Quantifying Causal Influences in the Presence of a Quantum Common Cause

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Quantum mechanics challenges our intuition on the cause-effect relations in nature. Some fundamental concepts, including Reichenbach's common cause principle or the notion of local realism, have to be reconsidered. Traditionally, this is witnessed by the violation of a Bell inequality. But are Bell inequalities the only signature of the incompatibility between quantum correlations and causality theory? Motivated by this question, we introduce a general framework able to estimate causal influences between two variables, without the need of interventions and irrespectively of the classical, quantum, or even postquantum nature of a common cause. In particular, by considering the simplest instrumental scenario—for which violation of Bell inequalities is not possible—we show that every pure bipartite entangled state violates the classical bounds on causal influence, thus, answering in negative to the posed question and opening a new venue to explore the role of causality within quantum theory.

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Introduction.—Estimating relations of cause and effect are central and, yet, one of the most challenging goals of science. Since long ago, it has been realized that correlations do not imply causation. The reason is that any correlation observed between two or more random variables can, at least in the classical regime, be explained by a potentially unobserved common cause. Understanding under which conditions such confounding factors can be controlled, such that empirical data can be turned into a causal hypothesis, has found a firm theoretical basis with the establishment of the mathematical theory of causality [1,2]. Today, concepts like interventions, randomized controlled experiments, and instrumental variables are common work tools in the estimation of causal influences in a variety of fields [3–7].

Despite its success, all such ideas and applications rely on the classical notion of causality that, since Bell's theorem [8], we know cannot be applied to quantum phenomena.

The violation of a Bell inequality shows that quantum correlations are incompatible with the joint assumption of the causal constraints of local realism and measurement independence ("free will") [9–18]. As it turns out, the phenomenon of quantum nonlocality can be seen as a particular case of a causal inference problem [19], a realization that has sparked a number of generalizations of nonlocality to causal networks of growing size and complexity [20–24]. Apart from the violation of Bell inequalities, are there any other consequences of quantum correlations to the theory of causality?

The standard manner for distinguishing between a common cause and direct causal influences among two variables is via an intervention [3]. However, in some cases, it might not be possible to intervene in the system, e.g., due to ethical reasons, or because one is interested in estimating causal effects in past experiments. As shown in Refs. [25,26], differently from the classical case, observed quantum correlations alone are sometimes enough to resolve the question. This has led to a formalization of a quantum common cause [27] and, more generally, quantum causal models [28–34]. However, the solution in Refs. [25,26] relies on causal tomography, that is, it depends on the precise knowledge of the physical system and the measurement apparatuses. Strikingly, as shown in the pioneering work [35] causal influences can also be estimated without interventions and, in a device-independent manner, via the introduction of an instrumental variable. This result, however, relies on the assumption that the unobserved hidden causes are classical and satisfy the property of local realism. In view of that, the instrumental scenario has started to be analyzed from a quantum perspective [36–38]; however, despite these initial attempts, it is not known how quantum effects can change the cause and effect relations that can be inferred from the instrumental data. That is precisely the question we resolve in this Letter.

We consider the problem of determining casual influences in quantum causal models. To this aim, we use the common measure known as the average causal effect (ACE) [1], defined in terms of interventions, which can either be measured directly or can be estimated from observational data with the help of an instrumental variable. As we show here, by considering the simplest instrumental scenario, every pure entangled state, as well as every pair of incompatible projective measurements, can generate correlations that violate the classical bounds on ACE, derived in Ref. [35]. Remarkably, in this simplest scenario, quantum correlations cannot violate any Bell-type inequality [28]. That is, our results imply that quantum correlations can generate nonclassical signatures going beyond the paradigmatic violation of Bell inequalities. Motivated by that, we also introduce a general framework for causal inference in the instrumental scenario, providing bounds for ACE and applicable to quantum theory and beyond.

We denote random variables by capital letters and their values by the corresponding lower-case letters. Additionally, we use the notation $p(a) \equiv p(A = a)$.

Quantifying causality and the instrumental scenario.— Given two variables A and B, our aim is to quantify how much of their correlations are due to direct causal influences from A to B, or due to some common cause described (classically) by a random variable, Λ. If we do not have empirical access to the common cause, one option is to intervene on the variable A, that is, fix its value to a value of our choice independent of Λ. The intervention erases any correlation between A and B mediated by Λ. Thus, any remaining correlation after such intervention can unambiguously be associated to the direct causal influence $A \to B$. Interventions are a natural choice for quantifying causality. In fact, one of the most widely used measures of causal influence is the ACE measure, defined in terms of interventions as

$$\mathrm{ACE}_{A\to B} = \max_{a,a',b}\Big(p(b|do(a)) - p(b|do(a'))\Big), \qquad (1)$$

where we used a notation, $p(b|do(a))$ to denote the probability of Bob's outcome b when variable A is set by force to be a. We refer to it as "do probability" in the text. The ACE measures the maximum change in the distribution of the variable B when the value of A is altered.

For a variety of reasons, however, it is not always possible to perform an intervention. With the aim of still being able to estimate causal influences based only on the observational data, the instrumental scenario has been developed [39,40]. The idea is to introduce a third variable in full control of the experimenter, the so-called instrumental variable X. The variable X is assumed to be independent from the common source variable Λ, that is, $p(x,\lambda) = p(x)p(\lambda)$. This is reminiscent of the measurement independence (free will) assumption in Bell's theorem [See Supplemental Material (SM) [41] for further details]. Furthermore, X is supposed to have a direct causal effect only over A and not B, that is, $p(b|a,x,\lambda) = p(b|a,\lambda)$. Such causal assumptions can be graphically represented via the directed acyclic graph shown in Fig. 1 (left). It implies that the observed probability distribution is given by

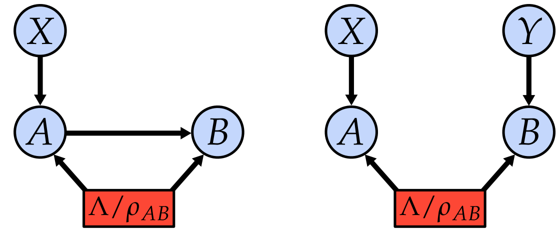

FIG. 1. Directed acyclic graphs depicting causal structures: (left) Instrumental scenario and (right) Bell scenario. In the quantum model, we consider, here, the classical common source, described by a random variable Λ, is replaced by a quantum (potentially entangled) state ρ_{AB}.

$$p(a,b|x) = \sum_\lambda p(a|x,\lambda)p(b|a,\lambda)p(\lambda). \qquad (2)$$

Do probabilities $p(b|do(a))$ are given by

$$p(b|do(a)) = \sum_\lambda p(b|a,\lambda)p(\lambda), \qquad (3)$$

where the conditional distribution $p(b|a,\lambda)$ as well as the distribution of $p(\lambda)$ are the same as in Eq. (2).

To understand the role of the instrumental variable, consider a simple linear relation between the variables given by $b = \kappa a + \lambda$. If we multiply both sides by x and compute the covariance given by $C(X,B) = \langle X,B\rangle - \langle X\rangle\langle B\rangle$, by using $C(X,\Lambda) = 0$, we see that $\kappa = C(X,B)/C(A,B)$. That is, simply combining the correlations of B with both A and X, we can estimate the causal influence κ without the need of any intervention. In this example, however, we assumed a prior knowledge of the functional dependencies among the variables. Nicely, causal influences can be estimated even without such assumptions, just as in the device-independent framework for quantum information [47], where we perform tasks without the precise knowledge of the underlying physical mechanisms.

In the particular case where all variables are binary $a,b,x \in \{0,1\}$, the classical ACE (CACE) can be tightly lower bounded by several expressions including only the observed probabilities $p(a,b|x)$ [35]. Here, we give one of the bounds that we often use in this Letter

$$\mathrm{CACE}_{A\to B} \geq 2p(0,0|0) + p(1,1|0) + p(0,1|1) + p(1,1|1) - 2. \qquad (4)$$

For more lower bounds on $\mathrm{CACE}_{A\to B}$ see Refs. [1,35] or SM [41].

We give another example that signifies the importance of lower bounds such as in Eq. (4). Consider that A stands for smoking or nonsmoking and B for cancer or no cancer. Clearly, intervening and forcing people to smoke is not possible. Strikingly, simply introducing an instrumental variable X standing, for example, for taxation or non-taxation of tobacco—that arguably will affect whether people smoke or not, but will not have a direct causal effect on the development of cancer—and using Eq. (4), we can estimate the effect of interventions and, thus, lower bound such causal influences.

Within the classical theory of causality, for the bound in Eq. (4) to be valid, one needs to assure that the instrumental causal assumptions are fulfilled. In other words, that the underlying causal structure is that described by Eq. (2). For that aim, the so-called instrumental inequalities have been devised [1,48,49].

In the instrumental scenario with binary variables, which we consider here, the only class of instrumental inequalities is given by $\sum_a \max_x p(a,b|x) \leq 1$ [48,49]. Curiously, these inequalities remain valid, if the common source is replaced by a quantum state or even postquantum box [28], in contrast to the simplest Bell scenario [50].

At first, this might seem to imply that the classical bound on ACE in Eq. (4) continues to hold even in the presence of quantum or postquantum sources. As we show next, this is not the case.

Quantifying causality with a quantum common source.—If the common source is a bipartite quantum state ρ_{AB}, the most general way to generate the classical binary variables A and B, is to perform local measurements, described by operators M_a^x and N_b^a, on each subsystem. Here, the value x is used to choose Alice's measurement setting, and the outcome a of Alice's measurement is used to determine Bob's measurement setting, accordingly. Quantum correlations in the instrumental scenario are then described by

$$p(a,b|x) = \mathrm{tr}[(M_a^x \otimes N_b^a)\rho_{AB}]. \qquad (5)$$

In full analogy with the classical case, one can then define quantum interventions as

$$p(b|do(a)) = \mathrm{tr}[(\mathbb{1} \otimes N_b^a)\rho_{AB}] = \mathrm{tr}[N_b^a \rho_B], \qquad (6)$$

where ρ_B is the reduced state of Bob's system. This implies that, if an actual intervention is made, the observed quantum average causal effect (QACE) is given by

$$\mathrm{QACE}_{A\to B} = \max_{a,a',b}\{\mathrm{tr}[(N_b^a - N_b^{a'})\rho_B]\}. \qquad (7)$$

As expected, if the shared state ρ_{AB} is separable, the classical and quantum definitions of ACE coincide (see SM [41]). That is, correlations mediated by a separable state comply with the classical bound in Eq. (4). As stated in our first result, the proof of which can be found in SM [41], the same does not hold true for entangled states.

Result 1: Every pure entangled state can generate correlations that violate the classical bound on ACE. Moreover, entanglement is necessary but not sufficient for such violations.

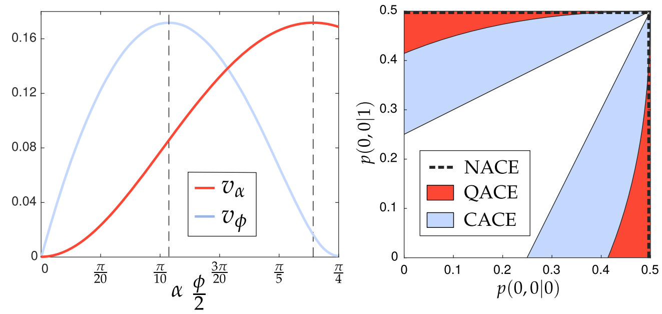

FIG. 2. (left) Violation v_α of the classical bound by an entangled two-qubit pure state with parameter α; and violation v_ϕ as a function of the angle ϕ between projective measurements of Bob. The dashed lines show that optimal states and measurements are different from the maximal entangled state ($\alpha = (\pi/4)$) and measurements in mutually unbiased bases ($\phi = (\pi/4)$). (right) Regions with nonzero lower bounds on CACE (Ref. [35]), QACE [Eq. (11)], and NACE [Eq. (12)].

This result implies that—even though, in the simplest instrumental scenario, quantum correlations admit classical explanation of the form in Eq. (2)—the amount of observable causal influence $\mathrm{QACE}_{A\to B}$ is strictly smaller than that required, if the correlations were classical. In other words, even if no instrumental inequality is violated, the nonclassicality of the correlations can be witnessed by interventions on the classical variable A.

In order to quantify the degree of violation v, we consider how much the classical bound in Eq. (4) overestimates the causal influence in the presence of an entangled source. In Fig. 2, we show violation v_α for an entangled two-qubit state $\rho_{AB} = |\psi\rangle\langle\psi|$, $|\psi\rangle = \cos(\alpha)|0,0\rangle + \sin(\alpha)|1,1\rangle$ for $\alpha \in [0, (\pi/4)]$. As detailed in the SM [41], a maximally entangled two-qubit state violates the classical bound by, at most, the amount $3(\sqrt{6}-2)/8 \approx 0.169$. However, this is not the optimal violation: nonmaximally entangled states give rise to a higher violation up to $3 - 2\sqrt{2} \approx 0.172$, a fact that, in the context of Bell inequalities, has been called nonlocality anomaly [51]. Moreover, one can easily see that entanglement is not sufficient for the violation. For example, a maximally entangled state mixed with white noise in the amount of p stays entangled for $p < 2/3$, however, it leads to a violation only if $p < 1 - \sqrt{2/3} \approx 0.1835$.

Violation of Bell inequalities [50] is not only a proof that the shared state is entangled, but also a witness of the fact that the measurements being performed should display some nonclassicality, as they should be incompatible [52–54]. As proven in the SM [41] and stated below, a similar result holds for the violation of the classical bounds on causal influence.

Result 2: Every pair of incompatible rank-1 projective qubit measurements can generate correlations that violate the classical bound on ACE. Moreover, incompatibility of both Alice's and Bob's observables is necessary but not sufficient for the violation.

In Fig. 2, we show violation of the bound in Eq. (4) as a function of the angle ϕ between the measurements of Bob that we consider to be $N_0^a = \frac{1}{2}[\mathbb{1} + \cos(\phi)\sigma_z + (-1)^a \sin(\phi)\sigma_x]$. In Fig. 2, the angle ϕ ranges between 0 and $\pi/2$ with 0 ($\pi/2$) corresponding to perfectly aligned (antialigned) σ_z (σ_x) measurements. The value $\phi = (\pi/4)$ corresponds to the case of measurements in mutually unbiased bases which are optimal for the violation of the simplest Bell inequality [50]. In our case, the optimal measurements of Bob correspond to $\phi = \arctan(\frac{2}{\sqrt{3\sqrt{2}+2}}) \simeq 0.2149\pi$.

So far, we have relied on interventions on the variable A and explicitly taken into account the quantum states and measurements. However, in the more general case, we are given some observational data $p(a,b|x)$, but do not know *a priori* which states and measurements have been employed. In this case, our aim is to be able to estimate QACE from the observational data $p(a,b|x)$, without actually needing to perform an intervention. That is, in order to find a device-independent bound on QACE, we have to optimize over all possible measurements and states generating the observed correlations $p(a,b|x)$. Our approach to this problem is to map the instrumental scenario to the more familiar and well-studied bipartite Bell scenario [37].

Let us consider a Bell scenario shown in Fig. 1 (right) that contains the same observed random variables A, B, and X as the instrumental scenario in Fig. 1 (left) and an additional classical variable Y that takes values from the same set as A, and has a causal effect only on B. We also take the hidden common cause, classical or quantum, to be the same for both scenarios. Let $p_{\mathrm{Bell}}(a,b|x,y)$ be the observed behavior in the considered Bell scenario. Local hidden-variable theories reproduce correlations of the following type:

$$p_{\mathrm{Bell}}(a,b|x,y) = \sum_{\lambda} p(a|x,\lambda)p(b|y,\lambda)p(\lambda). \quad (8)$$

Conversely, quantum behavior corresponding to measurement operators M_a^x and N_b^y and quantum state ρ_{AB} is $p_{\mathrm{Bell}}(a,b|x,y) = \mathrm{tr}[(M_a^x \otimes N_b^y)\rho_{AB}]$. The following mapping:

$$p(a,b|x) = p_{\mathrm{Bell}}(a,b|x,a), \quad \forall\ a,b,x \quad (9)$$

connects classical, quantum, and postquantum correlations in Bell and the instrumental scenarios in a unified manner. Indeed, one can directly see that the mapping in Eq. (9) transforms classical correlations in Eq. (8) to the ones in Eq. (2), and the same mapping connects their quantum counterparts. More importantly, we can compute the unobserved do probabilities $p(b|do(a))$ in terms of $p_{\mathrm{Bell}}(a,b|x,y)$ in the following way:

$$p(b|do(a)) = \sum_{a'} p_{\mathrm{Bell}}(a',b|x,a), \quad \forall\ a,b,x, \quad (10)$$

where the choice of x does not play any role as long as the correlations $p_{\mathrm{Bell}}(a,b|x,y)$ obey the nonsignaling constraints [55]. One can then see that expressing do probabilities with the map in Eq. (10) is equivalent to the previous definitions for do probabilities in classical and quantum cases. We remark that the mapping in Eqs. (9), (10) is not the same as the postprocessing on the events of $Y = A$, but is rather a projection from the space of $p_{\mathrm{Bell}}(a,b|x,y)$ to the space of $p(a,b|x)$ and $p(b|do(a))$. The mapping in Eqs. (9), (10) allows the use of known techniques for bounding the set of quantum correlations in a Bell scenario, in particular, the so-called Navascués-Pironio-Acín hierarchy [56], with a slight variation: Here, the probabilities $p_{\mathrm{Bell}}(a,b|x,a')$, $(a \neq a')$, with no analogy in the instrumental scenario, play the role of the "unobserved" variables of the semidefinite program [57]. Additionally, for binary A one should take into account the relation $p_{\mathrm{Bell}}(a,b|x,a') = p(b|do(a')) - p(a',b|x)$, that follows from $\sum_a p_{\mathrm{Bell}}(a,b|x,a') = p_{\mathrm{Bell}}(b|a')$.

In the following, we focus on the binary case $(a,b,x \in \{0,1\})$ and derive a number of analytical results.

Result 3: In the instrumental scenario with dichotomic measurements QACE is lower bounded as

$$\mathrm{QACE}_{A\to B} \geq \sum_{x=0,1} [p(0,0|x) + p(1,1|x)] - \zeta - 1,$$
$$\zeta = \max_{\pm} \sqrt{\prod_{a=0,1} \{1 \pm \sum_{x=0,1} (-1)^x [p(a,0|x) - p(a,1|x)]\}}. \quad (11)$$

The derivation of the above bound is presented in the SM [41]. In Fig. 2 (right), we compare the lower bounds in Eq. (11) and the one in Eq. (4) (along with the other bounds in Ref. [35]) by plotting the regions in which these bounds are nonzero, showing a clear gap between the classical and quantum descriptions. In Fig. 2 (right), a particular slice of the probability space is considered, corresponding to $p(1,0|x) = 0$, $p(0,1|x) = \frac{1}{2} - p(0,0|x)$, $p(1,1|x) = \frac{1}{2}$, $x = 0, 1$.

Quantifying causality in postquantum theories.—One might be interested about whether nontrivial lower bounds similar to Eqs. (4), (11) exist in generalized probabilistic theories. Here, we answer this question for correlations constrained only by the nonsignaling condition in a Bell scenario [55].

In order to do so, we map [using Eqs. (9), (10)] the nonsignaling constraints to the instrumental scenario and use linear programming techniques (see SM [41]) to find tight lower bounds on nonsignaling ACE (NACE)

$$\mathrm{NACE}_{A\to B} \geq \max_x [p(0,0|x)] + \max_x [p(1,1|x)] - 1. \quad (12)$$

In Fig. 2 (right), we also plot the region where $\mathrm{NACE}_{A\to B} \geq 0$, which is given by two lines with $p(0,0|0) = \frac{1}{2}$ and $p(0,0|1) = \frac{1}{2}$.

Discussion.—The incompatibility of quantum correlations with classical causal models is a cornerstone in the foundations of quantum theory. The paradigmatic manner of witnessing this nonclassicality is via the violation of Bell inequalities. There are causal scenarios, however, where violations of Bell-type inequalities are not possible [28]. At first, this might seem to imply that quantum common causes do have a classical explanation in such scenarios. As we show here, this intuition is false. Even in the absence of Bell violations, quantum correlations can violate the classical bounds for the causal influence between two variables in the presence of a quantum common cause. More precisely, every pure entangled state and a pair of incompatible projective measurements can violate such bounds. Motivated by this result we propose a general framework to put bounds on the average causal effect in the presence of quantum common causes and even nonsignaling boxes. We obtain several analytical results and compare the regions where the aforementioned bounds are nontrivial.

Here, we have focused on the scenario where all the observed variables are classical, but the common cause can be quantum. Generalizations where other variables in the instrumental causal structure are made quantum open an interesting venue for future research. For instance, the teleportation protocol [58] is an instrumental scenario where the instrumental variable X is the state to be teleported and the outcome B is the teleported quantum state. Other paradigmatic quantum information scenarios, such as the remote state preparation [59] and dense coding [60], also have an underlying instrumental causal structure. On the more foundational side, many physical principles have been developed for understanding why quantum correlations do not violate Bell inequalities up to the maximum allowed by special relativity [55]. In this Letter, we showed that quantum theory also imposes strict bounds on the causal influence between events that differ for generalized probabilistic theories. Can it be that there is an underlying causal principle explaining quantum correlations? Finally, we notice that, just as Bell's theorem [8], our conclusions also rely on the assumption of measurement independence. Recently, the relaxation of such an assumption in Bell scenarios has been given growing attention both theoretically [9–15] and experimentally [16–18]. How robust is the quantum violation of the classical causal bound under such relaxation? We hope that our results will trigger further developments in all these new directions.

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