043001 (2014).

[46] J. Struck, J. Simonet, and K. Sengstock, Phys. Rev. A **90**, 031601(R) (2014).

[47] Q. Zhou, J. V. Porto, and S. Das Sarma, Phys. Rev. B **83**, 195106 (2011).

[48] F. Pinheiro, G. M. Bruun, J. P. Martikainen, and J. Larson, Phys. Rev. Lett. **111**, 205302 (2013).

[49] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.245301 for more details on the derivation of the effective Hamiltonian, possible mean field phases, and detailed discussions, which includes Refs. [2,10,50,51].

[50] A. Eckardt, C. Weiss, and M. Holthaus, Phys. Rev. Lett. **95**, 260404 (2005).

[51] T. Boulier, J. Maslek, M. Bukov, C. Bracamontes, E. Magnan, S. Lellouch, E. Demler, N. Goldman, and J. V. Porto, Phys. Rev. X **9**, 011047 (2019).

[52] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

[53] W. Zwerger, J. Opt. B **5**, S9 (2003).

[54] C. Trefzger, C. Menotti, B. Capogrosso-Sansone, and M. Lewenstein, J. Phys. B **44**, 193001 (2011).

[55] D. Jaksch, V. Venturi, J. I. Cirac, C. J. Williams, and P. Zoller, Phys. Rev. Lett. **89**, 040402 (2002).

[56] U. Bissbort, S. Götze, Y. Li, J. Heinze, J. S. Krauser, M. Weinberg, C. Becker, K. Sengstock, and W. Hofstetter, Phys. Rev. Lett. **106**, 205303 (2011).

[57] U. R. Fischer and B. Xiong, Phys. Rev. A **84**, 063635 (2011).

[58] K. V. Krutitsky and P. Navez, Phys. Rev. A **84**, 033602 (2011).

[59] D. Pekker, B. Wunsch, T. Kitagawa, E. Manousakis, A. S. Sørensen, and E. Demler, Phys. Rev. B **86**, 144527 (2012).

[60] Dirk-Sören Lühmann, Phys. Rev. A **87**, 043619 (2013).

[61] B. K. Alavani, A. Das, and R. V. Pai, J. Phys. B **51**, 145302 (2018).

[62] V. V. Dodonov, I. A. Malkin, and V. I. Man'ko, Physica (Utrecht) **72**, 597 (1974).

[63] M. Hillery, Phys. Rev. A **36**, 3796 (1987).

[64] MSF has also been predicted in three different single-component bosonic systems. However, different types of symmetry breaking occur in these three systems, i.e., $U(1)$ symmetry breaking for atomic Bose gas near a Feshbach resonance, $U(1)/Z_2$ symmetry breaking for the attractive Bose-Hubbard model as well as the extended Bose-Hubbard model with pair hopping.

[65] M. F. Parsons, A. Mazurenko, C. S. Chiu, G. Ji, D. Greif, and M. Greiner, Science **353**, 1253 (2016).

[66] P. T. Brown, D. Mitra, E. Guardado-Sanchez, P. Schauß, S. S. Kondov, E. Khatami, T. Paiva, N. Trivedi, D. A. Huse, and W. S. Bakr, Science **357**, 1385 (2017).

[67] H. Ott, Rep. Prog. Phys. **79**, 054401 (2016).

[68] N. Gemelke, X. Zhang, C. Hung, and C. Chin, Nature (London) **460**, 995 (2009).

[69] W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Fölling, L. Pollet, and M. Greiner, Science **329**, 547 (2010).

[70] J. F. Sherson, C. Weitenberg, M. Endres, M. Cheneau, I. Bloch, and S. Kuhr, Nature (London) **467**, 68 (2010).

[71] D. Greif, M. F. Parsons, A. Mazurenko, C. S. Chiu, S. Blatt, F. Huber, G. Ji, and M. Greiner, Science **351**, 953 (2016).

[72] L. W. Cheuk, M. A. Nichols, K. R. Lawrence, M. Okan, H. Zhang, and M. W. Zwierlein, Phys. Rev. Lett. **116**, 235301 (2016).

[73] Z. Wu, L. Zhang, W. Sun, X. T. Xu, B. Z. Wang, S. C. Ji, Y. J. Deng, S. Chen, X. J. Liu, and J. W. Pan, Science **354**, 83 (2016).