

## Symmetry Breaking and Error Correction in Open Quantum Systems

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Symmetry-breaking transitions are a well-understood phenomenon of closed quantum systems in quantum optics, condensed matter, and high energy physics. However, symmetry breaking in open systems is less thoroughly understood, in part due to the richer steady-state and symmetry structure that such systems possess. For the prototypical open system—a Lindbladian—a unitary symmetry can be imposed in a “weak” or a “strong” way. We characterize the possible  $\mathbb{Z}_n$  symmetry-breaking transitions for both cases. In the case of  $\mathbb{Z}_2$ , a weak-symmetry-broken phase guarantees at most a classical bit steady-state structure, while a strong-symmetry-broken phase admits a partially protected steady-state qubit. Viewing photonic cat qubits through the lens of strong-symmetry breaking, we show how to dynamically recover the logical information after any gap-preserving strong-symmetric error; such recovery becomes perfect exponentially quickly in the number of photons. Our study forges a connection between driven-dissipative phase transitions and error correction.

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While an open quantum system typically evolves toward a thermal state [1], nonthermal steady states emerge in the presence of an external drive [2,3] or via reservoir engineering [4,5]. In particular, systems with multiple steady states have recently attracted much attention due to their ability to remember initial conditions [6–18]. For Markovian environments, this involves studying Lindblad superoperators (Lindbladians) [19–21] that possess multiple eigenvalues of zero [22].

On the one hand, Lindbladians with such degenerate steady states are the key ingredient for *passive* error correction [23–34]. In this paradigm, the degenerate steady-state structure of an appropriately engineered Lindbladian stores the logical information, and the Lindbladian passively protects this information from certain errors by continuously mapping any leaked information back into the structure without distortion. An important task remains to identify *generic* systems that host such protected qubit steady-state structures, and classify the errors that can be corrected in this way.

On the other hand, the presence of a ground-state degeneracy in the infinite-size limit of a closed system is a salient feature of symmetry breaking (e.g., the ferromagnetic ground states of the Ising model) [35]. While the study of analogous phase transitions in open systems has become a rich and active field [3,36–49] with significant experimental relevance [50–54], attention has focused on the steady-state degeneracy in symmetry-broken phases only recently [55–57].

Since steady-state degeneracy is a requirement for both passive error correction and symmetry breaking, it is natural to ask whether there are any connections between the two phenomena. Here, we begin to shed light on this interesting and important direction by (A) describing how the dimension and structure of the steady-state manifold changes across a dissipative phase transition, and (B) identifying any passive protection due to the symmetry-broken phase (we will often drop the word symmetry below).

To this end, we emphasize an important distinction between “weak” and “strong” transitions which is unique to open systems. This difference stems from the dissipative part of the Lindbladian which can respect a symmetry in two separate ways, as first noted by Buča and Prosen [6]. We show that the  $\mathbb{Z}_2$  strong-broken phase encodes a qubit in its steady-state structure in the infinite-size limit, and that errors preserving this structure can be passively corrected. Our analysis is made concrete by considering a driven-dissipative photonic mode—a minimal model for the study of both nonequilibrium transitions [55] and bosonic error-correcting codes [25].

*Generic  $\mathbb{Z}_n$  symmetry breaking.*—We consider open systems governed by a Lindblad master equation

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H, \rho] + \sum_i (2L_i \rho L_i^\dagger - \{L_i^\dagger L_i, \rho\}), \quad (1)$$

with density matrix  $\rho$ , Hamiltonian  $H$ , dissipators  $L_i$ , and Lindbladian  $\mathcal{L}$ . A *strong* symmetry is satisfied if there

exists an operator  $P$  such that  $[H, P] = [L_i, P] = 0$ ,  $\forall i$ . A *weak* symmetry is satisfied if  $[\mathcal{L}, \mathcal{P}] = 0$ , where  $\mathcal{P}(\cdot) = P(\cdot)P^\dagger$ . A strong symmetry necessarily implies a weak symmetry but the converse is not true. For example, the dissipators only need to commute up to a phase ( $L_i P = e^{i\theta_i} P L_i$ ) for the weak condition to be met. We will showcase differences between previously studied weak-symmetry transitions and the strong-symmetry ones we introduce here, focusing on changes to the dimension *and* structure of the steady-state manifold.

Let us review [55] weak  $\mathbb{Z}_2$ -symmetry breaking, which is similar to conventional closed-system symmetry breaking and is ubiquitous in open systems [36,45,46]. Here,  $P$  is a parity operator that satisfies  $P|\pm\rangle = \pm|\pm\rangle$  with parity eigenvalues  $\pm 1$  and sets of eigenstates  $\{|\pm\rangle\}$ . Its superoperator version,  $\mathcal{P}(\cdot) = P(\cdot)P^\dagger$ , possesses  $+1$  and  $-1$  “superparity” eigenvalues, belonging respectively to eigenoperators  $|\pm\rangle\langle\pm|$  and  $|\pm\rangle\langle\mp|$ . A weak  $\mathbb{Z}_2$  symmetry  $\mathcal{P}$  can thus be used to block diagonalize  $\mathcal{L}$  into two sectors,  $\mathcal{L} = \text{diag}[\mathcal{L}_+, \mathcal{L}_-]$ , one for each superparity. Since the  $-1$  superparity sector contains only traceless eigenoperators, the (trace-one) steady state of a finite-size system will necessarily have superparity  $+1$  and be an eigenoperator of  $\mathcal{L}_+$ . If a symmetry-broken order parameter is to acquire a nonzero steady-state expectation value in the infinite-size limit,  $\mathcal{L}_-$  must also pick up a zero-eigenvalue eigenoperator, and positive or negative mixtures of the original steady state and this new eigenoperator will become the two steady states of the system (a “1-to-2” transition).

In the strong case, there are two superparity superoperators,  $\mathcal{P}_l(\cdot) = P(\cdot)$  and  $\mathcal{P}_r(\cdot) = (\cdot)P^\dagger$ , that commute with each other as well as with  $\mathcal{L}$ . Their eigenvalues further resolve the states  $|+\rangle\langle+|$  from  $|-\rangle\langle-|$  (and similarly  $|+\rangle\langle-|$  from  $|-\rangle\langle+|$ ), yielding the finer block diagonalization  $\mathcal{L} = \text{diag}[\mathcal{L}_{++}, \mathcal{L}_{--}, \mathcal{L}_{+-}, \mathcal{L}_{-+}]$ . The key observation is that both  $\mathcal{L}_{++}$  and  $\mathcal{L}_{--}$  have to admit steady-state eigenoperators, since their respective sectors house eigenoperators with nonzero trace. A strong transition is therefore a 2-to-4 transition: the dimension of the steady-state manifold increases from 2 to 4 as  $\mathcal{L}_{-+}$  and  $\mathcal{L}_{+-}$  pick up zero eigenvalues in the broken phase. This reasoning generalizes to  $\mathbb{Z}_n$  symmetries (see Table I).

*Steady-state structure in different  $\mathbb{Z}_2$  phases.*—Apart from differences in the dimension of the steady-state manifold, a weak-broken  $\mathbb{Z}_2$  phase can yield at most a

TABLE I. Comparison of a strong vs weak  $\mathbb{Z}_n$  symmetry of  $\mathcal{L}$ . The final column describes transitions in the dimension of the steady state ( $D_{\text{SS}}$ ) manifold (number of zero eigenvalues of  $\mathcal{L}$ ) when going from the unbroken phase to the broken phase.

$\mathbb{Z}_n$ symmetry	Definition	Sufficient condition	$D_{\text{SS}}$ transition
Strong	$[\mathcal{L}, \mathcal{P}_{l,r}] = 0$	$[H, P] = [L_i, P] = 0$	$n$ -to- $n^2$
Weak	$[\mathcal{L}, \mathcal{P}] = 0$	$[H, P] = \{L_i, P\} = 0$	1-to- $n$

classical bit structure, while a strong-broken phase can yield a qubit steady-state manifold. To see this, we express the steady state of a  $\mathbb{Z}_2$ -symmetric model in the parity basis,  $|\vec{\pm}\rangle = (|\pm\rangle_1, |\pm\rangle_2, \dots)$ , as

$$\rho_{\text{ss}} = \begin{pmatrix} s_{++} & s_{+-} \\ s_{-+} & s_{--} \end{pmatrix}. \quad (2)$$

Table II lists the “degrees of freedom” for the steady state in each phase, i.e., which part of the matrix is allowed to change depending on the initial condition  $\rho_i$ . The strong-broken phase can remember both the relative magnitude and phase of an initial state, which guarantees that a qubit can be encoded into the steady state. The strong-unbroken and weak-broken phases both host a classical bit structure, where classical mixtures remain stable. The weak-unbroken phase will generically possess a unique steady state.

*$\mathbb{Z}_2$ -symmetric model.*—We make this general analysis more concrete by focusing on a minimal driven-dissipative example that exhibits both strong and weak versions of  $\mathbb{Z}_2$  symmetry-breaking transitions in an infinite-size limit. Consider the rotating-frame Hamiltonian for a photonic cavity mode subject to a coherent two-photon drive:

$$H = \omega a^\dagger a + \lambda[a^2 + (a^\dagger)^2], \quad (3)$$

where  $\omega, \lambda \in \mathbb{R}$  [10,25,58–60]. The Hamiltonian possesses a  $\mathbb{Z}_2$  symmetry with respect to Bose parity:  $[H, P] = 0$ , where  $P = \exp(i\pi a^\dagger a)$ . Dissipation can be introduced in ways that respect strong or weak versions of the parity symmetry. We present our strong case along with the previously studied weak case [55], further developing the latter.

In the strong case, we consider two-photon loss  $L_2 = \sqrt{\kappa_2} a^2$  and dephasing  $L_d = \sqrt{\kappa_d} a^\dagger a$ . In the weak case, we add one-photon loss  $L_1 = \sqrt{\kappa_1} a$  in addition to  $L_2$  and  $L_d$ . Note that  $[L_2, P] = [L_d, P] = 0$  and  $\{L_1, P\} = 0$ , which justifies our classification. The competition between non-commuting terms in the Hamiltonian results in a symmetry-broken phase for large ratios of  $\lambda/\omega$ , as described below.

We uncover the phase diagram using two independent methods that agree: (1) a solution for the order parameter and (2) an expression for the dissipative gap. The expectation value of the order parameter  $a$  satisfies

TABLE II. The structure and participating degrees of freedom of the steady state (SS) matrix in Eq. (2) for different  $\mathbb{Z}_2$  phases.

$\mathbb{Z}_2$ phase	SS freedom	SS structure
Strong, broken	$s_{++}, s_{--}, s_{+-}, s_{-+}$	Qubit
Strong, unbroken	$s_{++}, s_{--}$	Classical bit
Weak, broken	$s_{+-}, s_{-+}$	Classical bit
Weak, unbroken	None	Unique

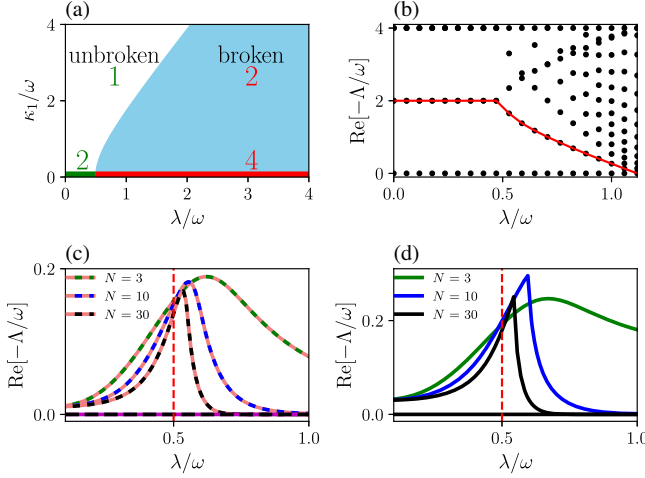


FIG. 1. (a) Phase diagram for the model in Eq. (3) with one-photon loss  $\kappa_1$ ,  $\kappa_d = 0$ , in the thermodynamic limit  $\lambda/\kappa_2 \rightarrow \infty$ . Integers indicate the dimension of the steady-state manifold. (b) Analytical expression for the dissipative gap (red line) and numerical spectrum (black dots) in the unbroken phase for  $\kappa_2 = \kappa_d = 0$ ,  $\kappa_1/\omega = 2$ . The dissipative gap closes as the phase boundary at  $\lambda/\omega = \sqrt{5}/2 \approx 1.1$  is approached. (c) Strong transition: decay rate of the four modes with the longest lifetime; two modes are always pinned to zero and the other two are degenerate (textured colors indicate twofold degeneracy of all modes in this subfigure). A 2-to-4 transition occurs near  $\lambda/\omega = 0.5$  (red dashed line) in the limit  $N \rightarrow \infty$ , in agreement with the phase diagram.  $\kappa_1 = 0$ ,  $\lambda/\kappa_2 = N$ ,  $\kappa_d/\omega = 0.01$ . (d) Weak transition: decay rate of the two modes with the longest lifetime. Full lines emphasize a lack of exact twofold degeneracy present in (c). A 1-to-2 transition is observed.  $\lambda/\kappa_2 = N$ ,  $\kappa_1/\omega = 0.02$ ,  $\kappa_d/\omega = 0.01$ .

$$\frac{d}{dt} \langle a \rangle = -2i\lambda \langle a^\dagger \rangle - (i\omega + \kappa_1 + \kappa_d) \langle a \rangle - 2\kappa_2 \langle a^\dagger a^2 \rangle, \quad (4)$$

where the right-hand side follows from  $\partial_t \langle a \rangle = \text{Tr}[a\mathcal{L}(\rho)]$ . To determine the steady-state expectation value, we set  $\partial_t \langle a \rangle_{ss} = 0$  and check which parameter regime produces nontrivial solutions for  $\langle a \rangle_{ss} \equiv \alpha$ . In the mean-field approximation,  $\langle a^\dagger a^2 \rangle \approx |\alpha|^2 \alpha$ , which is justified when  $|\alpha|^2$  (the cavity photon population) is large. The critical boundary satisfies  $(\kappa_1 + \kappa_d)/\omega = \sqrt{4(\lambda/\omega)^2 - 1}$ , with a cavity photon population  $|\alpha|^2 = [\sqrt{4\lambda^2 - \omega^2} - (\kappa_1 + \kappa_d)] / (2\kappa_2)$  and  $\exp(2i \cdot \arg[\alpha]) = (-\omega - i\sqrt{4\lambda^2 - \omega^2}) / (2\lambda)$  in the broken phase. The steady-state population of photons diverges as  $\lambda/\kappa_2 \equiv N \rightarrow \infty$ , which represents the thermodynamic limit for this model [55,57,61,62]. Figure 1(a) presents the phase diagram for  $\lambda/\kappa_2 \rightarrow \infty$ ; the mean-field equation is exact in this limit. Both weak ( $\kappa_1 \neq 0$ ) and strong ( $\kappa_1 = 0$ ) models indeed exhibit a transition characterized by a  $\mathbb{Z}_2$ -broken order parameter  $\langle a \rangle_{ss}$ .

We show that the dissipative gap closes at the critical boundary for  $\kappa_2 = \kappa_d = 0$ . In this (thermodynamic) limit,  $\mathcal{L}$  is quadratic in Bose operators, hence we can calculate the dissipative gap in the unbroken phase:

$\Delta_g = -\text{Re}[\kappa_1 + \sqrt{4\lambda^2 - \omega^2}]$  [see Supplemental Material (SM) [63]]. Setting  $\Delta_g = 0$  leads to a phase boundary which is identical to the mean-field analysis plotted in Fig. 1(a). Figure 1(b) plots the expression for  $\Delta_g$  along with a numerical calculation of the Lindblad spectrum  $\{\Lambda\}$ , defined via  $\mathcal{L}(e_j) = \Lambda_j e_j$  where  $e_j$  are eigenoperators of  $\mathcal{L}$  with eigenvalues  $\Lambda_j$ . We expect an extensive number of modes to touch zero at the critical point  $\lambda \approx 1.1$ , but our numerics are limited by a finite Hilbert space. Similar results were recently reported in a related model [67].

Away from this exactly solvable limit, i.e.,  $\kappa_2 \neq 0$  and/or  $\kappa_d \neq 0$ , we use numerical exact diagonalization to examine the steady-state dimension across the boundary. Figure 1(c) probes the strong transition by plotting the four spectral eigenvalues with the smallest decay rate. Indeed, two of these are always pinned to zero due to the strong symmetry, but two additional zero eigenvalues appear in the broken phase. The transition occurs near values predicted by the phase diagram as the system approaches the thermodynamic limit  $\lambda/\kappa_2 = N \rightarrow \infty$ . We repeat the analysis for the weak transition in Fig. 1(d) by plotting the two modes with the longest lifetimes and observe a 1-to-2 transition. This confirms our general analysis in Table I. The degeneracy at zero in the broken phase is split by an exponentially small term  $\sim \exp(-N)$  (see SM [63]).

The rest of our analysis will focus on the strongly symmetric model, setting  $\kappa_1 = 0$ . We inspect the nature of the steady states by writing down their exact expressions in extreme limits. First consider the unbroken phase  $\omega \neq 0$ ,  $\kappa_2 \neq 0$ ,  $\lambda = \kappa_d = 0$ . There are only two eigenoperators of  $\mathcal{L}$  with zero eigenvalue  $|0\rangle\langle 0|$  and  $|1\rangle\langle 1|$ . The steady-state manifold reads  $\rho_{ss}(x) = x|0\rangle\langle 0| + (1-x)|1\rangle\langle 1|$  for  $x \in [0, 1]$ . This represents a classical bit of information, since only relative magnitudes of an initial superposition are remembered, in agreement with Table II.

Next, consider the broken limit  $\omega = \kappa_d = 0$ ,  $\lambda \neq 0$ ,  $\kappa_2 \neq 0$ . Define the following coherent states  $|\pm\alpha\rangle = \sum_{n=0}^{\infty} (\pm\alpha)^n |n\rangle / \sqrt{n!}$  where  $\pm\alpha = \pm e^{-i\pi/4} \sqrt{\lambda/\kappa_2}$ .  $\alpha$  matches the mean-field result, defined up to a minus sign degeneracy. Then any pure state of the form  $|\psi\rangle = c_e |\alpha\rangle_e + c_o |\alpha\rangle_o$  will be a steady state, where we define normalized even and odd “cat” coherent states  $|\alpha\rangle_{e,o} \propto |\alpha\rangle \pm |-\alpha\rangle$  [68]. An arbitrary superposition of these cat states is a steady state, an example of a decoherence-free subspace (DFS) [23].

*Passive error correction for cat qubits.*—We now show that a qubit encoded in the steady-state subspace of the strong-broken phase benefits from passive error correction in the thermodynamic limit  $\lambda/\kappa_2 = N \rightarrow \infty$ . We have just seen that the limit  $\kappa_1 = \kappa_d = \omega = 0$  hosts a DFS spanned by cat states. We define  $\mathcal{L}_0$  to be the Lindbladian at this point. Previous studies have suggested that this coherent subspace could serve as a platform for universal quantum computation that is intrinsically protected against dephasing errors [25]. Reference [25] found that, as  $|\alpha|^2 \rightarrow \infty$ , an

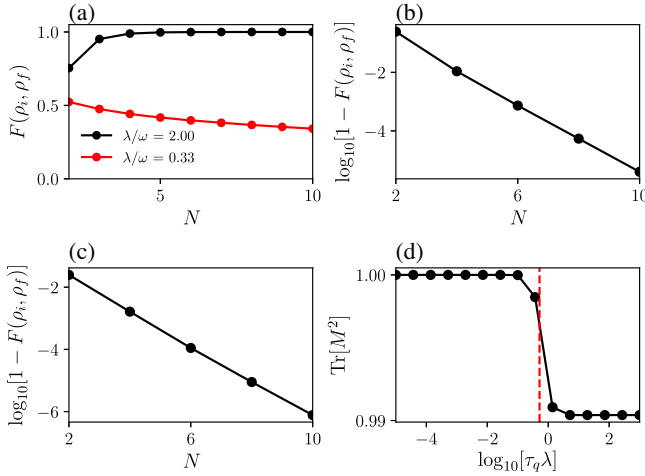


FIG. 2. (a) Fidelity of the initial and final states for the quench protocol given in the main text with  $\lambda/\kappa_2 = N$ ,  $\kappa_d = 0$ ,  $\tau_q \lambda = 10$ ,  $F(\rho_i, \rho_f) = \text{Tr}[\sqrt{\sqrt{\rho_i} \rho_f \sqrt{\rho_i}}]$ . Quenches to the strong-broken phase (black dots) have a fidelity that tends to one in the thermodynamic limit, while quenches to the strong-unbroken phase (red dots) do not. (b) Same parameters as in (a) with  $\lambda/\omega = 2$ ; the fidelity tends to one exponentially fast in  $N$ . (c) A dephasing error  $\kappa_d/\lambda = 0.03$ ,  $\omega = 0$ ,  $\lambda/\kappa_2 = N$ ,  $\tau_q \lambda = 10$ ; again the fidelity is exponentially close to one. (d) Purity of  $M$  [see Eq. (5)] for different quench times with the same parameters as in (b) and  $N = 15$ . The dashed line is the timescale set by the dissipative gap  $\tau_g = \Delta_g^{-1}$  of  $\mathcal{L}_0 + \mathcal{L}'$ . ( $\Delta_g$  is the decay rate of the longest-lived excitation above the four steady-state solutions.) Short quenches keep the system approximately pure, while long quenches evolve the system to a mixed NS steady state. Errors are correctable in both cases. For all figures,  $c_e = 1/\sqrt{2}$ ,  $c_o = i/\sqrt{2}$ .

initially pure cat qubit, which encounters a dephasing term in the Lindbladian for a short time (with respect to the inverse dissipative gap) will return to its initial pure state after evolving the system with  $\mathcal{L}_0$ . In this context, our analysis allows us to (1) extend the protection to errors that last an *arbitrary* amount of time (cf. [69]), (2) understand the dynamics of the state throughout the error process, and (3) classify the types of errors that self correct via the environment.

We consider the following protocol: Initialize the system in a pure state  $\rho_i = |\psi\rangle\langle\psi|$ ,  $|\psi\rangle = c_e|\alpha\rangle_e + c_o|\alpha\rangle_o$ , which represents the qubit and satisfies  $\mathcal{L}_0(\rho_i) = 0$ . Then quench the state with an “error” for an arbitrary time  $\tau_q$  to obtain  $\rho_m = \exp[(\mathcal{L}_0 + \mathcal{L}')\tau_q](\rho_i)$ . Finally, turn off the error and evolve the system with  $\mathcal{L}_0$  for a long time such that it reaches its steady state:  $\rho_f = \lim_{t \rightarrow \infty} \exp[\mathcal{L}_0 t](\rho_m)$ . For what types of perturbations  $\mathcal{L}'$  will  $\rho_f$  and  $\rho_i$  be equal?

In Figs. 2(a) and 2(b), we plot the fidelity  $F$  between the initial state and the final state for the protocol described above with an error in the frequency, i.e.,  $H' = \omega a^\dagger a$ , which either keeps the system in the strong-broken phase (black dots) or moves it to the strong-unbroken phase (red dots). The fidelity tends to one exponentially in cavity photon

number for a long quench time  $\tau_q$  *only* if the perturbation kept the system in the broken phase. Figure 2(c) shows a similar behavior in the presence of a dephasing error: The qubit is able to perfectly correct itself as  $N \rightarrow \infty$ .

We can understand this striking behavior by recalling that the system is guaranteed to host a qubit steady state structure in the  $N \rightarrow \infty$  limit of the strong-broken phase. Away from the special point  $\mathcal{L}_0$  but within the strong-broken phase, our numerics suggest that the steady-state structure is a noiseless subsystem (NS) [70]: a qubit in any state tensored with a fixed mixed state. In other words, at any time after the introduction of the error, the state has the form

$$\rho_m(\tau_q) = \begin{pmatrix} |c_e|^2 & c_e c_o^* \\ c_e^* c_o & |c_o|^2 \end{pmatrix} \otimes M(\tau_q), \quad (5)$$

where the qubit factor remains perfectly encoded in the even-odd parity basis, while the state  $M(\tau_q)$  interpolates between the (pure) DFS steady state and the (mixed) NS steady state. The purity of  $M(\tau_q)$  for different quench times is given in Fig. 2(d), corroborating this interpretation: Short quenches leave  $M$  approximately pure, while long quenches allow it to equilibrate to a mixed steady state (cf. [34]). In both cases, the initial qubit state can be restored via evolution by  $\mathcal{L}_0$ , with most of the recovery (up to exponentially small corrections) occurring after a time of order of the inverse dissipative gap. This decoupling of the qubit from auxiliary modes is reminiscent of the decoupling used in quantum-information-preserving sympathetic cooling of trapped ions [71] and neutral atoms [72], as well as in the nuclear-spin-preserving manipulation of electrons in alkaline-earth atoms [73,74]. The SM [63] provides numerical evidence for the structure in Eq. (5), including the NS steady state of  $\mathcal{L}_0 + \mathcal{L}'$ . The SM [63] also shows perfect recovery of the fidelity for long quenches via an independent method, i.e., asymptotic projections [8].

The argument above relies on the presence of a qubit steady-state structure for  $\mathcal{L}_0 + \mathcal{L}'$  in the large- $N$  limit. In its absence, the error will immediately cause the state to lose information about the relative magnitude and/or phase of  $c_e, c_o$ , which define the qubit. We conjecture that any error  $\mathcal{L}'$  which keeps the model in the strong-broken phase can

TABLE III. Examples of errors that can and cannot be passively corrected via evolution by  $\mathcal{L}_0$  for the protocol given in the main text. An error must preserve the strong symmetry *and* keep the model in the broken phase in order for the final state to match the initial one.

Error	Strong?	Broken?	Correcting?
$L_1 = \sqrt{\kappa_1} a, \lambda/\kappa_1 > 0.5$	No	Yes	No
$H' = \omega a^\dagger a, \lambda/\omega < 0.5$	Yes	No	No
$H' = \omega a^\dagger a, \lambda/\omega > 0.5$	Yes	Yes	Yes
$L'_d = \sqrt{\kappa_d} a^\dagger a, \lambda/\kappa_d > 0.5$	Yes	Yes	Yes



be passively corrected, which agrees with Fig. 2(a). Table III provides a list of potential errors. Our framework allows us to classify the terms that are expected to self correct via  $\mathcal{L}_0$ . Analytical proof of this conjecture requires an exact solution for the steady states in the entire strong-broken phase—an open direction for future work.

*Discussion and outlook.*—Recent experiments have made progress on the stabilization and manipulation of photonic cat qubits encoded into superconducting resonators [58,75–80]. Our study shows that certain errors which arise via coherent Hamiltonian terms can be passively corrected. For example, Ref. [81] proposes that  $H = \omega a^\dagger a$  processes are useful for parity checks, Toffoli gates, and  $X$  gates. If this term unintentionally acts on some other qubit, then an error occurs. Such errors get passively corrected via  $\mathcal{L}_0$  once the manipulation ends. Further, our analysis shows that logical information can be stored in the steady state even in the presence of terms which are beyond experimental control, e.g.,  $\kappa_d \neq 0$ .

Although single-photon loss can induce qubit errors which are not correctable passively (the dominant decoherence mechanism in experiments), in the SM [63] we show that a *classical* bit encoded in  $\mathcal{L}_0$  will recover from errors which keep  $\mathcal{L}_0 + \mathcal{L}'$  in the *weak-broken* phase. Our setup thus admits a tunable classical-quantum steady-state structure. For qubits, this implies that modest single-photon loss induces passively correctable bit-flip errors, as well as phase-flip errors that require active correction.

While we have studied a  $\mathbb{Z}_2$ -symmetric system, a  $\mathbb{Z}_n$ -symmetric model should host a similarly protected qubit in the strong-broken phase. Our symmetry-breaking analysis should also apply to examples in Dicke-model physics [36], multimode systems [82], molecular platforms [83], and trapped ions [4,84].

In closed quantum systems, symmetry-breaking transitions can be dual to topological transitions. Various aspects of topological matter have been generalized to open systems [85–90], e.g., zero-frequency edge modes with a finite lifetime can be protected via a *frequency gap* [91]. An open question remains whether edge modes with zero decay rate can be protected by a *dissipative gap*, resulting in a qubit steady state robust against local errors.

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- [1] H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
- [2] C. Noh and D. G. Angelakis, *Rep. Prog. Phys.* **80**, 016401 (2016).
- [3] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, *Nat. Phys.* **4**, 878 (2008).
- [4] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **77**, 4728 (1996).
- [5] M. B. Plenio and S. F. Huelga, *Phys. Rev. Lett.* **88**, 197901 (2002).
- [6] B. Buča and T. Prosen, *New J. Phys.* **14**, 073007 (2012).
- [7] V. V. Albert and L. Jiang, *Phys. Rev. A* **89**, 022118 (2014).
- [8] V. V. Albert, B. Bradlyn, M. Fraas, and L. Jiang, *Phys. Rev. X* **6**, 041031 (2016).
- [9] B. Buča, J. Tindall, and D. Jaksch, *Nat. Commun.* **10**, 1730 (2019).
- [10] D. Roberts and A. A. Clerk, *Phys. Rev. X* **10**, 021022 (2020).
- [11] K. Macieszczak, M. Guta, I. Lesanovsky, and J. P. Garrahan, *Phys. Rev. Lett.* **116**, 240404 (2016).
- [12] E. I. R. Chiacchio and A. Nunnenkamp, *Phys. Rev. Lett.* **122**, 193605 (2019).
- [13] S. Dutta and N. R. Cooper, arXiv:2004.07981.
- [14] Z.-P. Cian, G. Zhu, S.-K. Chu, A. Seif, W. DeGottardi, L. Jiang, and M. Hafezi, *Phys. Rev. Lett.* **123**, 063602 (2019).
- [15] M. van Caspel and V. Gritsev, *Phys. Rev. A* **97**, 052106 (2018).
- [16] M. Gau, R. Egger, A. Zazunov, and Y. Gefen, *Phys. Rev. B* **102**, 134501 (2020).
- [17] Z. Zhang, J. Tindall, J. Mur-Petit, D. Jaksch, and B. Buča, *J. Phys. A* **53**, 215304 (2020).
- [18] R. A. Santos, F. Iemini, A. Kamenev, and Y. Gefen, arXiv:2002.00237.
- [19] G. Lindblad, *Commun. Math. Phys.* **48**, 119 (1976).
- [20] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, *J. Math. Phys. (N.Y.)* **17**, 821 (1976).
- [21] A. Belavkin, B. Y. Zeldovich, A. Perelomov, and V. Popov, *Sov. Phys. JETP* **56**, 264 (1969).
- [22] V. V. Albert, Lindbladians with multiple steady states: Theory and applications, Ph.D. Thesis, Yale University, 2017.
- [23] D. A. Lidar, I. L. Chuang, and K. B. Whaley, *Phys. Rev. Lett.* **81**, 2594 (1998).
- [24] B. M. Terhal, *Rev. Mod. Phys.* **87**, 307 (2015).
- [25] M. Mirrahimi, Z. Leghtas, V. V. Albert, S. Touzard, R. J. Schoelkopf, L. Jiang, and M. H. Devoret, *New J. Phys.* **16**, 045014 (2014).
- [26] S. Puri, S. Boutin, and A. Blais, *npj Quantum Inf.* **3**, 18 (2017).
- [27] E. Kapit, *Phys. Rev. Lett.* **116**, 150501 (2016).
- [28] J. P. Paz and W. H. Zurek, *Proc. R. Soc. Ser. A* **454**, 355 (1998).
- [29] J. P. Barnes and W. S. Warren, *Phys. Rev. Lett.* **85**, 856 (2000).
- [30] C. Ahn, A. C. Doherty, and A. J. Landahl, *Phys. Rev. A* **65**, 042301 (2002).
- [31] M. Sarovar and G. J. Milburn, *Phys. Rev. A* **72**, 012306 (2005).

- [32] O. Oreshkov and T. A. Brun, *Phys. Rev. A* **76**, 022318 (2007).
- [33] J. Kerckhoff, H. I. Nurdin, D. S. Pavlichin, and H. Mabuchi, *Phys. Rev. Lett.* **105**, 040502 (2010).
- [34] J.-M. Lihm, K. Noh, and U. R. Fischer, *Phys. Rev. A* **98**, 012317 (2018).
- [35] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, Cambridge, England, 2011).
- [36] P. Kirton, M. M. Roses, J. Keeling, and E. G. Dalla Torre, *Adv. Quantum Technol.* **2**, 1800043 (2019).
- [37] A. Mitra, S. Takei, Y. B. Kim, and A. J. Millis, *Phys. Rev. Lett.* **97**, 236808 (2006).
- [38] D. Nagy, G. Szirmai, and P. Domokos, *Phys. Rev. A* **84**, 043637 (2011).
- [39] J. Marino and S. Diehl, *Phys. Rev. Lett.* **116**, 070407 (2016).
- [40] E. G. Dalla Torre, E. Demler, T. Giamarchi, and E. Altman, *Phys. Rev. B* **85**, 184302 (2012).
- [41] E. G. Dalla Torre, S. Diehl, M. D. Lukin, S. Sachdev, and P. Strack, *Phys. Rev. A* **87**, 023831 (2013).
- [42] M. F. Maghrebi and A. V. Gorshkov, *Phys. Rev. B* **93**, 014307 (2016).
- [43] J. T. Young, A. V. Gorshkov, M. Foss-Feig, and M. F. Maghrebi, *Phys. Rev. X* **10**, 011039 (2020).
- [44] R. Lundgren, A. V. Gorshkov, and M. F. Maghrebi, *Phys. Rev. A* **102**, 032218 (2020).
- [45] C. Joshi, F. Nissen, and J. Keeling, *Phys. Rev. A* **88**, 063835 (2013).
- [46] J. Jin, A. Biella, O. Viyuela, C. Ciuti, R. Fazio, and D. Rossini, *Phys. Rev. B* **98**, 241108(R) (2018).
- [47] M. H. Szymanska, J. Keeling, and P. B. Littlewood, *Phys. Rev. Lett.* **96**, 230602 (2006).
- [48] R. Rota, F. Minganti, C. Ciuti, and V. Savona, *Phys. Rev. Lett.* **122**, 110405 (2019).
- [49] W. Verstraelen, R. Rota, V. Savona, and M. Wouters, *Phys. Rev. Research* **2**, 022037 (2020).
- [50] S. R. K. Rodriguez, W. Casteels, F. Storme, N. Carlon Zambon, I. Sagnes, L. Le Gratiet, E. Galopin, A. Lemaître, A. Amo, C. Ciuti, and J. Bloch, *Phys. Rev. Lett.* **118**, 247402 (2017).
- [51] C. Carr, R. Ritter, C. G. Wade, C. S. Adams, and K. J. Weatherill, *Phys. Rev. Lett.* **111**, 113901 (2013).
- [52] M. Fitzpatrick, N. M. Sundaresan, A. C. Y. Li, J. Koch, and A. A. Houck, *Phys. Rev. X* **7**, 011016 (2017).
- [53] J. Klinder, H. Keßler, M. Wolke, L. Mathey, and A. Hemmerich, *Proc. Natl. Acad. Sci. U.S.A.* **112**, 3290 (2015).
- [54] F. Brennecke, R. Mottl, K. Baumann, R. Landig, T. Donner, and T. Esslinger, *Proc. Natl. Acad. Sci. U.S.A.* **110**, 11763 (2013).
- [55] F. Minganti, A. Biella, N. Bartolo, and C. Ciuti, *Phys. Rev. A* **98**, 042118 (2018).
- [56] H. Wilming, M. J. Kastoryano, A. H. Werner, and J. Eisert, *J. Math. Phys. (N.Y.)* **58**, 033302 (2017).
- [57] E. M. Kessler, G. Giedke, A. Imamoglu, S. F. Yelin, M. D. Lukin, and J. I. Cirac, *Phys. Rev. A* **86**, 012116 (2012).
- [58] Z. Leghtas, S. Touzard, I. M. Pop, A. Kou, B. Vlastakis, A. Petrenko, K. M. Sliwa, A. Narla, S. Shankar, M. J. Hatridge *et al.*, *Science* **347**, 853 (2015).
- [59] N. Bartolo, F. Minganti, W. Casteels, and C. Ciuti, *Phys. Rev. A* **94**, 033841 (2016).
- [60] F. Minganti, N. Bartolo, J. Lolli, W. Casteels, and C. Ciuti, *Sci. Rep.* **6**, 26987 (2016).
- [61] H. J. Carmichael, *Phys. Rev. X* **5**, 031028 (2015).
- [62] J. B. Curtis, I. Boettcher, J. T. Young, M. F. Maghrebi, H. Carmichael, A. V. Gorshkov, and M. Foss-Feig, [arXiv:2006.05593](https://arxiv.org/abs/2006.05593).
- [63] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.125.240405> for calculations supporting dissipative-gap closure at the phase boundary, numerical evidence for a noiseless subsystem in the strong-broken phase, and a discussion on passive protection of a classical bit in the weak-broken phase, which includes Refs. [64–66].
- [64] T. Prosen, *New J. Phys.* **10**, 043026 (2008).
- [65] T. Prosen and T. H. Seligman, *J. Phys. A* **43**, 392004 (2010).
- [66] R. Blume-Kohout, H. K. Ng, D. Poulin, and L. Viola, *Phys. Rev. A* **82**, 062306 (2010).
- [67] X. H. Zhang and H. U. Baranger, [arXiv:2007.01422](https://arxiv.org/abs/2007.01422).
- [68] L. Gilles, B. M. Garraway, and P. L. Knight, *Phys. Rev. A* **49**, 2785 (1994).
- [69] J. Cohen, Autonomous quantum error correction with superconducting qubits, Ph.D. Thesis, Ecole Normale Supérieure, 2017.
- [70] E. Knill, R. Laflamme, and L. Viola, *Phys. Rev. Lett.* **84**, 2525 (2000).
- [71] Y. Wang, M. Um, J. Zhang, S. An, M. Lyu, J.-N. Zhang, L. M. Duan, D. Yum, and K. Kim, *Nat. Photonics* **11**, 646 (2017).
- [72] R. Belyansky, J. T. Young, P. Bienias, Z. Eldredge, A. M. Kaufman, P. Zoller, and A. V. Gorshkov, *Phys. Rev. Lett.* **123**, 213603 (2019).
- [73] I. Reichenbach and I. H. Deutsch, *Phys. Rev. Lett.* **99**, 123001 (2007).
- [74] A. V. Gorshkov, A. M. Rey, A. J. Daley, M. M. Boyd, J. Ye, P. Zoller, and M. D. Lukin, *Phys. Rev. Lett.* **102**, 110503 (2009).
- [75] R. W. Heeres, P. Reinhold, N. Ofek, L. Frunzio, L. Jiang, M. H. Devoret, and R. J. Schoelkopf, *Nat. Commun.* **8**, 94 (2017).
- [76] N. Ofek, A. Petrenko, R. Heeres, P. Reinhold, Z. Leghtas, B. Vlastakis, Y. Liu, L. Frunzio, S. M. Girvin, L. Jiang, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, *Nature (London)* **536**, 441 (2016).
- [77] R. Lescanne, M. Villiers, T. Peronnin, A. Sarlette, M. Delbecq, B. Huard, T. Kontos, M. Mirrahimi, and Z. Leghtas, *Nat. Phys.* **16**, 509 (2020).
- [78] S. Touzard, A. Grimm, Z. Leghtas, S. O. Mundhada, P. Reinhold, C. Axline, M. Reagor, K. Chou, J. Blumoff, K. M. Sliwa, S. Shankar, L. Frunzio, R. J. Schoelkopf, M. Mirrahimi, and M. H. Devoret, *Phys. Rev. X* **8**, 021005 (2018).
- [79] A. Grimm, N. E. Frattini, S. Puri, S. O. Mundhada, S. Touzard, M. Mirrahimi, S. M. Girvin, S. Shankar, and M. H. Devoret, *Nature (London)* **584**, 205 (2020).
- [80] J. M. Gertler, B. Baker, J. Li, S. Shirol, J. Koch, and C. Wang, [arXiv:2004.09322](https://arxiv.org/abs/2004.09322).
- [81] J. Guillaud and M. Mirrahimi, *Phys. Rev. X* **9**, 041053 (2019).
- [82] V. V. Albert, S. O. Mundhada, A. Grimm, S. Touzard, M. H. Devoret, and L. Jiang, *Quantum Sci. Technol.* **4**, 035007 (2019).
- [83] V. V. Albert, J. P. Covey, and J. Preskill, *Phys. Rev. X* **10**, 031050 (2020).

- [84] S. C. Burd, R. Srinivas, J. J. Bollinger, A. C. Wilson, D. J. Wineland, D. Leibfried, D. H. Slichter, and D. T. C. Allcock, *Science* **364**, 1163 (2019).
- [85] E. J. Bergholtz, J. C. Budich, and F. K. Kunst, [arXiv:1912.10048](#).
- [86] A. Altland, M. Fleischhauer, and S. Diehl, [arXiv:2007.10448](#).
- [87] F. Song, S. Yao, and Z. Wang, *Phys. Rev. Lett.* **123**, 170401 (2019).
- [88] C.-H. Liu, K. Zhang, Z. Yang, and S. Chen, *Phys. Rev. Research* **2**, 043167 (2020).
- [89] T. Yoshida, K. Kudo, H. Katsura, and Y. Hatsugai, *Phys. Rev. Research* **2**, 033428 (2020).
- [90] C. Gneiting, A. Koottandavida, A. V. Rozhkov, and F. Nori, [arXiv:2007.05960](#).
- [91] S. Lieu, M. McGinley, and N. R. Cooper, *Phys. Rev. Lett.* **124**, 040401 (2020).

*Correction:* Two inline equations contained minor errors and have been fixed: the last equation in the third complete sentence after Eq. (4) and the last equation in the second sentence in the fourth paragraph after Eq. (4).