Bosonic Bott Index and Disorder-Induced Topological Transitions of Magnons

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We investigate the role of disorder on the various topological magnonic phases present in deformed honeycomb ferromagnets. To this end, we introduce a bosonic Bott index to characterize the topology of magnon spectra in finite, disordered systems. The consistency between the Bott index and Chern number is numerically established in the clean limit. We demonstrate that topologically protected magnon edge states are robust to moderate disorder and, as anticipated, localized in the strong regime. We predict a disorderdriven topological phase transition, a magnonic analog of the "topological Anderson insulator" in electronic systems, where the disorder is responsible for the emergence of the nontrivial topology. Combining the results for the Bott index and transport properties, we show that bulk-boundary correspondence holds for disordered topological magnons. Our results open the door for research on topological magnonics as well as other bosonic excitations in finite and disordered systems.

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Topological phases in nature have attracted intense interest and research in recent decades. Initially, the research was mainly on electronic systems such as topological insulators [1,2]. Currently, the concept of topology has been extended to many research areas that span condensed-matter physics. Different kinds of bosonic low-energy excitations, e.g., phonons [3], photons [4], magnons [5–21], and even macroscopic motions [22–24], host topological states. Topological phases are characterized by certain topological indices that remain unchanged under smooth deformations. Nontrivial topology is usually associated with the appearance of robust edge states immune to disorder, known as "bulk-boundary correspondence," which is one of the most exotic features of topological matters and invokes many potential applications [25–29].

Among various kinds of excitations, research on topological states in magnonic systems has increased in recent years. Many models have been proposed to support topological magnons. Some of them can be mapped to known electronic models [7,10,12,17,18], while some are exclusive in bosonic systems [19,20,28,29]. Nevertheless, most previous studies on topological magnons focused on clean systems and did not consider disorder, which is ubiquitous and unavoidable in nature. The role of disorder in topological systems is a crucial issue since it is related to one of the fundamental features of topological systems: the robustness of the edge states. In electronic systems research, there is plenty of discussion on this issue. Not only have transport properties been studied [30-33], but also the Chern number in real space [34,35], and the Bott index [36–40] has been used to label the topology of finite or disordered systems. However, how disorders affect topological magnons is still underexplored.

In this Letter, we consider a honeycomb ferromagnet with nearest-neighbor (NN) pseudodipolar interaction [41], whose magnons can be topologically nontrivial, and we study the effect of disordered on site anisotropy. To label the magnon topology in finite or disordered magnets, we generalize the real-space Bott index [36-40] to bosonic systems. We first show that the Bott index agrees with the Chern number in the clean limit. We then demonstrate the bulk-boundary correspondence in disordered magnonic systems by comparing their transport properties and topological indices. We find that, for the topologically nontrivial phase, the topology as well as the protected edge states are quite robust against disorder, unless the disorder is more than 3 times larger than the gap. For the topologically trivial phase, we identify a disorder-induced nontrivial phase, which is the magnonic analogy of the topological Anderson insulator [30,31]. Our findings reveal that the Bott index is a useful tool in research on topological bosonic systems without translational symmetry.

We consider a ferromagnetic material with localized spins on a deformed two-dimensional honeycomb lattice formed by heavy metal atoms having strong spin-orbit coupling. The spin Hamiltonian we consider is given by

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - F \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{e}_{ij}) (\mathbf{S}_j \cdot \mathbf{e}_{ij}) - \frac{1}{2} \sum_i K_i S_{zi}^2 - \mu_B B \sum_i S_{zi}, \qquad (1)$$

where J > 0 is the NN ferromagnetic exchange coupling and F is the NN pseudodipolar interaction resulting from the spin-orbit coupling, with \mathbf{e}_{ij} being the unit vector



FIG. 1. (a) Schematic of a normal honeycomb magnet. The ground state is out of plane. (b) From left to right: the spin-wave spectra for infinite samples (along k_x for $k_y = 0$), zigzag strips of width $N_y = 100$ along x, and finite samples of $N_x = 20$ and $N_y = 20$. (c) Schematic of a squeezed honeycomb magnet of $\theta = 5\pi/12$. (d) Spin-wave spectra for (c).

connecting lattice sites *i* and *j* along one of the NN lattice vectors $\mathbf{a}_{1,2,3}$ [41]. This Hamiltonian can be mapped to the Kitaev model [34] in the linear regime. The easy-axis anisotropy K_i at the A(B) sublattice consists of two parts: a homogeneous part $K_{A(B)}$ and a random part K_{ri} , where K_{ri} is statistically independent for different *i* and uniformly distributed on the interval [-W, W]. For convenience, we define $K = (K_A + K_B)/2$ and $\Delta K = (K_A - K_B)/2$. B is the applied magnetic field along the \hat{z} direction (μ_B is the Bohr magneton). Since we are interested in the strong disorder limit, a sufficiently large easy-axis anisotropy and/or a magnetic field is assumed so that the spins align out of plane in the ground state [29]. The angles between $\mathbf{a}_{1,2,3}$ and the x direction are $\theta_1 = \pi/2, \ \theta_2 = \pi/2 - \theta$, and $\theta_3 = \pi/2 + \theta$. Figures 1(a) and 1(c) show a perfect $(\theta = 2\pi/3)$ and deformed $(\theta = 5\pi/12)$ honeycomb lattice, respectively.

We first consider the magnon spectra on honeycomb ferromagnets in the absence of disorder. The linearexcitation **k**-space magnon Hamiltonian is written in terms of bosonic creation (annihilation) operators, $a^{\dagger}(a)$ and $b^{\dagger}(b)$ on each sublattice A and B [42] as $\mathcal{H} = \frac{1}{2}x^{\dagger}H_{\mathbf{k}}x$, where $x = (a_{\mathbf{k}}, a_{-\mathbf{k}}^{\dagger}, b_{\mathbf{k}}, b_{-\mathbf{k}}^{\dagger})^{T}$ (see the Supplemental Material [43] for the explicit form of $H_{\mathbf{k}}$). We diagonalize $H_{\mathbf{k}}$ to obtain the magnon spectrum by employing the Bogoljubov transformation [44]. The transformation matrix $\mathcal{T}_{\mathbf{k}}$ that diagonalizes $H_{\mathbf{k}}$ satisfies the generalized eigenvalue problem (GEP) [45]

relations
$$\eta_{ij} = [x_i^i, x_j]$$
 so that $\eta = \mathbb{1}_{2\times 2} \otimes \sigma_z (\sigma_{x,y,z})$ are the
Pauli matrices), and E_k is the diagonal matrix whose
diagonal elements $\varepsilon_n(\mathbf{k})$ are the eigenvalues of \mathcal{H} . The
GEP has a particle-hole symmetry in that the ε_n 's are
artificially doubled in positive-negative pairs $\varepsilon_n(\mathbf{k}) =$
 $-\varepsilon_n(-\mathbf{k})$. Therefore, it is sufficient to consider the positive
solutions of ε_n only. Note that Eq. (2) is equivalent to the
result from the linearized classical Landau-Lifshitz-Gilbert
equation [28]. This system is known to be gapped and
topologically nontrivial when $\theta \neq (\pi/2)$ [23,28]. In Fig. 1,
we show the spin-wave spectra of three different samples for
the normal [Fig. 1(a)] and deformed [Fig. 1(c)] honeycomb
lattices. From left to right, in Figs. 1(b) and 1(d), the spec-
trum is plotted for the infinite system (along k_x with $k_y = 0$),
the zigzag strip along the x direction and width $N_y = 100$,
and the finite samples with dimensions $N_x = N_y = 20$,
assuming periodic boundary conditions (PBCs) and open
boundary conditions (OBCs). The parameters are $F = 7J$,
 $K = 20J$, $\Delta K = 0$, and $B = 0$, and N_x and N_y are the
number of units cells in the x and y directions, respectively
($K = 20J$ and $B = 0$ are used throughout this Letter). Both
Figs. 1(b) and 1(d) show gapped bulk spectra in infinite and
periodic systems and gapless (crossing) edge states for an
open strip, indicating a nontrivial topology.

where η is a metric matrix reflecting the commutation

In infinite translationally symmetric systems, the Chern number of the *n*th band is [19,20]

$$C_n = \frac{i}{2\pi} \int_{\text{B.Z.}} P_n \left(\frac{\partial P_n}{\partial k_x} \frac{\partial P_n}{\partial k_y} - \frac{\partial P_n}{\partial k_y} \frac{\partial P_n}{\partial k_x} \right).$$
(3)

$$\eta H_{\mathbf{k}} \mathcal{T}_{\mathbf{k}} = \mathcal{T}_{\mathbf{k}} \eta E_{\mathbf{k}},\tag{2}$$

Here, $P_n(\mathbf{k})$ is the bosonic projector defined by $P_n = \mathcal{T}_{\mathbf{k}}\eta\Gamma_n\mathcal{T}_{\mathbf{k}}^{\dagger}\eta$, where Γ_n is a diagonal matrix taking a value of 1 for the *n*th diagonal components and zero otherwise. The Chern numbers of the upper (lower) magnon bands, $C_u(C_l)$, are labeled in Figs. 1(b) and 1(d). A topological transition occurs at $\theta = \pi/2$, where C_u and C_l flip their signs. When $\theta \ge (\pi/2)$, $C_u = -C_l = \pm 1$ [23]. When the magnetic anisotropy at each sublattice differs, i.e., $\Delta K \ne 0$, one of the gaps at *K* or *K'* points closes and reopens, and the system becomes topologically trivial ($C_u = C_l = 0$) [28,29].

In the presence of disorder or in finite samples, the periodicity of the lattice is broken so that the **k**-space Chern number is invalid. We need a real-space index to label the topology. In electronic systems, the Bott index was introduced to study nonperiodic systems, such as disordered topological insulators [38] and quasicrystals [39,40]. The Bott index quantifies the obstruction to construct an orthogonal basis of localized Wannier functions that span the occupied states [37], and it has been proven to be equivalent to the Chern number in the large-system limit [46].

We now extend the definition of the Bott index to bosonic systems. For a finite system of size $N_x \times N_y$ (in total, there are $N = N_x N_y$ unit cells), dual to the **k**-space representation, the GEP in real space is $\eta HT = T\eta E$, where *H* is the $4N \times 4N$ real-space Hamiltonian, $\eta =$ $\mathbb{1}_{2N \times 2N} \otimes \sigma_z$ is the metric due to the bosonic commutation relation in real space, and *E* is the diagonal matrix of eigenenergies. T is the matrix diagonalizing the Hamiltonian. For a set of eigenstates $\{\varepsilon_n\}$, its bosonic Bott index is given by

$$\mathcal{B}\{\varepsilon_n\} = \frac{1}{2\pi} \operatorname{Im}\{\operatorname{tr}[\log(VUV^{\dagger}U^{\dagger})]\},\qquad(4)$$

where the two matrices U and V are defined from

$$Pe^{2\pi i X}P = \mathcal{T}\eta \begin{pmatrix} 0 & 0\\ 0 & U \end{pmatrix} \mathcal{T}^{\dagger}\eta,$$
 (5)

$$Pe^{2\pi iY}P = \mathcal{T}\eta \begin{pmatrix} 0 & 0\\ 0 & V \end{pmatrix} \mathcal{T}^{\dagger}\eta, \tag{6}$$

where $P = \mathcal{T}\eta\Gamma\mathcal{T}^{\dagger}\eta$ is the projector on states $\{\varepsilon_n\}$. $X = i_x/N_x$ and $Y = i_y/N_y$ are the rescaled coordinates, where $i_{x,y}$ are spatial indices of the unit cells. Γ is a diagonal matrix taking a value of 1 for the *j*th diagonal elements when $j \in \{\varepsilon_n\}$, and 0 otherwise. Note that for fermionic systems, $\eta = 1$, and the above definition returns to the electronic Bott index [40,46]. \mathcal{B} is always an integer as long as $VUV^{\dagger}U^{\dagger}$ is nonsingular [38,46], and specifically, $\mathcal{B} = 0$ when the matrices U and V commute. For a clean system with well-defined gaps, the Bott index of each band separated by gaps is well defined.

We then compared the Bott index and Chern number in the absence of disorder (W = 0). In Figs. 1(b) and 1(d)



FIG. 2. Comparison between the Chern number (C_u) and Bott index (\mathcal{B}_u) for finite and clean systems. The equivalence is established as a function of the staggered anisotropy ΔK and for a system size of 40×40 .

(third panel), we label the Bott indices for the upper and lower magnon bands (\mathcal{B}_u and \mathcal{B}_l , respectively) of clean 40×40 samples with PBCs. The results are consistent with the Chern number for infinite systems. A systematic comparison is shown in Fig. 2 in terms of ΔK for the upper band of the normal [Fig. 1(a)] and deformed [Fig. 1(c)] honeycomb lattices. The vertical dashed line represents the ΔK values for gap closing, resulting in a topological phase transition from nontrivial to a trivial magnon spectrum. Both the Bott index and Chern number consistently describe the topology of the system. Note that, near the topological transition point, the Berry curvature is ill defined, so the numerically calculated Chern numbers are not integers. Although the Bott indices are still integers in the case, a larger system size and thus higher computational cost are necessary to obtain accurate results.

Next, let us consider the presence of disorder in the conventional honeycomb lattice system [Fig. 1(a)]. Because of disorder, the gap is filled with states, even though PBCs are used. We define the Bott indices as functions of energy [47], $\mathcal{B}_u(\varepsilon)$ and $\mathcal{B}_l(\varepsilon)$, where $\mathcal{B}_u(\varepsilon)$ [$\mathcal{B}_l(\varepsilon)$] is the Bott index of all the states with higher (lower) energy than ε . In the following, we calculate the ensemble-averaged Bott indices over 100 uncorrelated random configurations, denoted by $\overline{\mathcal{B}}$, for system size of $N_x = N_y = 40$.

First, we consider systems that are topologically nontrivial in clean limits. In Fig. 3(a), $\bar{B}_u(\varepsilon_0)$ is plotted against the disorder strength W for $\Delta K = 0$, F = 3J, 5J, and 7J, where ε_0 is the energy at the midpoint of the gap in the clean limit [43]. For moderate disorder, the Bott index is still one, meaning that the system is topologically nontrivial. When the disorder is strong enough, a topological transition occurs and the system becomes topologically trivial. This phenomenon is consistent with the common wisdom that the topology is quite robust since very strong disorder (approximately 3 times the gap) is needed to break the topology. A more remarkable phenomenon occurs when the disorder affects a topologically trivial system. In Fig. 3(b), we consider an originally trivial system (at W = 0) with F = 7J and $\Delta K = 1.35J$ and plot $\bar{B}_u(\varepsilon_0)$



FIG. 3. Comparison between the Bott index and the total transmission as a function of the disorder strength W/J, when the clean-limit system is (a) topologically nontrivial ($\Delta K = 0$) and (b) trivial ($\Delta K = 1.35J$). The inset in (b) is the band structure near the gap of a 100-wide zigzag strip. The real-space wave function is depicted for F = 7J and W = 6J in (c) $\Delta K = 0$ and (d) $\Delta K = 1.35J$, corresponding to the circled data points in (a) and (b), respectively. This is represented by the expectation of the in plane spin components $\langle S_x \rangle$ and $\langle S_y \rangle$, the spatial distribution of the eigenstate whose energy is closest to ε_0 . The size of the circles indicates the amplitude, and the color encodes the azimuthal angle.

against *W*. The band structure of a strip near the gap in the clean limit is shown in the inset. There are no gapless edge states inside the bulk gap. Surprisingly, as the disorder strength increases, the Bott index increases from zero and reaches a plateau of $\mathcal{B}_u(\varepsilon_0) = 1$ and then drops to zero at W > 8J. This finding indicates that there exists a disorderinduced topological phase, similar to the topological Anderson insulator phase in electronic systems [30–32].

Now, we demonstrate the bulk-boundary correspondence in our topological magnon model by studying the transport properties. We consider a disordered strip sample with two identical (clean) leads attached to its left- and righthand sides. We evaluate the total transmission probability $T(\varepsilon = \varepsilon_0)$ from left to right for $N_x = N_y = 200$ samples [43]. The results are plotted in Figs. 3(a) and 3(b) (averaged over 100 disorder realizations). In the clean limit, the total transmission equals the total number of propagating channels according to the Landauer-Büttiker formula [48]. For the nontrivial phase, since there is one rightward edge channel, at zero disorder, we have T = 1, as shown in Fig. 3(a). The topologically protected edge channel remains robust as the disorder increases; however, at certain values, the magnonic modes become localized, and thus, the topology is destroyed. For the trivial phase [Fig. 3(b)], since there is no channel inside the gap, we shift the band of the leads upward by 2J to make full use of the bulk channels [32]. As *W* increases, the transmission increases from zero to a plateau (T = 1) and then decays to zero at very large disorder, following the topological transition [49].

The existence of edge states in strongly disordered magnets is further confirmed by the calculation of the real-space wave functions. Eigenstates whose energies are closest to ε_0 for a certain disorder configuration were considered. For clarity of representation, we use a smaller system size $N_x = N_y = 20$. We plot the expectations of the in plane spin components $\langle S_x \rangle$ and $\langle S_y \rangle$ in Figs. 3(c) and 3(d) for the originally nontrivial phase ($\Delta K = 0$) and disorder-induced nontrivial phase ($\Delta K = 1.35J$), respectively. The parameters F = 7J and W = 6J were used for both plots; see the circled data points in Figs. 3(a) and 3(b). Clear features of the edge states can be observed. However, for $\Delta K = 1.35J$, the penetration depth is larger, so interedge backscattering is more likely; see the Supplemental Material [43] for details.

To further understand the emergence of the disorderdriven topological transition, we consider the self-energy Σ induced by the disorder, defined by $(\varepsilon_0 - H_{\mathbf{k}} - \Sigma)^{-1} =$ $\langle (\varepsilon_0 - H_{\mathbf{k}}^{\text{eff}})^{-1} \rangle$, where $H_{\mathbf{k}}^{\text{eff}}$ is the disorder-renormalized effective Hamiltonian. We numerically calculate Σ in the self-consistent Born approximation [31] for the parameters used in Fig. 3(b). The result is a 4×4 matrix that can be decomposed into three Hermitian components $\Sigma^{0\sim 2}$ and one anti-Hermitian component. Σ^0 is proportional to identity matrix $\mathbb{1}_{4\times 4}$, which shifts the whole spectrum. Σ^1 is proportional to $\sigma_x \otimes \mathbb{1}_{2 \times 2}$, which shifts only the position of the valley. Σ^2 is proportional to $\sigma_z \otimes \mathbb{1}_{2\times 2}$, which has the same structure as the ΔK term in $H_{\mathbf{k}}$ and is responsible for the topological transition. The randomness on the anisotropy effectively reduces ΔK and drives the system back to the nontrivial phase. The non-Hermitian component reflects the inverse lifetime of the magnon states. By letting Σ^2 be the critical value of the topological transition, we can solve for the critical disorder strength W = 3.7J, which is consistent with the numerical result [43].

We have numerically demonstrated very good agreement between the total transmission and the Bott index, which indicates that the bulk-boundary correspondence holds in the disordered topological magnon system. Our numerical studies pave the way for a rigorous mathematical proof of the equivalence between the bosonic Bott index and Chern number [46], as well as the bulk-boundary correspondence in topological magnonic systems, which are open issues for future research.

All the discussions above also apply for other deformed honeycomb lattices, provided that the clean system is gapped [23]. Note that a further increase in W destroys the ferromagnetic ground state, and the system enters a spin-glass-like state [50], which is not the purpose of this

Letter. We expect our definition of the bosonic Bott index to be applied to any bosonic system [43], as long as the metric matrix η is modified according to the commutation relations of creation (annihilation) operators, which will benefit many research areas, such as topological phononics, photonics, and superconductors. AB_3 -type 2D honeycomb magnetic materials such as CrI₃ and OsCl₃ are possible experimental platforms for our model. There are already first-principles and experimental indications of strong pseudodipolar interaction [51,52].

In conclusion, we introduced a bosonic Bott index as an integer-valued real-space topological invariant in bosonic systems and used it to study the magnon topology in a disordered honeycomb ferromagnet. In the clean limit, the topological phase is controlled by the bond angle and staggered anisotropy, and the Bott index is consistent with the Chern number. In the presence of disorder, the edge states in the nontrivial phase are robust to moderate disorder. In the trivial phase, the disorder can induce a phase transition to a nontrivial topology, which is the magnonic counterpart of the topological Anderson insulator phase in electronic systems. Our findings open the door for the investigation of the topology of disordered bosonic systems.

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