Mott-Glass Phase of a One-Dimensional Quantum Fluid with Long-Range Interactions

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We investigate the ground-state properties of quantum particles interacting *via* a long-range repulsive potential $\mathcal{V}_\sigma(x) \sim 1/|x|^{1+\sigma}$ $(-1 < \sigma)$ or $\mathcal{V}_\sigma(x) \sim -|x|^{-1-\sigma}$ $(-2 \leq \sigma < -1)$ that interpolates between the Coulomb potential $\mathcal{V}_0(x)$ and the linearly confining potential $\mathcal{V}_{-2}(x)$ of the Schwinger model. In the absence of disorder the ground state is a Wigner crystal when $\sigma \leq 0$. Using bosonization and the nonperturbative functional renormalization group we show that any amount of disorder suppresses the Wigner crystallization when $-3/2 < \sigma \leq 0$; the ground state is then a Mott glass, i.e., a state that has a vanishing compressibility and a gapless optical conductivity. For $\sigma < -3/2$ the ground state remains a Wigner crystal.

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Introduction.—The ground state of a one-dimensional quantum fluid with short-range interactions is generically a Luttinger liquid. This corresponds to a metallic state, which is, however, not described by Landau's Fermi liquid theory, for fermions and to a superfluid state, but without Bose-Einstein condensation, for bosons [1]. In the presence of disorder, the ground state either remains a Luttinger liquid or becomes an Anderson insulator (fermions) or a Bose glass (bosons), i.e., an insulating state with a vanishing dc conductivity, a gapless optical conductivity and a nonzero compressibility [2–4].

Whether one-dimensional disordered quantum fluids can exhibit other phases besides the Luttinger liquid and the Anderson-insulator or Bose-glass phases has been the subject of debate for a long time. In particular, several works have addressed the existence of a Mott-glass phase but no firm positive conclusion has been reached so far. The Mott glass is intermediate between the Mott insulator and the Anderson insulator or Bose glass, and is characterized by a vanishing compressibility and a gapless conductivity; it would result from the coexistence of gapped single-particle excitations (which imply a vanishing compressibilty) and gapless particle-hole excitations (hence the absence of gap in the conductivity).

On the one hand it has been proposed that the interplay between disorder and a commensurate periodic potential could stabilize a Mott glass [5,6], but this conclusion, when the interactions are short range, has been challenged [7,8]. On the other hand, the existence of a Mott glass in a disordered system with linearly confining interactions mediated by a $(1+1)$-dimensional gauge field (disordered Schwinger model) has been predicted by the Gaussian variational method [9] and the perturbative functional renormalization group (FRG) [6], but this conclusion is in conflict with a recent study based on the nonperturbative FRG [10]. The only system that seems to certainly satisfy the basic properties of the Mott glass is the one-dimensional electron gas with (unscreened) Coulomb interactions [11].

In this Letter we determine the phase diagram of a one-dimensional quantum fluid where the particles interact with both a short-range potential and a long-range potential

$$\mathcal{V}_\sigma(x) = \begin{cases} \frac{e^2}{(x^2+a^2)^{(1+\sigma)/2}} & \text{if } -1 < \sigma, \\ -e^2 \ln|x/a| & \text{if } \sigma = -1, \\ -e^2|x|^{-1-\sigma} & \text{if } -2 \leq \sigma < -1 \end{cases} \tag{1}$$

(a is a short-distance cutoff [12]) that interpolates between the Coulomb potential $\mathcal{V}_0(x)$ and the linearly confining potential $\mathcal{V}_{-2}(x)$ of the Schwinger model [13,14]. Although our conclusions hold for both fermions and bosons, we use the terminology of the Bose fluid in the following.

Our main results are summarized in Fig. 1. The ground state of the pure fluid is a Luttinger liquid for $\sigma > 0$ (in that

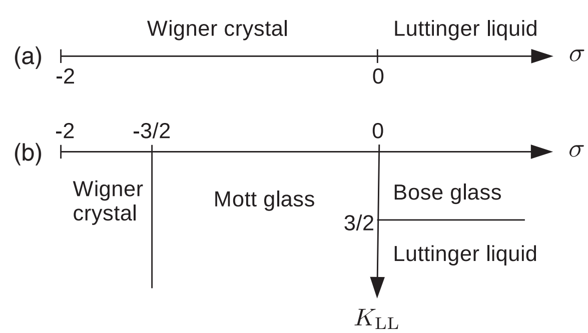

FIG. 1. Phase diagram of a pure (a) or disordered (b) one-dimensional Bose fluid with short-range interactions and a long-range interaction potential $\mathcal{V}_\sigma(x)$ [Eq. (1)]. K_{LL} denotes the Luttinger parameter associated with short-range interactions; it determines the ground state when $\sigma > 0$ (in that case $\mathcal{V}_\sigma$ is effectively short range) but plays no important role for long-range interactions.

case $\mathcal{V}_\sigma$ is effectively short range) and a Wigner crystal for $\sigma \leq 0$ as first shown by Schulz [15–19] in the case of Coulomb interactions (true long-range crystalline order however occurs only for $\sigma < 0$). In the presence of disorder, the Wigner crystal is stable if the interactions are sufficiently long range, i.e., $\sigma < -3/2$, but is unstable against a Mott glass when $-3/2 < \sigma \leq 0$. Apart from the vanishing compressibility, we find that the Mott glass is described by a fixed point of the FRG flow equations similar to the one describing the Bose-glass phase. Besides the finite localization length and the gapless conductivity, this fixed point is characterized by a renormalized disorder correlator that assumes a cuspy functional form whose origin lies in the existence of metastable states associated with glassy properties [20,21].

Model and FRG formalism.—The low-energy Hamiltonian of the pure Bose fluid in the presence of the long-range interaction potential $\mathcal{V}_\sigma(x)$ can be written as

$$\hat{H} = \hat{H}_{\mathrm{LL}} + \frac{1}{2}\sum_q \hat{\rho}(-q)\mathcal{V}_\sigma(q)\hat{\rho}(q), \tag{2}$$

where $\hat{\rho}(q)$ and $\mathcal{V}_\sigma(q)$ are the Fourier transforms of the density operator $\hat{\rho}(x)$ and $\mathcal{V}_\sigma(x)$, respectively, and a UV momentum cutoff Λ is implied. The Luttinger-liquid Hamiltonian $\hat{H}_{\mathrm{LL}}$ includes the kinetic energy of the particles and their short-range interactions. In the bosonization formalism [1],

$$\begin{aligned}\hat{H} = &\sum_q \frac{v_{\mathrm{LL}} q^2}{2\pi}\left\{\frac{1}{K_{\mathrm{LL}}}\hat{\varphi}(-q)\hat{\varphi}(q) + K_{\mathrm{LL}}\hat{\theta}(-q)\hat{\theta}(q)\right\} \\ &+ \frac{1}{2\pi^2}\sum_q q^2 \mathcal{V}_\sigma(q)\hat{\varphi}(-q)\hat{\varphi}(q),\end{aligned} \tag{3}$$

where $\hat{\theta}$ is the phase of the boson operator $\hat{\psi}(x) = e^{i\hat{\theta}(x)}\hat{\rho}(x)^{1/2}$. $\hat{\varphi}$ is related to the density operator *via*

$$\hat{\rho}(x) = \rho_0 - \frac{\partial_x \hat{\varphi}(x)}{\pi} + 2\sum_{m=1}^{\infty}\rho_{2m}\cos[2m\pi\rho_0 x - 2m\hat{\varphi}(x)], \tag{4}$$

where ρ_0 is the average density and the ρ_{2m}'s are nonuniversal parameters that depend on microscopic details. $\hat{\varphi}$ and $\hat{\theta}$ satisfy the commutation relations $[\hat{\theta}(x), \partial_y\hat{\varphi}(y)] = i\pi\delta(x-y)$. v_{LL} denotes the velocity of the sound mode when $\mathcal{V}_\sigma = 0$ and the dimensionless parameter K_{LL}, which encodes the strength of the short-range interactions, is the Luttinger parameter.

In the absence of long-range interactions ($\mathcal{V}_\sigma(q) = 0$), the system is a Luttinger liquid, characterized by a nonzero compressibility $\kappa = K_{\mathrm{LL}}/\pi v_{\mathrm{LL}}$ and a nonzero charge stiffness or Drude weight [defined as the Dirac peak $\delta(\omega)$ in the conductivity] $D = v_{\mathrm{LL}}K_{\mathrm{LL}}$ [22]. The superfluid correlation function $\langle\hat{\psi}(x)\hat{\psi}^\dagger(0)\rangle \sim 1/|x|^{1/2K_{\mathrm{LL}}}$ and the density correlation function $\langle\hat{\rho}(x)\hat{\rho}(0)\rangle_{|q|\sim 2\pi\rho_0} \sim \cos(2\pi\rho_0 x)/|x|^{2K_{\mathrm{LL}}}$ decay algebraically; the former dominates for $K_{\mathrm{LL}} > 1/2$, the latter for $K_{\mathrm{LL}} < 1/2$ (all other correlation functions are subleading).

The long-range interaction potential $\mathcal{V}_\sigma(q)$ can be simply taken into account by introducing momentum-dependent velocity and Luttinger parameter defined by

$$v(q)K(q) = v_{\mathrm{LL}}K_{\mathrm{LL}}, \qquad \frac{v(q)}{K(q)} = \frac{v_{\mathrm{LL}}}{K_{\mathrm{LL}}} + \frac{\mathcal{V}_\sigma(q)}{\pi}. \tag{5}$$

The long-range potential in Eq. (3) can then be simply taken into account by replacing, in the Luttinger-liquid Hamiltonian, v_{LL} and K_{LL} by $v(q)$ and $K(q)$ [1]. For $\sigma > 0$, since $\mathcal{V}_\sigma(q)$ has a finite limit for $q \to 0$, $v(q=0)$ and $K(q=0)$ are finite; this essentially leads to a mere renormalization of v_{LL} and K_{LL} and the ground state remains a Luttinger liquid. By contrast, for $\sigma \leq 0$, in the small-momentum limit $\mathcal{V}_\sigma(q) \sim |q|^\sigma$ so that $v(q) \sim |q|^{\sigma/2}$ and $K(q) \sim |q|^{-\sigma/2}$ are determined by the long-range part of the interactions (for $\sigma = 0$, $|q|^\sigma$ should be interpreted as $-\ln|q|$), which drastically modifies the ground state and the low-energy properties. The sound mode $\omega = v_{\mathrm{LL}}|q|$ of the Luttinger liquid is replaced by a collective mode with dispersion $\omega = v(q)|q| \sim |q|^{1+\sigma/2}$ ($\omega \sim |q|\sqrt{-\ln|q|}$ for $\sigma = 0$) and the compressibility $\kappa = \lim_{q\to 0} K(q)/\pi v(q)$ vanishes. Algebraic superfluid correlations are suppressed whereas translation invariance is spontaneously broken by the formation of a Wigner crystal with period $1/\rho_0$: $\langle\hat{\rho}(q = 2m\pi\rho_0)\rangle = \rho_{2m}\langle e^{2mi\hat{\varphi}(x)}\rangle \neq 0$ (m integer); for $\sigma = 0$, the order is only quasi-long-range [23]. The Wigner crystal has a nonzero charge stiffness $D = \lim_{q\to 0} v(q)K(q) = v_{\mathrm{LL}}K_{\mathrm{LL}}$ independent of the long-range interactions.

From now on, we restrict ourselves to genuine long-range interactions, i.e., $\sigma \leq 0$. A weak disorder contributes to the Hamiltonian a term

$$\hat{H}_{\mathrm{dis}} = \int dx\left\{-\frac{1}{\pi}\eta\partial_x\hat{\varphi} + \rho_2[\xi^* e^{2i\hat{\varphi}} + \mathrm{H.c.}]\right\}, \tag{6}$$

where we distinguish the so-called forward (η) and backward (ξ) scatterings; their Fourier components are near 0 and $\pm 2\pi\rho_0$, respectively [2,3]. The forward scattering potential η can be eliminated by a shift of $\hat{\varphi}$, i.e., $\hat{\varphi}(x) \to \hat{\varphi}(x) + \alpha(x)$ with a suitable choice of $\alpha(x)$, and is therefore discarded in the following (it does, however, play a role in some of the correlation functions discussed below). The average over disorder can be done using the replica method, i.e., by considering n copies of the model. Assuming that $\xi(x)$ is Gaussian distributed with zero mean and variance $\overline{\xi^*(x)\xi(x')} = (\mathcal{D}/\rho_2^2)\delta(x-x')$ (an overline indicates disorder averaging), we obtain the following low-energy Euclidean action (after integrating out the field θ),

$$S[\{\varphi_a\}] = \frac{1}{2}\sum_{Q,a}\varphi_a(-Q)\left(Z_x q^2 f_q + \frac{\omega^2}{\pi v_{\rm LL} K_{\rm LL}}\right)\varphi_a(Q) - \mathcal{D}\sum_{a,b}\int dx \int_0^\beta d\tau d\tau' \cos[2\varphi_a(x,\tau) - 2\varphi_b(x,\tau')], \tag{7}$$

where $\varphi_a(x,\tau)$ is a bosonic field with $\tau \in [0,\beta]$ an imaginary time ($\beta = 1/T \to \infty$), and $a, b = 1 \cdots n$ are replica indices. We use the notation $Q = (q, i\omega)$ with $\omega \equiv \omega_n = 2n\pi T$ (n integer) a Matsubara frequency. In Eq. (7), $Z_x f_q = v(q)/\pi K(q)$ and in the following we use the low-momentum approximation $Z_x f_q \simeq v_{\rm LL}/\pi K_{\rm LL} + Z_x |q|^\sigma$ (or $Z_x f_q \simeq v_{\rm LL}/\pi K_{\rm LL} + Z_x \ln|\Lambda/q|$ for $\sigma = 0$) valid when $|q| a \ll 1$. We can now identify two characteristic length scales. The first one, $L_x = (Z_x \pi K_{\rm LL}/v_{\rm LL})^{1/\sigma}$ is a crossover length beyond which the long-range potential $\mathcal{V}_\sigma$ dominates over the short-range interactions. The second one, the Larkin length $L_c \sim (Z_x^2/\mathcal{D})^{1/(3+2\sigma)}$, signals the breakdown of perturbation theory with respect to disorder [24]. The divergence of L_c when $\sigma \to -3/2$ suggests, as will be confirmed below, that the Wigner crystal is stable when $\sigma < -3/2$.

Most physical quantities can be obtained from the partition function $\mathcal{Z}[\{J_a\}]$ or, equivalently, from the effective action (or Gibbs free energy)

$$\Gamma[\{\phi_a\}] = -\ln \mathcal{Z}[\{J_a\}] + \sum_a \int dx \int_0^\beta d\tau J_a \phi_a, \tag{8}$$

defined as the Legendre transform of the free energy $-\ln \mathcal{Z}[\{J_a\}]$. Here J_a is an external source which couples linearly to the field φ_a and allows us to obtain the expectation value $\phi_a(x,\tau) = \langle \varphi_a(x,\tau)\rangle = \delta \ln \mathcal{Z}[\{J_f\}]/\delta J_a(x,\tau)$. We compute $\Gamma[\{\phi_a\}]$ using a Wilsonian nonperturbative FRG approach [25–27], where fluctuation modes are progressively integrated out. In practice we consider a scale-dependent effective action $\Gamma_k[\{\phi_a\}]$ which incorporates fluctuations with momenta (and frequencies) between a running momentum scale k and the UV scale Λ. The effective action of the original model, $\Gamma_{k=0}[\{\phi_a\}]$, is obtained when all fluctuations have been integrated out whereas $\Gamma_\Lambda[\{\phi_a\}] = S[\{\phi_a\}]$. Γ_k satisfies a flow equation which allows one to obtain $\Gamma_{k=0}$ from Γ_Λ but which cannot be solved exactly [28–30].

Following previous FRG studies of one-dimensional disordered boson systems [10,20,21,31], we consider the following truncation of the effective action,

$$\Gamma_k[\{\phi_a\}] = \sum_a \Gamma_{1,k}[\phi_a] - \frac{1}{2}\sum_{a,b}\Gamma_{2,k}[\phi_a,\phi_b], \tag{9}$$

with the ansatz

$$\Gamma_{1,k}[\phi_a] = \frac{1}{2}\sum_Q \phi_a(-Q)[Z_x q^2 f_q + \Delta_k(i\omega)]\phi_a(Q),$$
$$\Gamma_{2,k}[\phi_a,\phi_b] = \int dx \int_0^\beta d\tau d\tau' V_k[\phi_a(x,\tau) - \phi_b(x,\tau')], \tag{10}$$

and the initial conditions $\Delta_\Lambda(i\omega) = \omega^2/\pi v_{\rm LL} K_{\rm LL}$ and $V_\Lambda(u) = 2\mathcal{D}\cos(2u)$. The π-periodic function $V_k(u)$ can be interpreted as a renormalized second cumulant of the disorder. The form of the ansatz (10) is strongly constrained by the so-called statistical tilt symmetry (STS) [21,39]. In particular, the term $Z_x q^2 f_q$ is not renormalized and no other space-derivative terms can be generated. The self-energy $\Delta_k(i\omega)$ is *a priori* arbitrary but satisfies $\Delta_k(i\omega = 0) = 0$. It is convenient to define k-dependent velocity and Luttinger parameter from $Z_x = v_k/\pi K_k f_k$ and $\Delta_k(i\omega) = Z_x \omega^2 f_k / v_k^2 + \mathcal{O}(\omega^4)$. In the absence of disorder, $\Gamma_k[\{\phi_a\}] = S[\{\phi_a\}]$ and one has $v_k \sim f_k^{1/2}$ and $K_k \sim f_k^{-1/2}$ in agreement with the momentum-dependent quantities $v(q)$ and $K(q)$ [Eq. (5)].

$\Gamma_{1,k}$ and $\Gamma_{2,k}$ contain all the necessary information to characterize the ground state of the system. From the disorder-averaged density-density correlation function

$$\chi_{\rho\rho}(q, i\omega) = \frac{q^2/\pi^2}{Z_x q^2 f_q + \Delta_{k=0}(i\omega)}, \tag{11}$$

we deduce that the compressibility

$$\kappa = \lim_{q\to 0}\chi_{\rho\rho}(q,0) = \lim_{q\to 0}\frac{1}{\pi^2 Z_x f_q} = 0 \tag{12}$$

vanishes so that the system remains incompressible in the presence of disorder. The determination of the conductivity $\sigma(\omega) = \lim_{q\to 0}(-i\omega/q^2)\chi_{\rho\rho}(q, \omega + i0^+)$ requires us to determine the self-energy $\Delta_k(i\omega)$ whose low-frequency behavior depends on $V_k(u)$. Incidentally, the importance of disorder is best characterized by the dimensionless disorder correlator defined by $\delta_k(u) = -K_k^2 V_k''(u)/v_k^2 k^3$. We refer to the Supplemental Material for more details about the implementation of the FRG approach and the derivation of the flow equations for $\Delta_k(i\omega)$ and $\delta_k(u)$ [32].

FRG flow and phase diagram.—By solving numerically the flow equations, we find that for $\sigma > -3/2$ the flow trajectories are attracted by a fixed point characterized by a vanishing Luttinger parameter $K_k \sim k^{-\sigma/2+\theta} \to 0$ (Fig. 2). The velocity behaves as $v_k \sim k^{\theta+\sigma/2}$ and vanishes in the limit $k \to 0$ if $\sigma > -2\theta$ but diverges (as in the Wigner crystal) if $\sigma < -2\theta$. Whether the latter case actually occurs (which requires $\theta < 3/4$ since $\sigma > -3/2$ in the Mott glass) depends on the value of θ which, for reasons explained in Ref. [21], cannot be accurately determined from the flow equations. The charge stiffness $D_k = v_k K_k \sim k^{2\theta}$ vanishes for $k \to 0$ and the system is insulating.

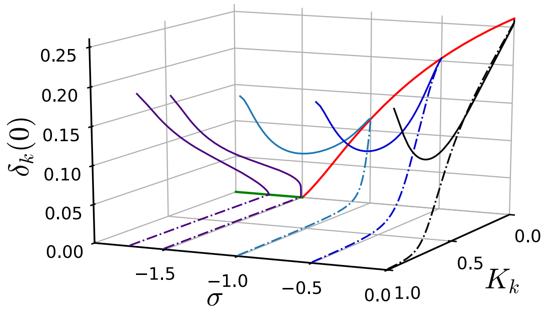

FIG. 2. Flow trajectories $[K_k, \delta_k(0)]$ for various values of σ. The solid and dash-dotted lines are obtained for different values of the disorder strength. The red solid line for $-3/2 < \sigma \le 0$ corresponds to the Mott-glass fixed point defined by $K^* = 0$ and $\delta^*(u)$ [Eq. (13)]. Disorder is irrelevant for $\sigma < -3/2$ and the solid green line corresponds to the Wigner-crystal fixed point.

On the other hand, the disorder correlator $\delta_k(u)$ reaches a nontrivial fixed point in the limit $k \to 0$ when $\sigma > -3/2$ (see Fig. 3):

$$\delta^*(u) = \frac{3+2\sigma}{6\pi \bar{l}_2}\left[\left(u - \frac{\pi}{2}\right)^2 - \frac{\pi^2}{12}\right] \quad (u \in [0, \pi]), \tag{13}$$

where $\bar{l}_2$ is a nonuniversal constant. Apart from the σ-dependent prefactor, $\delta^*(u)$ is identical to the fixed-point solution in the Bose-glass phase [20,21]. It exhibits cusps at $u = n\pi$ (n integer). For any nonzero momentum scale this cusp singularity is rounded into a quantum boundary layer (QBL) as shown in Fig. 3: For u near $n\pi$, $\delta_k(n\pi) - \delta_k(u) \propto |u - n\pi|$ except in a boundary layer of size $|u - n\pi| \sim K_k$, and the curvature $|\delta_k''(n\pi)| \sim 1/K_k \sim k^{-\theta+\sigma/2}$ diverges when $k \to 0$. The cusp singularity and the QBL describes the physics of rare low-energy metastable states and their coupling to the ground state by quantum fluctuations

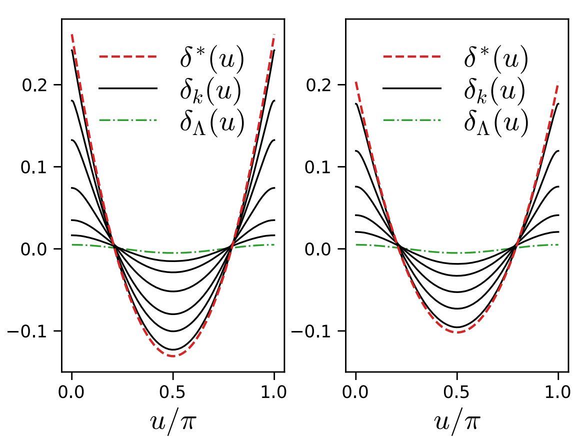

FIG. 3. Disorder correlator $\delta_k(u)$ for various values of k and $\sigma = 0$ (left), $\sigma = -0.5$ (right). The green dash-dotted curve shows the initial condition $\delta_\Lambda(u) \propto \cos(2u)$ and the red dashed one the fixed-point solution (13).

[20,21]. This is characteristic of disordered systems with glassy properties [40].

The behavior of the self-energy $\Delta_k(i\omega)$ when $\sigma > -3/2$ is also reminiscent of the Bose-glass phase. For small k, there is a frequency regime, where $\Delta_k(i\omega)$ is compatible with a linear dependence $A + B|\omega|$, which implies that the real part of the conductivity,

$$\begin{aligned} \sigma(\omega) &= -\frac{i\omega}{\pi^2 \Delta_{k=0}(\omega + i0^+)} \\ &= \frac{1}{\pi A^2}(-iA\omega + B\omega^2) + \mathcal{O}(\omega^3), \end{aligned} \tag{14}$$

vanishes as ω^2 [41]. However, when $2 - \theta + 3\sigma/2$ becomes negative, which necessarily occurs when σ varies between 0 and $-3/2$ since $\theta > 0$, the constant A grows and seems to diverge for $k \to 0$. This could indicate that the conductivity vanishes with an exponent larger than 2: $\Re[\sigma(\omega)] \ll \omega^2$ [32]. Thus, for $\sigma > -3/2$, we essentially recover the physical properties of the Bose-glass phase with the notable exception that the compressibility vanishes: The ground state is a Mott glass.

In the Mott glass, the backward scattering destroys the long-range crystalline order: $\langle \hat\rho(q = 2\pi\rho_0)\rangle = \rho_2 \langle e^{2i\hat\varphi(x)}\rangle = 0$, and the corresponding correlation function $\chi(x) = \langle e^{2i\hat\varphi(x)} e^{-2i\hat\varphi(0)}\rangle$ decays algebraically. Taking into account the forward scattering, we find [32]

$$\chi(x) \sim \begin{cases} \frac{e^{-C|x|^{1+2\sigma}}}{|x|^{\gamma_\sigma}} & \text{if } \sigma > -1/2, \\ \frac{1}{|x|^{\gamma_\sigma}} & \text{if } \sigma < -1/2 \end{cases} \tag{15}$$

(C is a positive constant), where $\gamma_\sigma = \pi^2(3 + 2\sigma)/9 + \theta - \sigma/2$. Forward scattering is relevant for $\sigma > -1/2$ and yields an exponential suppression of crystalline order but becomes irrelevant for $\sigma < -1/2$ [32].

When $\sigma < -3/2$, both forward and backward scatterings are irrelevant and the Wigner crystal is stable against a weak disorder as shown by the flow trajectories in Fig. 2. Thus, for sufficiently long-range interactions, the Wigner crystal is sufficiently rigid to survive the detrimental effect of disorder. The case $\sigma = -2$ (disordered Schwinger model) requires a separate study since $Z_x q^2 f_q$ does not vanish for $q \to 0$. Although there are contradicting results in the literature regarding the possible existence of a Mott glass in the disordered Schwinger model [6,9,10], our results regarding the stability of the Wigner crystal against disorder when $-2 < \sigma < -3/2$ are in line with a recent FRG study predicting the absence of a Mott glass when $\sigma = -2$, the ground state being similar to a Mott insulator (vanishing compressibility and gapped conductivity) [10].

The phase diagram of a one-dimensional disordered Bose fluid with the long-range interaction potential $\mathcal{V}_\sigma$ [Eq. (1)] is shown in Fig. 1. In the absence of disorder, the ground state is a Luttinger liquid for effectively short-range

TABLE I. Some of the physical properties of the phases shown in the phase diagrams of Fig. 1: crystalline order, compressibility κ, and low-frequency optical conductivity $\sigma(\omega)$ (QLRO stands for quasi-long-range order).

	Crystallization	κ	$\Re[\sigma(\omega)]$
Luttinger liquid $\sigma > 0$	No	> 0	$D\delta(\omega)$
Wigner crystal $\sigma = 0$	QLRO	0	$D\delta(\omega)$
Wigner crystal $-2 < \sigma < 0$	LRO	0	$D\delta(\omega)$
Wigner crystal $\sigma = -2$	LRO	0	Gapped
Bose glass	No	> 0	ω^2
Mott glass	No	0	ω^2

interactions ($\sigma > 0$) and a Wigner crystal for genuine long-range interactions ($\sigma \leq 0$). The Luttinger liquid is unstable against infinitesimal disorder and becomes a Bose glass when the Luttinger parameter satisfies $K_{\mathrm{LL}} < 3/2$ [with $K_{\mathrm{LL}} = \lim_{q\to 0} K(q)$ including the effect of the potential $\mathcal{V}_\sigma$] [2,3]. On the other hand, disorder transforms the Wigner crystal into a Mott glass when $\sigma > -3/2$. Some of the physical properties of these various phases are summarized in Table I.

Conclusion.—We have shown that a one-dimensional disordered Bose fluid with long-range interactions exhibits a rich phase diagram which includes the long-sought Mott-glass phase. Since the Hamiltonian studied in this Letter also describes the charge degrees of freedom of fermions, a similar phase diagram is expected for a one-dimensional Fermi fluid.

On the experimental side, long-range interactions have been realized in various cold-atom systems, e.g., trapped ions [43–45] or dipolar quantum gases [46], and we may hope that one-dimensional quantum fluids with long-range interactions will be realized in the near future. Of particular interest are cold-atom systems in an optical lattice and using an optical cavity to realize the Hubbard model with an additional infinite-range (cavity-mediated) interaction [47,48]. In the presence of disorder this system, in one dimension, would be described by the low-energy model studied in this Letter. But the scaling of the long-range interaction with the system size, the so-called Kac prescription [49], prevents a direct comparison with the results of this Letter [48]. On the other hand we note that the Schwinger model has already been realized [50] and allows for a check of our prediction regarding the stability of the Wigner crystal when $\sigma = -2$.

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[1] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).

[2] T. Giamarchi and H. J. Schulz, Localization and interaction in one-dimensional quantum fluids, Europhys. Lett. **3**, 1287 (1987).

[3] T. Giamarchi and H. J. Schulz, Anderson localization and interactions in one-dimensional metals, Phys. Rev. B **37**, 325 (1988).

[4] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Boson localization and the superfluid-insulator transition, Phys. Rev. B **40**, 546 (1989).

[5] E. Orignac, T. Giamarchi, and P. Le Doussal, Possible New Phase of Commensurate Insulators with Disorder: The Mott Glass, Phys. Rev. Lett. **83**, 2378 (1999).

[6] T. Giamarchi, P. Le Doussal, and E. Orignac, Competition of random and periodic potentials in interacting fermionic systems and classical equivalents: The Mott glass, Phys. Rev. B **64**, 245119 (2001).

[7] T. Nattermann, A. Petković, Z. Ristivojevic, and F. Schütze, Absence of the Mott Glass Phase in 1D Disordered Fermionic Systems, Phys. Rev. Lett. **99**, 186402 (2007).

[8] P. Le Doussal, T. Giamarchi, and E. Orignac, Comment on "Absence of the Mott Glass Phase in 1D Disordered Fermionic Systems", arXiv:0809.4544.

[9] Y.-Z. Chou, R. M. Nandkishore, and L. Radzihovsky, Mott glass from localization and confinement, Phys. Rev. B **97**, 184205 (2018).

[10] N. Dupuis, Is there a Mott-glass phase in a one-dimensional disordered quantum fluid with linearly confining interactions?, Europhys. Lett. **130**, 56002 (2020).

[11] B. I. Shklovskii and A. L. Efros, Zero-phonon ac hopping conductivity of disordered systems, Zh. Éksp. Teor. Fiz. **81**, 406 (1981) [Sov. Phys. JETP **54**, 218 (1981)].

[12] The short-distance cutoff a is necessary to make the Fourier transform of $\mathcal{V}_\sigma(x)$ well defined when $\sigma \geq 0$.

[13] J. Schwinger, Gauge invariance and mass. II, Phys. Rev. **128**, 2425 (1962).

[14] S. Coleman, More about the massive Schwinger model, Ann. Phys. (N.Y.) **101**, 239 (1976).

[15] H. J. Schulz, Wigner Crystal in One Dimension, Phys. Rev. Lett. **71**, 1864 (1993).

[16] G. Fano, F. Ortolani, A. Parola, and L. Ziosi, Unscreened Coulomb repulsion in the one-dimensional electron gas, Phys. Rev. B **60**, 15654 (1999).

[17] M. M. Fogler, Ground-State Energy of the Electron Liquid in Ultrathin Wires, Phys. Rev. Lett. **94**, 056405 (2005).

[18] M. Casula, S. Sorella, and G. Senatore, Ground state properties of the one-dimensional Coulomb gas using the lattice regularized diffusion Monte Carlo method, Phys. Rev. B **74**, 245427 (2006).

[19] G. E. Astrakharchik and M. D. Girardeau, Exact ground-state properties of a one-dimensional Coulomb gas, Phys. Rev. B **83**, 153303 (2011).

[20] N. Dupuis, Glassy properties of the Bose-glass phase of a one-dimensional disordered Bose fluid, Phys. Rev. E **100**, 030102(R) (2019).

[21] N. Dupuis and R. Daviet, Bose-glass phase of a one-dimensional disordered Bose fluid: Metastable states, quantum tunneling, and droplets, Phys. Rev. E **101**, 042139 (2020).

[22] At zero temperature, the charge stiffness D is related to the superfluid density $\rho_s = D/\pi$.

[23] In the case of the Coulomb interaction ($\sigma = 0$), the density-density correlation function $\langle\hat{\rho}(x)\hat{\rho}(x)\rangle_{|q|\sim 2\pi\rho_0} \sim \cos(2\pi\rho_0 x)e^{-C\sqrt{\ln|x|}}$ decays much slower than any power

law while superfluid correlations $\langle\hat{\psi}(x)\hat{\psi}^\dagger(0)\rangle \sim e^{-C' \ln^{3/2}|x|}$ decay faster than any power law [15].
[24] For sufficiently small disorder, one always has $L_c > L_x$.
[25] J. Berges, N. Tetradis, and C. Wetterich, Non-perturbative renormalization flow in quantum field theory and statistical physics, Phys. Rep. **363**, 223 (2002).
[26] B. Delamotte, An introduction to the nonperturbative renormalization group, in *Renormalization Group and Effective Field Theory Approaches to Many-Body Systems*, Lecture Notes in Physics Vol. 852, edited by A. Schwenk and J. Polonyi (Springer, Berlin Heidelberg, 2012), pp. 49–132.
[27] N. Dupuis, L. Canet, A. Eichhorn, W. Metzner, J. M. Pawlowski, M. Tissier, and N. Wschebor, The nonperturbative functional renormalization group and its applications, arXiv:2006.04853.
[28] C. Wetterich, Exact evolution equation for the effective potential, Phys. Lett. B **301**, 90 (1993).
[29] U. Ellwanger, Flow equations for n point functions and bound states, Z. Phys. C **62**, 503 (1994).
[30] T. R. Morris, The exact renormalization group and approximate solutions, Int. J. Mod. Phys. A **09**, 2411 (1994).
[31] See Ref. [32] for a discussion of the truncation (9).
[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.235301, which includes Refs. [33–38], for more details about one-dimensional quantum fluids with long-range interactions and the implementation of the nonperturbative functional renormalization-group approach in the disordered case.
[33] Z. Ristivojevic, A. Petković, P. Le Doussal, and T. Giamarchi, Phase Transition of Interacting Disordered Bosons in One Dimension, Phys. Rev. Lett. **109**, 026402 (2012).
[34] Z. Ristivojevic, A. Petković, P. Le Doussal, and T. Giamarchi, Superfluid/Bose-glass transition in one dimension, Phys. Rev. B **90**, 125144 (2014).
[35] G. Tarjus and M. Tissier, Random-field Ising and O(N) models: Theoretical description through the functional renormalization group, Eur. Phys. J. B **93**, 50 (2020).
[36] F. Rose, F. Léonard, and N. Dupuis, Higgs amplitude mode in the vicinity of a $(2+1)$-dimensional quantum critical point: A nonperturbative renormalization-group approach, Phys. Rev. B **91**, 224501 (2015).
[37] F. Rose and N. Dupuis, Nonperturbative functional renormalization-group approach to transport in the vicinity of a $(2+1)$-dimensional O(N)-symmetric quantum critical point, Phys. Rev. B **95**, 014513 (2017).
[38] I. Balog, G. Tarjus, and M. Tissier, Benchmarking the nonperturbative functional renormalization group approach on the random elastic manifold model in and out of equilibrium, J. Stat. Mech. (2019) 103301.
[39] U. Schulz, J. Villain, E. Brézin, and H. Orland, Thermal fluctuations in some random field models, J. Stat. Phys. **51**, 1 (1988).
[40] L. Balents, J.-P. Bouchaud, and M. Mézard, The large scale energy landscape of randomly pinned objects, J. Phys. I **6**, 1007 (1996).
[41] This result differs from the low-frequency conductivity $\sigma(\omega) \sim |\omega|$ (ignoring logarithmic corrections) obtained by Shklovskii and Efros for the electron gas with Coulomb interactions [11]. This expression was, however, obtained in the strong-disorder limit where the localization length is much smaller than the interparticle distance whereas the bosonization approach is valid in the opposite, weak-disorder, limit [42].
[42] H. Maurey and T. Giamarchi, Transport properties of a quantum wire in the presence of impurities and long-range Coulomb forces, Phys. Rev. B **51**, 10833 (1995).
[43] J. W. Britton, B. C. Sawyer, A. C. Keith, C. C Joseph Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins, Nature (London) **484**, 489 (2012).
[44] R. Islam, C. Senko, W. C. Campbell, S. Korenblit, J. Smith, A. Lee, E. E. Edwards, C. C. J. Wang, J. K. Freericks, and C. Monroe, Emergence and frustration of magnetism with variable-range interactions in a quantum simulator, Science **340**, 583 (2013).
[45] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feig, S. Michalakis, A. V. Gorshkov, and C. Monroe, Non-local propagation of correlations in quantum systems with long-range interactions, Nature (London) **511**, 198 (2014).
[46] M. A. Baranov, M. Dalmonte, G. Pupillo, and P. Zoller, Condensed matter theory of dipolar quantum gases, Chem. Rev. **112**, 5012 (2012).
[47] R. Landig, L. Hruby, N. Dogra, M. Landini, R. Mottl, T. Donner, and T. Esslinger, Quantum phases from competing short- and long-range interactions in an optical lattice, Nature (London) **532**, 476 (2016).
[48] T. Botzung, D. Hagenmüller, G. Masella, J. Dubail, N. Defenu, A. Trombettoni, and G. Pupillo, Effects of energy extensivity on the quantum phases of long-range interacting systems, arXiv:1909.12105.
[49] M. Kac, G. E. Uhlenbeck, and P. C. Hemmer, On the van der Waals theory of the vapor–liquid equilibrium. I. Discussion of a one–dimensional model, J. Math. Phys. (N.Y.) **4**, 216 (1963).
[50] E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, and R. Blatt, Real-time dynamics of lattice gauge theories with a few-qubit quantum computer, Nature (London) **534**, 516 (2016).